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Abstract

By a concept of normal mode theory, the realistic seismograms of Love waves are synthesized for the CIT11A spherical earth model. The synthetic seismograms, which include the effects of instrumental response and attenuation, are computed for fundamental and higher modes up to the third order. The source is assumed to be a double couple of body forces varying stepwise in time applied to a point. Calculations are made for various focal depths ranging from 30 to 530 km. It is demonstrated that a strike-slip source excites Love waves by an order of magnitude larger in amplitude than a dip-slip source, and that the higher mode contribution relative to that of the fundamental mode is significantly large for a dip-slip source at any depth. The synthesized seismograms thus obtained facilitate the determination of various source parameters. The results are applied to an intermediate-depth earthquake in Eastern New Guinea on February 26, 1963 (d=182 km). The seismic moment is estimated as $2.5 \times 10^{27}$ dyne cm.

1. Introduction

For mantle waves whose period and wave length are long as compared with the time duration and the dimension of the source respectively, the source can be assumed as a point, at least to the first approximation, where a double couple force is exerted stepwise in time. The moment of the component couple is called “seismic moment” and it is equivalent to the product of the rigidity, the dislocation area and the average displacement discontinuity across the dislocation surface (Maruyama, 1963).

Extensive analyses of mantle Love and Rayleigh waves have been made in order to estimate the seismic moment (e.g. Aki, 1966; Brune and King, 1967; Brune and Engen, 1969). Their studies are made in the frequency domain using the theories by Haskell (1963, 1964) and Ben-Menahem and Harkrider (1964). They assumed that only a fundamental mode is excited. This assumption is valid only for very shallow
earthquakes.

In the present paper the study of the excitation of Love waves is made in the time domain by a concept of normal mode theory. The fundamental and higher modes up to the third order are taken into account. Synthesized seismograms are calculated according to Saito (1967). Saito’s method has been successfully applied to surface waves by Kanamori (1970 a, b) and to free oscillations by Abe (1970). In a similar manner, Jobert (1962, 1964) and Satô et al. (1968) calculated the seismograms of Love waves for several focal depths; however, their simple source models are inadequate for the interpretation of the actual seismograms of natural earthquakes.

The higher mode contribution to Love wave seismograms is more easily understood in the time domain than in the frequency domain. The synthesized seismograms including higher modes make it possible to estimate the seismic moment of deep and intermediate-depth earthquakes. We shall apply them to the Eastern New Guinea earthquake of February 26, 1963 (d=182 km).

2. Basic Formula

Let the spatial parameters of a source be specified by a unit vector \( \mathbf{n} \) normal to the fault plane and by a unit vector \( \mathbf{v} \) which designates the direction of the displacement on this plane. The right-hand Cartesian coordinates \( x, y, \) and \( z \) with the origin at the source are taken to coincide with the dip direction, the strike direction of the fault plane and the vertically upward direction respectively. The azimuthal angle \( \phi \) is measured counterclockwise from positive \( x \) axis.

The displacement field due to this fault is equivalent to that due to a double couple with forces parallel to \( \mathbf{n} \) and \( \mathbf{v} \) in the absence of the fault (Maruyama, 1963). According to Saito (1967), the azimuthal component, \( U_\phi \), of the surface displacement of the torsional oscillations is expressed by

\[
U_\phi(\theta, \phi, t) = W_d \left( \frac{n_x v_x - n_y v_y}{n_z v_x + n_y v_y} \right) \left( \frac{\cos \phi}{\sin \phi} \right) - W_s \left( \frac{n_x v_x + n_y v_y}{n_z v_x + n_y v_y} \right) \left( \frac{\cos 2\phi}{\sin 2\phi} \right).
\]

Here \( W_d \) and \( W_s \) are the excitation functions independent of \( n, v \) and \( \phi \). The formula (1) becomes \( U_\phi = W_d \sin \phi \) for a dip slip on a vertical fault and \( U_\phi = W_s \cos 2\phi \) for a strike slip on a vertical fault. If the couple is exerted step-wise in time at the source located at a radial distance \( r = r_s \), the excitation functions \( W_s \) and \( W_d \) are given by

\[
W_s = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} A_l(r_s) \frac{dP_l}{d\theta} \exp \left( - \frac{\omega t}{2 \eta} \right) \cos \omega t,
\]

