Numerical Simulation Of Complex Turbulent Jets: 
Origin Of Axis-Switching
複雑乱流噴流の数値シミュレーション（軸伸張の発生）

Ayodeji O. DEMUREN*, Robert V. WILSON**, and Toshio KOBAYASHI*
アヨデジ オー デムレン・ロバート ヴィ ウィルソン・小 林 敏 雄

INTRODUCTION

Turbulent jets are present in many physical processes and technological applications. In most cases, efficient mixing of the jets with ambient fluid is desired. Experiments [1-4] have shown that three-dimensional (3-D) jets can be used to enhance mixing and entrainment rates in comparison to nominally two-dimensional (2-D), plane or axisymmetric, jets. A fundamental understanding of the dynamics of complex, turbulent jets is required for their prediction and control. The present study is concerned with the understanding of the roles of vorticity and turbulence in the spatial evolution of incompressible 3-D jets in the near to medium field and the effects of external forcing and Reynolds number thereupon.

Some analytical studies have used vortex-induction arguments [5, 6] or stability analysis [7, 8] to explain some observed phenomena in complex jets, such as axis-switching and enhanced mixing. In the former category, the Biot-Savart law was used to predict how elliptic vortex rings would distort and switch axes. Jets are of course not vortex rings. On the other hand, conventional RANS approach by McGuirk and Rodi [9] failed to predict observed axis-switching and saddle-shaped velocity profiles in turbulent rectangular jets.

MATHEMATICAL FORMULATION

Governing Equations

The partial differential equations governing the incompressible jet fluid flow are the Navier-Stokes equations which can be written in Cartesian tensor form, for dimensionless variables as:

\[ \frac{\partial \bar{u}_i}{\partial \tau} + \bar{u}_i \frac{\partial \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{Re_0} \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} \]

where, \( \bar{u}_i \) are the Cartesian velocity components in the Cartesian coordinate directions \( x_i \), \( p \) is the pressure and \( Re_0 \) is the Reynolds number based on the equivalent diameter. These equations must be solved in conjunction with the continuity equation:

\[ \frac{\partial \bar{u}_i}{\partial x_i} = 0 \]

which expresses the divergence-free velocity condition.

At higher Reynolds numbers, all scales present in the flow cannot be resolved on computational grids that present resources would allow, so a large eddy simulation (LES) must be utilized. The application of a grid filter, \( \tilde{\mathcal{G}} \), to equations (1) and (2) results in the filtered equations of motion:

\[ \frac{\partial \tilde{u}_i}{\partial \tau} + \tilde{u}_i \frac{\partial \tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re_0} \frac{\partial^2 \tilde{u}_i}{\partial x_i \partial x_j} \frac{\partial \tilde{r}_{ij}}{\partial x_j} \]

and

\[ \frac{\partial \tilde{u}_i}{\partial x_i} = 0 \]

which \( \tilde{r}_{ij} = \tilde{u}_i \tilde{u}_j - \bar{u}_i \bar{u}_j \) is the subgrid scale (SGS) stress which must be modeled in terms of the resolved velocity field. The Smagorinsky eddy-viscosity model is utilized in the present study to approximate the SGS stress as:

\[ \tau_{ij} - \frac{2}{3} \tau_{kk} X_i \tilde{s} \tilde{s}_j \]

where \( \Delta \) is a length scale associated with the grid size, \( \tilde{s}_i = \frac{1}{2} \left[ \frac{\partial \tilde{u}_i}{\partial x_i} + \frac{\partial \tilde{u}_j}{\partial x_j} \right] \) is the resolved strain rate tensor, and \( |\tilde{s}| = \sqrt{2 \tilde{s}_i \tilde{s}_j} \). The models coefficient, \( C_\alpha \), is set to the constant value of 0.01.

Time-averaged budget of Eq. (3), suggests that this treatment is adequate since the SGS stress is considerably smaller than the resolved Reynolds stress. The Navier-Stokes Eq. (1) or the filtered Eq. (3) are dis-
cretized temporally with an explicit third-order Runge-Kutta (RK) scheme and spatially with implicit fourth-order compact finite difference schemes [10].

MODEL PROBLEMS

Spatial simulations of various 3D jet flows are performed in this study in which a fixed region of the flow is computed and disturbances grow in the streamwise direction. Table 1 summarizes the test cases. Discrete or broad made instabilities are present in the shear layers [11].

All computations are performed on a $10^3$ domain, with $104 \times 128^2$ uniform grid points in the streamwise and cross-plane directions, respectively. In all cases, $D_e = 1$. Additional 24 streamwise grid points lie in a 2 diameter long buffer zone.

