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2. *Maximum Amplitude and Epicentral Distance. II.*
—*Approach to Source Spectrum of Earthquakes.*—

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1. Introduction

In a previous paper[1], $A \sim \varDelta$ and $T \sim \varDelta$ relations of several destructive earthquakes of medium class were studied, and formulas of a kind of spectrum at the source were derived. The problem is studied again in the hope of getting a standard formula that may give in effect a mean source spectrum of "shallow and destructive" earthquakes and may be related to the mechanism of radiation of earthquake energy. To arrive at any decisive conclusion, there are as yet too many unknown or ambiguous factors in observations. In this paper, a way of thinking is presented which is expected to be improved in the future.

The results in the previous paper are as follows:

$$B(T) = C. \exp(-sT^2), \tag{1}$$

$$C = 7.25 \times 10^4 \, \mu.\text{km}^{1/2}, \qquad s = 0.06 \, \text{sec}^{-2},$$

for Shizuoka earthquake of 1935, whose Magnitude is 6.3 by J.M.A. For Shionomisaki earthquake of 1948, whose Magnitude is 7.2 by J.M.A. or 7.3 by Gutenberg,

$$B(T) = C. \exp(-\alpha T^{1/2}) \tag{2}$$

$$C = 2 \times 10^6 \, \mu.\text{km}^{1/2}, \qquad \alpha = 1.2 \, \text{sec}^{-1/2}.$$

The periods T for which (1) and (2) are applicable are roughly from 2 to 4 sec. for the former and from 3 to 6 sec. for the latter, since those are the calculated periods of "Maximum amplitude" in the range of

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[1] R. YOSHIYAMA, "Maximum Amplitude and Epicentral Distance.—Proposed a Theoretical Elucidation of Empirical Formulas and Some Development.—" *Bull. Earthq. Res. Inst.*, **37** (1959), 389-404.

epicentral distances concerned in the analysis. As to the band of short period, which contributes to the "Maximum amplitude" at small distances, neither (1) nor (2) seems reasonable, because particle velocity and acceleration calculated thereof are divergent at $T=0$. The uppermost limit of particle velocity of a wave is not free from the compressive or shearing strength of the medium; it must be given by ev, where e is an ultimate strain and v velocity of propagation of the wave, and is equal to 150 cm/sec, if we put $e=5\cdot10^{-4}$ and $v=3$ km/sec. Though observations are not sufficient to estimate the ultimate strain for an oscillatory motion of high frequency, 150 cm/sec would be a maximum possible value, and a velocity of particle motion less than this would be presumable in general. Moreover, so long as we assume the earthquake-generating forces in purely internal forces, a certain limitation of the particle velocity, subject to a solid friction of the medium, is reasonable. So that, if we put $B(T)=B_0T^n$ at $T=0$, $n\geqq1$. Anyway, (1) and (2) will give decidedly too large a value for the band of short period, though the observations of "Maximum amplitudes" at short epicentral distances to confirm it are not sure at present.

On the other hand, judging from the observations of "Maximum amplitudes" at large distances, amplitudes of wave of long period calculated from (2), not to speak of (1), seem too small.

2. General view of spectrum

Reflecting upon the process in the previous paper, those exponential functions of (1) and (2) were derived so that the empirical formula of $T\sim\Delta$ relation may be given as simply as possible without any general reasoning; it was conventionally assumed that $T\propto\Delta^n$, a linear relation in a logarithmic scale. However, it is plausible neither that the period of the waves of "Maximum amplitude" is equal to zero at the epicentre nor that it gets so large as Δ^n with increasing Δ; there will practically be some limits on both sides that respectively depend on the Magnitude of the earthquake as is schematically shown in Fig. 1. In Fig. 10~12 of the previous paper, we can see that, at small distances, the deviation of the observations from a linear relation is in the sense expected from Fig. 1. At large distances, whether the deviation is in the sense expected or not, it is not yet clear. A presumable amplitude spectrum at the origin, that accords with the $T\sim\Delta$ relation given in Fig. 1, is such like that shown in Fig. 2, instead of the formula (1) or (2), either

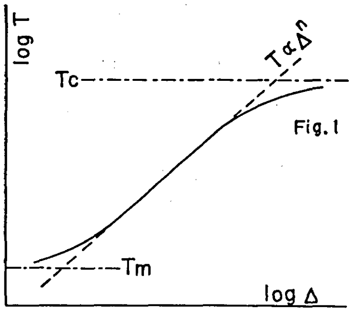

Fig. 1. Schematic representation of presumable, expected to be observed, $T \sim \Delta$ relation.
——: assumption in this paper.
- - -: assumption in the previous paper.