(2)
\[ W_a = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} A_l(r_s) \frac{dP_l}{d\theta} \exp\left(-\frac{\omega_l t}{2\pi Q_l}\right) \cos \omega_l t , \]  

(2)

at the colatitude \( \theta \) and at time \( t \) after the origin time. The quantities \( \omega_l \) and \( Q_l \) are the eigen angular frequency and the dimensionless quality factor respectively. The suffix \( l \) is the order number of the associated Legendre function and \( n \) determines the number of internal nodal surfaces. For example, the modes denoted by \( n=0 \) and 1 represent the fundamental and the first higher mode respectively. Here

\[ A_l(r_s) = \frac{2l+1}{4\pi l(l+1)} \frac{1}{r_s} \frac{y_l(r_s)}{\omega_l I_l} , \]

\[ B_l(r_s) = \frac{2l+1}{4\pi l(l+1)} \frac{1}{\mu_s} \frac{y_l(r_s)}{\omega_l I_l} , \]

(3)

where \( \mu_s \) is the rigidity at \( r=r_s \). The functions \( y_l(r) \) and \( y_{l1}(r) \) represent the radial factors of the displacement and stress of the torsional oscillations. The energy integral defined by

\[ I_l = \int_0^a \rho(r) r^2 y_l(r) dr , \]

(4)

is proportional to the kinetic energy of each mode of oscillation [see also, Abe (1970)], where \( a \) is the earth's radius and \( \rho(r) \) is the density as a function of \( r \).

3. Numerical Results

The oceanic model CIT11A constructed by Kovach and Anderson (1964) is used in this study. Phase- and group-velocity curves of the fundamental \((n=0)\) and the first three higher modes \((n=1-3)\) for this model are shown in Fig. 1. They are obtained from the eigen frequencies and energy integrals by the ordinary procedure (e.g. Alteman et al., 1959; Takeuchi et al., 1962). Calculations are made at 34 points between \( l=10 \) (\( T=617 \) sec) and \( l=1280 \) (\( T=7.7 \) sec) for the fundamental mode and at 24 points between \( l=15 \) and 1280 for higher modes. The functions \( A_l(r_s) \) and \( B_l(r_s) \) at these points are calculated for six depths, 31.5, 101.5, 201.5, 301.5, 401.5, and 531.5 km. The successive values of \( \omega_l, A_l, \) and \( B_l \) are determined by 4-point Lagrangean interpolation. They are checked against the rigorous calculations at several points.

Specific quality factor \( Q_l \) is experimentally known only for a limited range of frequency and mode. Following Anderson and Archambeau (1964), Love wave \( Q \) is calculated as a function of period for the
MM8 model (Anderson et al., 1965) and for $n=0$ to 3. The results are shown in Fig. 2. The $Q$ values of the four modes are not very different from one another. Their values lie between 100 and 150 in the frequency range where the group velocity curves are flat. If $Q$-structure constructed by Tsai and Aki (1969) is used together with the CIT11A velocity structure, much lower $Q$ is obtained in the same range of frequency for all the higher modes. Here we tentatively take $Q_1 = Q = 100$, but this value may be changed when the study of $Q$ within the earth is much advanced. Instrumental correction was made by using the theoretical formula derived by Hagiwara (1958) for electromagnetic seismographs. We take the constants as follows: the natural period of pendulum, 30 sec; the natural period of galvanometer, 100 sec; the damping factors of pendulum and galvanometer, 1.0; the coupling factor $\sigma$, 0.15.

The excitation functions $W_e$ and $W_d$ in which the instrumental re-
Fig. 3. Synthetic seismograms of the first four Love-modes at $\theta=90^\circ$ for a double couple strike-slip source varying stepwise in time. The seismic moment is $10^{22}$ dyne-cm. The vertical scale gives the trace amplitude on the standard 30–100 seismograph records with magnification of 1500. The beginning time of all traces is 35 min after the origin time. The focal depths are taken as 31.5, 101.5, 201.5, 301.5, 401.5, and 531.5 km.