RESULTS AND DISCUSSION

Low Reynolds Number Jets

Figures 1 to 3 show computed instantaneous vorticity fields for the low Reynolds number cases 1 to 3, respectively. In case 1, the flow remains symmetrical about the major and minor axes, just like the inlet mean flow and perturbation velocities, hence the flow has remained laminar and transition to turbulence did not occur in the 10 diameters computed. The phenomenon of axis switching is observed to be taking place near the end of the domain. The source of this is seen to be clearly in the complex vorticity field. It is most likely due to self-induction, since there is no streamwise vorticity at the inlet plane. It appears that the effect of the induced streamwise vorticity field is to pump fluid from the major axis to the minor axis. This process can be explained with deformation of vortex rings [6]. Spanwise rollers are seen in Figs. 1 (a) and (b) which form super-elliptical vortex rings. Through self-induction, the narrow ends will move faster in the streamwise direction than the wider sides; a process which will eventually lead to axis-switching. In case 2, the vortex field is even more complex. Self-induction must now compete with the instability mechanism which produces vortex pairing in 2D mixing layers. In the latter (see Wilson and

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Re$_D$</th>
<th>Jet Geometry</th>
<th>Shear-Layer Instability Mode</th>
<th>DNS or LES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>750</td>
<td>2:1 Rectangular</td>
<td>Fundamental</td>
<td>DNS</td>
</tr>
<tr>
<td>2</td>
<td>750</td>
<td>2:1 Rectangular</td>
<td>Fundamental + 1st sub-harmonic</td>
<td>DNS</td>
</tr>
<tr>
<td>3</td>
<td>750</td>
<td>2:1 Rectangular</td>
<td>Broad</td>
<td>DNS</td>
</tr>
<tr>
<td>4</td>
<td>75,000</td>
<td>2:1 Rectangular</td>
<td>Broad</td>
<td>LES</td>
</tr>
<tr>
<td>5</td>
<td>75,000</td>
<td>2:1 Elliptic</td>
<td>Broad</td>
<td>LES</td>
</tr>
<tr>
<td>6</td>
<td>75,000</td>
<td>Circular</td>
<td>Broad</td>
<td>LES</td>
</tr>
</tbody>
</table>

Figure 1. Contours of vorticity magnitude, (a)-(e) and streamwise vorticity, (f)-(h) for case 1 at $t = 2$ flow through times for fundamental forcing function. (a) minor axis plane, $z/D_e = 0$, (b) major axis plane, $y/D_e = 0$, (c) cross flow plane, $x/D_e = 2.5$, (d) $x/D_e = 5$, (e) $x/D_e = 7.5$, and (f) $x/D_e = 10$. 

Figure 2. Contours of vorticity magnitude, (a)-(e) and streamwise vorticity, (f)-(h) for case 2 (see Fig. 1 for legend).

Figure 3. Contours of vorticity for case 3 at $t = 2$ flow through times for broad mode forcing function. (a) minor axis plane, $z/D_e = 0$, (b) major axis plane, $y/D_e = 0$, (c) cross flow plane, $x/D_e = 2.5$, (d) $x/D_e = 5$, (e) $x/D_e = 7.5$, and (f) $x/D_e = 10$. 

Table 1. Parameter for Test Cases

Figure 3. Countours of vorticity for case (3) at $t = 2$ flow through times for broad mode forcing function. (a) minor axis plane, $z/D_e = 0$, (b) major axis plane, $y/D_e = 0$, (c) cross flow plane, $x/D_e = 2.5$, (d) $x/D_e = 5$, (e) $x/D_e = 7.5$, and (f) $x/D_e = 10$. 

just like the inlet mean flow and perturbation velocities, hence the flow has remained laminar and transition to turbulence did not occur in the 10 diameters computed. The phenomenon of axis switching is observed to be taking place near the end of the domain. The source of this is seen to be clearly in the complex vorticity field. It is most likely due to self-induction, since there is no streamwise vorticity at the inlet plane. It appears that the effect of the induced streamwise vorticity field is to pump fluid from the major axis to the minor axis. This process can be explained with deformation of vortex rings [6]. Spanwise rollers are seen in Figs. 1 (a) and (b) which form super-elliptical vortex rings. Through self-induction, the narrow ends will move faster in the streamwise direction than the wider sides; a process which will eventually lead to axis-switching. In case 2, the vortex field is even more complex. Self-induction must now compete with the instability mechanism which produces vortex pairing in 2D mixing layers. In the latter (see Wilson and
In the near field. There is then a competition between largely inviscid mechanisms and turbulent mechanisms in the subsequent evolution of the jet. The question is which mechanism dominates, under what conditions? Figures 4-6 show computed streamwise velocity for cases 4 to 6, respectively, which have been averaged over several time steps. We see clearly that axis-switching occurred in both the rectangular and the elliptic jets.