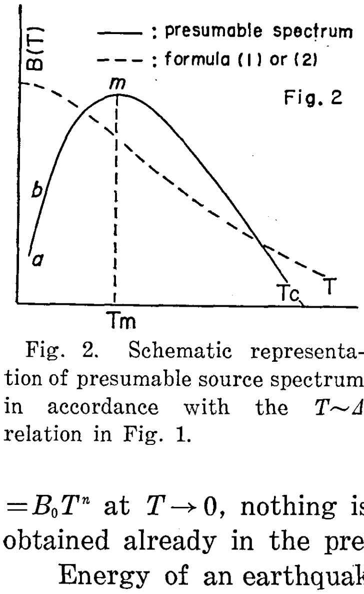

Fig. 2. Schematic representation of presumable source spectrum in accordance with the $T \sim \Delta$ relation in Fig. 1.

of which being ever decreasing with increasing T. Another theoretical weak point of the formula (1) or (2) is that, each of them being separately derived, there is no general guiding principle in the mathematical formulation of the presumable spectrum. The number of peaks in the spectrum may not be necessarily single; there would be some or many, proper to each earthquake, even if confined to remarkable ones; Fig. 2 would be accepted to give a general view of the spectrum at the source. Some presumable points of its quality can be deduced from various information and reasoning. Though, as yet, information may not be enough to develop the reasoning, some of them are pointed out in the following.

The height of the peak and the corresponding period T_m depend chiefly on the Magnitude of the earthquake and depth of its origin; the larger the earthquake, the higher is the peak and the longer the period. At a very short period, i. e., at " a " in Fig. 2, very large acceleration is observed in some cases at the epicentre; therefore, we can not presume a limit of the acceleration from observations. Hence, if we put $B(T) = B_0 T^n$ at $T \rightarrow 0$, nothing is added at present to that $n \geqq 1$, that was obtained already in the preceding section.

Energy of an earthquake would spread out from the origin as various kinds of seismic waves: a part of it, with a short period, may go out as bodily waves, P or S waves; the remaining part as surface waves, Rayleigh or Love waves. The limit, if any, of period of waves where the one is separated from the other would depend on the depth and the extension of the earthquake origin. Then, that shown in Fig. 2 is a resultant of two kinds of spectrum, those of bodily waves and surface waves, or, speaking in detail still more, four kinds of spectrum, those of P, S, Rayleigh and Love waves; each of them would be similar in feature to that of Fig. 2 with each peak and each period of the peak. It is natural to assume at present that, in a destructive earthquake of shallow depth, the main part of the energy is in the surface waves. Strictly speaking, if earthquake energy is really given by the generally used Magnitude-energy formula, we might say that it is transmitted by a wave of "Maximum amplitude", the nature of which is still unknown, but, certainly, the wave is a resultant of S wave and surface wave at short epicentral distances and is a surface wave at large distances.

Another remarkable information of the spectrum is a conclusion of "constant velocity" obtained by seismological engineers. Accuracies and generality of the analysis are not clear enough to accept the conclusion as one of principle in seismology. If it were established by future analysis of high accuracy, it is not theoretically appropriate to regard it as, though it looks like, a principle of the same significance with the principle of "equipartition of energy" in physics, because the principle in physics is concerned with a microscopic law, while the conclusion above stated, so it seems, is concerned with macroscopic observations. If any principle analogical to that in physics is presumable in seismology, too, assuming at the earthquake origin a number of oscillators, or a number of wave sources, subject to the principle, and assuming a law of probability of being in phase or off phase at the time of occurrence of an earthquake for the interference among the waves therefrom, a theoretical curve from "a" to "b" in Fig. 2 may be traced. However, the process of the reasoning needs numerous assumptions perhaps far beyond our knowledge of observations at present. In short, the conclusion among the seismological engineers may at present merely suggest a comparatively smooth increase of the curve from a to the peak m in Fig. 2, without, however, indicating where the peak is; the range of period of seismic waves where the law of "constant velocity" is expected among the seismological engineers is not clear. At present, therefore, it is preferable to set some plausible and basic assumptions to build a presumable spectrum at the source of earthquake in accordance with Fig. 2. Those assumptions are expected, on the one side, to explain the observations of $T \sim \Delta$ and $A \sim \Delta$ relations, and, on the other side, to be induced in future to a fundamental law of radiation of earthquake energies.

3. Assumption of the mechanism of energy radiation of an earthquake

It is already and generally assumed elsewhere that, to a seismic wave with a period T, a source of radius vT is necessarily correspondent, where v is approximately equal to the velocity of propagation of the wave. However, any assumption is not yet presented of the relation between the source energy and the "Maximum amplitude" of the wave. The expected assumption is clearly related to the time rate or mechanism of energy radiation in earthquakes and to the physics of an initial or a boundary condition which is imposed on mathematically in a theoretical study of generation of a wave. As to observations related to the problem, various kinds of empirical formula among "Maximum amplitude", its period, its time of duration and so on are presentend from which Gutenberg and Richter worked out their Magnitude-energy formula; those formulas implicate in nature the relation in question. However, judging partly from the fact that gradual changes of the Magnitude-energy formula are still going on, it seems those formulas are not yet sure enough to deduce the expected relation. So considered in four steps, we assume as follows:

1) $\dot{u}(T)=\xi(\bar{\varepsilon})$

where $\dot{u}(T)$ is particle velocity due to the wave with a period T, $\bar{\varepsilon}$ is density of energy calculated from earthquake energy E and the volume of the wave source by, when T is not very small,

$$\bar{\varepsilon}=3E/4\pi(vT)^3, \tag{3}$$

and, if we use the notation in the previous paper, $\dot{u}(T)=2\pi B(T)/T$.