Fig. 4. Synthetic seismograms of the first four Love-modes at $\theta=90^\circ$ for a double couple dip-slip source varying stepwise in time (see the caption for Fig. 3).
response is included are shown in Figs. 3 and 4. They represent the displacement for a strike-slip on a vertical fault and for a dip-slip on a vertical fault. It is noted that the synthesized seismograms for an arbitrary double couple source is represented by a linear combination of \( W_b \) and \( W_a \) as shown in the formula (1). The seismograms are synthesized at \( \theta = 90^\circ \) from \( t = 35 \) to 41.25 min after the origin time. The calculations are made for a seismic moment of \( 10^{20} \text{ dyne} \cdot \text{cm} \).

Owing to the existence of the flat portion of the dispersion curves of the group velocity, the waves of the fundamental and the first higher modes appear as an isolated shape. When all the modes are superimposed, rather complicated features in the Love wave seismograms result owing to nearly the same group arrivals of these modes. At greater focal depths the excitation of the second and third higher modes overcomes that of the first two modes, and the seismograms become extremely dispersed. This result seems to be inconsistent with the actual observations; we often observe pulse-like Love waves even for very deep earthquakes as \( d = 500-650 \text{ km} \). It is indicative of the incompleteness of either the CIT11A model, the MM8 model or, probably, both.

In general the strike-slip source excites the Love waves by an order of magnitude larger in amplitude than the dip-slip source. The higher modes excited by the strike-slip source do not significantly disturb the fundamental mode for focal depths from the surface to 200 km. For depths greater than 200 km, the excitation of the first higher mode exceeds that of the fundamental mode. In contrast, the higher modes, especially the first higher mode, due to the dip-slip source are excited with the amplitude nearly equal to or even larger than that of the fundamental mode for any focal depths. This distinctive feature of excitation arises from the difference of the displacement-versus-depth and the stress-versus-depth curves among these modes in the vicinity of the low-velocity channel.

4. Estimation of Seismic Moment for an Intermediate-Depth Earthquake

The equalization method has been used for the estimation of seismic moment from mantle waves; Aki (1966) equalized Love waves to the origin in the frequency domain, while Kanamori (1970 a, b) equalized Love and Rayleigh waves to a uniform epicentral distance in the time domain. However, if higher-mode surface-waves are superposed on surface waves, this method becomes difficult to apply. We therefore employ an alternative method. Instead of equalizing the observed seismograms to a certain distance, we calculate the synthetic seismo-
gram for the distance of each station and compare it with the observed seismogram. For the calculation of the synthetic seismograms we employ, instead of using formula (2), a simpler method.

The Love wave seismogram obtained at $\theta_0$ is written as

$$f(\theta_0, t) = \int_0^\infty G(\theta_0) \cos [\omega t + \varphi(\theta_0)] d\omega,$$  \hspace{1cm} (5)

for a single mode. During the propagation from $\theta_0$ to $\theta_0 + \theta_1$, the phase and amplitude of the waves are distorted due to the dispersion, geometrical spreading and attenuation. The seismogram at a distance $\theta_0 + \theta_1$ is approximately given by

$$f(\theta_0 + \theta_1, t) = \frac{1}{\sqrt{\cos \theta_1}} \int_0^\infty G(\theta_0) \exp \left(-\frac{\omega \theta_1}{2UQ}\right) \times \cos \left[\omega \left(t - \frac{\alpha \theta_1}{C}\right) + \varphi(\theta_0) + \frac{N\pi}{2}\right] d\omega.$$  \hspace{1cm} (6)

Here $N\pi/2$ represents the polar phase shift (Brune et al., 1961). Fig. 5 illustrates the synthetic fundamental-mode Love waves for a strike-slip source at distances from $\theta = 70^\circ$ to $250^\circ$ with $20^\circ$ interval. The focal depth is taken as $31.5$ km. In this figure $\theta_0$ is $90^\circ$, and the seismograms start at $t = t_0 + \frac{a\theta_1}{U_0}$ where $t_0 = 35$ min and $U_0 = 4.45$ km/sec. The dispersion, attenuation, and polar phase shift are clearly visualized.