The round jet, of course, remained circular with no preferential growth direction. Jet evolution can be explained with the mean streamwise vorticity equation:

\[ 0 = -U_1 \frac{\partial \Omega_1}{\partial x_1} - U_2 \frac{\partial \Omega_1}{\partial x_2} - U_3 \frac{\partial \Omega_1}{\partial x_3} - \Omega_1 \frac{\partial U_1}{\partial x_1} + \Omega_2 \frac{\partial U_1}{\partial x_2} + \Omega_3 \frac{\partial U_1}{\partial x_3} \]

\[ + A_1 + A_2 \]

\[ A_3 + A_4 \]

\[ \frac{\partial^2}{\partial x_1^2} \frac{\partial}{\partial x_2} \frac{\partial}{\partial x_3} u_2 y_3 + \frac{1}{Re} \left( \frac{\partial^2}{\partial x_1^2} u_1^2 + \frac{\partial^2}{\partial x_2^2} u_2^2 + \frac{\partial^2}{\partial x_3^2} u_3^2 \right) \Omega \]

where \( A_1 \) represents convection, \( A_2 \) vortex tilting and stretching, \( A_3 - A_4 \) are terms resulting from the anisotropy of turbulent stresses, and \( A_5 \) is the diffusion term. The last plays little role in high Reynolds number flows. Figure 7 shows the terms obtained by post-processing the LES results of case 4, at \( x/D_0 = 3.75 \). The mean streamwise vorticity, \( \Omega \), and velocity, \( U \), are also shown. (Negative values are shown as dashed lines.) This is the location at which streamwise vorticity starts to form and the distortion of the mean velocity contours from super-ellipses commences. It is clear that the two terms involving the

Figure 4. Contours of time-averaged streamwise velocity for LES of the rectangular jet, case 4; (a) \( x/D_0 = 0 \), (b) \( x/D_0 = 4.88 \), (c) \( x/D_0 = 7.31 \), (d) \( x/D_0 = 9.755 \), and (e) minor axis plane \( x/D_0 = 0 \), (f) major axis plane, \( y/D_0 = 0 \).

Figure 5. Contours of time-averaged streamwise velocity for LES of the elliptic jet, case 5 (See Figure 4 for Legend.)
CONCLUDING REMARKS

Numerical simulation of jets with complex cross-sections have been performed using a third-order Runge-Kutta scheme for temporal integration and a fourth-order compact scheme for spatial discretization. Cases at low Reynolds numbers are direct simulations and those at high Reynolds number are large eddy simulations.

The phenomenon of axis-switching is observed to be dependent on instability waves present in the inlet boundary layers, and could be induced in both laminar and turbulent jets by single-mode forcing. It is confirmed that this is based on self-induction of the vorticity field. The presence of a discrete sub-harmonic modes led to its suppression and there was a tendency towards bifurcation.

Self-induction cannot produce axis-switching in jets which have a broad mode of instabilities in their shear layers. In non-turbulent jets, no axis-switching occurs. In turbulent jets, turbulence-induced secondary flow is responsible for axis-switching.

(Manuscript received, October 23, 1997)

REFERENCES


Figure 6. Contours of time-averaged streamwise velocity for LES of the round jet, case 6 (See Figure 4 for legend)

Figure 7. Balance of terms in the time-averaged streamwise vorticity equation for LES of the rectangular jet, case 4.

anisotropy of the secondary Reynolds stresses \( (A_4 + A_5) \) are responsible for the generation of the streamwise vorticity. They balance almost exactly the convection terms \( A_3 \). These are the same terms responsible for the generation of secondary motion in turbulent flow in non-circular ducts [13]. The inviscid, vortex tilting and stretching term, \( A_2 \), plays little or no role. That is, for natural turbulent jets, with a broad-mode of instabilities in the shear-layer, jet evolution is controlled by turbulence-induced secondary motion, i.e., of Prandtl's second kind, and not by self-induction as in forced jets. Hence, axis-switching in complex jets can be predicted with RANS, so long as the turbulence model utilized can reproduce the anisotropy of the turbulence. McGuirk and Rodi [9] had utilized a k-\( \varepsilon \) turbulence model which is inadequate for this purpose. Therefore, their inability to predict axis-switching is not now surprising. Some form of Reynolds stress model (full or algebraic) would be required.