The method is applied to the Eastern New Guinea earthquake whose focal coordinates given by ISS (International Seismological Summary) are as follows: Date, February 26, 1963; Time, 20 h 14 m 10s(GMT); Location, $7.6^\circ$S, $146.2^\circ$E; Depth, 182 km; Magnitude, 7 1/2 (Pasadena). Records were obtained from the 30–100 long-period instruments of the USCGS world-wide standard network (WWSSN). The first-motion data of long-period $P$

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**Fig. 5.** The travelling Love waves of the fundamental mode. The source model is a double couple strike-slip. The focal depth is taken as $31.5$ km. The seismic moment is $10^{27}$ dyne·cm.
waves are plotted in Fig. 6, using the Wulff-net projection of the lower hemisphere of the focal sphere. One nodal plane which is vertical and has a strike of N 5° W is determined almost uniquely. The strike azimuth of the second nodal plane is, therefore N 85° E. We cannot determine the dip angle uniquely. However, we can give a possible bound for the dip angle: the dip angle should be smaller than ±20°. *Isacks and Molnar* (1971) obtained a fault plane solution of this earthquake from probably the same data. The first nodal plane of course coincides with ours. The second plane dips towards N 5° W with a dip angle of 20°. It will be shown, however, that their solution is inconsistent with the observed radiation pattern of Love waves.

The five stations of WWSSN are selected so that the entire propagation path is included in the oceanic region, because the earth model used here represents a pure oceanic-structure. Their geographical distribution referred to the epicenter is illustrated in Fig. 7. The seismograms as presented in Fig. 8 are taken from the NS component of the long-period records. Since the azimuth of the epicenter at each station listed in Table 1 is close to either 90° or 270°, the NS component registers almost the transverse motion. Note that no appreciable Love waves
are recorded at TRN.

The theoretical radiation pattern of the Love-wave amplitude at \( \theta = 90^\circ \) is illustrated in Fig. 7 for the mechanism solution obtained by Isacks and Molnar (1971). The absence of clear Love waves at TRN rejects their solution because their solution predicts a large Love-wave amplitude at this station. Alternatively we assume the second nodal plane to be horizontal. The solution is a pure dip-slip on either a vertical or a horizontal plane. The theoretical radiation pattern of Love waves for this solution is shown in Fig. 7. This solution is much more consistent with the observed Love-wave radiation pattern than the solution obtained by Isacks and Molnar (1971). It may be noticed that the radiation pattern of Love waves is drastically changed by an addition of a small amount of strike-slip component to a dip-slip type fault.

The direction of motion of Love waves at two stations GOL and PRE is also to be noted. These two stations have nearly the same epicentral distance and are located at almost opposite azimuths. The NS motion of Love waves recorded at these stations are in the same direction, i.e., the transverse component as defined in this paper is of opposite sign to each other. This result is consistent with our solution (pure dip-slip), since the radiation pattern of Love wave phase is antisymmetric with respect to the epicenter for a pure dip-slip type fault. The preferred focal mechanism solution of this earthquake is, therefore, as follows: the first nodal plane is vertical and strikes N5° W while the second plane is almost horizontal and strikes N85° E. The dip angle of the second plane should be much smaller than 20°.

The synthesized seismograms corresponding to each

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Fig. 8. Synthetic and observed Love waves of the Eastern New Guinea earthquake of February 26, 1963 (\( d = 182 \) km). All the synthetic seismograms for \( M_s = 10^{27} \) dyne-cm are calculated at the distance of the respective stations. No appreciable Love waves are observed at TRN.
station are computed according to the formula (6) for the focal depth of 201.5 km, assuming that the second nodal plane is horizontal. Only the fundamental and first higher mode excitations are taken into account in the computation, since the excitation of other modes are negligibly small as seen in Fig. 4. The resulting seismograms are compared with the observed Love-wave records in Fig. 8. The agreement of the wave form is rather poor between the observed and synthesized seismograms if only the fundamental mode is considered. If the first higher mode is superimposed, the agreement becomes much better. This agreement enables one to estimate the seismic moment from the amplitude ratio. The results are shown in Table 1. The average seismic moment is $2.5 \times 10^{37}$ dyne·cm.

<table>
<thead>
<tr>
<th>Station</th>
<th>$d$ (deg)</th>
<th>$\alpha_E$ (deg)</th>
<th>$\alpha_S$ (deg)</th>
<th>$\phi$ (deg)</th>
<th>$M_s$ ($\times 10^{37}$ dyne·cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSC</td>
<td>100.0</td>
<td>55.5</td>
<td>267.8</td>
<td>29.5</td>
<td>2.0</td>
</tr>
<tr>
<td>DUG</td>
<td>103.3</td>
<td>50.6</td>
<td>271.2</td>
<td>34.4</td>
<td>1.0</td>
</tr>
<tr>
<td>GOL</td>
<td>108.9</td>
<td>50.7</td>
<td>276.0</td>
<td>34.3</td>
<td>3.1</td>
</tr>
<tr>
<td>PRE</td>
<td>111.3</td>
<td>238.8</td>
<td>109.9</td>
<td>206.2</td>
<td>3.8</td>
</tr>
<tr>
<td>TRN</td>
<td>152.6</td>
<td>81.6</td>
<td>274.0</td>
<td>3.4</td>
<td>—</td>
</tr>
</tbody>
</table>

$M_s = 2.5 \times 10^{37}$ dyne·cm

$\alpha_E$: epicentral distance,
$\alpha_E$: epicenter to station azimuth,
$\alpha_S$: station to epicenter azimuth,
$\phi$: azimuthal angle referred to the source geometry (see the text),
$M_s$: seismic moment.

5. Conclusions

A convenient method for the estimation of seismic moment from the mantle Love waves is devised for arbitrary focal depths and earthquake mechanisms. First, the synthetic seismograms of the fundamental and higher modes up to the third order are computed, for a seismic moment of $10^{37}$ dynd·cm, by the concept of normal mode theory at an epicentral distance of 90°. Second, they are translated to an arbitrary epicentral distance in order to compare with the actual record. Calculations are made for six different focal depths for the CIT11A spherical earth model. A strike-slip source excites the Love waves by an order of magnitude larger in amplitude than a dip-slip source. The excitations of all the higher modes are much smaller than those of the fundamental mode for a strike-slip source if the source depth is
shallower than 200 km. In the case of a pure dip-slip source, the contribution from the higher modes, especially from the first higher mode, cannot be neglected for any focal depth.

The method is applied to the Eastern New Guinea earthquake of February 26, 1963 (d=182 km). The fault plane solution obtained by using $P$ wave first-motions and Love wave amplitudes represents a nearly pure dip-slip fault. It is shown that the first higher mode Love-waves are contained by a large amount in the observed Love-wave seismograms. The seismic moment is estimated to be $2.5 \times 10^{27}$ dyne·cm.

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References

Abe, K., Determination of seismic moment and energy from the earth's free oscillation, Phys. Earth Planet. Interiors, 4, 49-61, 1970.
1. マントル地震によるラブ波の励起問題

現実的な地球モデルと震源モデルをもってい、ラブ波の合成地震記象を作り、excitation に関する基本的な性質を調べ、実際の地震に応用してその有用性を確かめた。球状地球モデルとしては、CIT11A を採用し、断層モデルと等価な震源モデルとし、点震源に時間間隔がステップ状に変化する力より構成される複数力源を考えた。合成地震記象は基本および 1 次から 3 次までの高次の基準性のモード解を周期 7 秒から 600 秒の範囲で加えあわせることにより作られた。伝播媒質の減衰および地震計の特性を考慮することにより記象を現実的なものとした。震源の深さとしては 30 km より 500 km までの範囲にわたって選んだ。

一般的な力系に対するラブ波の excitation は震源を複数力源とし、pure strike-slip 型と pure dip-slip 型の二種のモデルの一次結合で論じられる。各モードの excitation と震源の力系・深さとの関係は第 3 番および第 4 番の各モードの合成地震記象に明確に示されている。特に顕著されるのは次の点である。strike slip 型によるラブ波の最大振幅は dip slip 型のものにくらべ約 10 倍大きい。strike slip 型の場合震源の深さが 200 km よりも深いときは、高次モードが基本モードよりも励起され易くなり高次モードの影響を無視できなくなる。高次モードの excitation では 1 次のモードが比較的良く励起される。dip slip 型の場合は、いずれの深さにおいても高次モード、特に 1 次のモードの影響を無視できない。

上記の結果を東南アジアの中深発地震（深さ 182 km）の発震機構および地震モーメントの推定に応用した。得られた発震機構は pure dip-slip 型で地震モーメントは 2.5×10^{27} dyne·cm である。