2) Main term of $\xi(\bar{\varepsilon})$ is $(\bar{\varepsilon})^{1/2}$ in accordance with the dimensions of $\dot{u}$ and energy.

3) Even when $\bar{\varepsilon}\to\infty$, $\dot{u}(T)$ tends to a certain limiting value.

4) There is a lower limit of energy density for a radiation of earthquake energy, $\varepsilon_0(T)$, and we put $\varepsilon_0(T)=\beta'/T^n$.

As one of the functions, as simple as possible, satisfying these assumptions, we put

$$\xi(\bar{\varepsilon})=\gamma'\sqrt{\frac{\bar{\varepsilon}-\varepsilon_0(T)}{1+\alpha'\bar{\varepsilon}}} \tag{4}$$

and $\varepsilon_0(T)=\beta'/T$, where γ', α' and β' are certain constants, expected to be universal for earthquakes, if the assumptions here introduced have any physical meaning. Putting (3) into (4), we get a final result,

$$B(T)=\gamma T\sqrt{\frac{1-\beta T^2}{1+\alpha T^3}} \tag{5}$$

where $\gamma=\gamma'/2\pi\sqrt{\alpha'}$, $\alpha=4\pi v^3/3\alpha' E$ and $\beta=4\pi v^3\beta'/3E$. It is expected from the assumptions that γ is a universal constant, or, at least, shifts but slightly from earthquake to earthquake, while α and β are reciprocally proportional to the energy of the earthquake.

So long as $|\alpha T^3|\ll 1$ and $|\beta T^2|\ll 1$, (5) gives a spectrum of "velocity constant". If the source spectrum of an earthquake is given by (5) and the law of attenuation of a wave with a period T by $\exp\{-f(T)\Delta\}/\Delta^{n_0}$, "Maximum amplitude A and its period T" in relation to the epicentral distance will be given by the following equations as already deduced in the previous paper.

$$\frac{d}{dT}\log(V\cdot B)=\Delta\frac{df}{dT}, \tag{6}$$

$$\log\Delta^{n_0}A=\log(V\cdot B)-f(T)\Delta, \tag{7}$$

where $V=1/\sqrt{(1-X^2)^2+4h^2X^2}$, $X=T/T_s$, T_s being the free vibration period of the seismograph used in the observation. The only difference between these formulas and those in the previous paper, (7) and (8), lies in that the effect V of the seismograph's constant is taken into consideration. Strictly speaking, circumstances of observations of "Maximum amplitude and its period" are too complicated and confusing even at present to correct the effect with a few factors. In this paper, it is assumed that $T_s=5$ sec and $h^2=0.35$, damping ratio thereof being 10:1. These figures will be accepted as a mean value of the constants of seismograph used in routine observation in Japan, though it seems doubtful that those characteristics of the seismograph were kept always constant. As to $f(T)$ and n_0, it is reasonable to assume $f(T)=k/T$, $k=(5\sim10)\times10^{-3}$, $n_0=1/2$ for large T and $n_0=1$ for small T. The last assumption may be substituted by that $n_0=1/2$ for large epicentral distances and $n_0=1$ for small ones. In this paper, $n_0=1/2$ is used.

4. Cut-off period and period of "Maximum amplitude" at the epicentre

From (5), the largest, or, say, a cut-off period T_c is introduced such that

$$T_c = (1/\beta)^{1/2}$$

$$\propto (E)^{1/2} \tag{8}$$

in an earthquake with energy E. At present, we have no information about T_c; moreover, according to the way how to understand the "source spectrum of an earthquake", there may be no cut-off period at all. However, if we put aside the natural breadth of the spectrum attached to a phenomenon of a limited time of duration, it seems reasonable to presume a practical upper limit of period in the spectrum of waves for each earthquake according to its Magnitude, especially when it is a small one. T_c reveals itself as a period of "Maximum amplitude" at a large distance where the amplitude is as small as a noise.

From (6), period of "Maximum amplitude" at the epicentre is, putting $\Delta=0$, given by

$$T_0 = (2/\alpha)^{1/3}$$

$$\propto (E)^{1/3} \tag{9}$$

To cover all the cases, from destructive earthquakes to minor shallow ones, by (9), an effective difference of velocity v of propagation of the wave in each earthquake should not be neglected; therefore, T_0 of a very small shallow earthquake may be a little longer than that expected at once from (9). T_0 is also a practical limit of period of wave for which the "constant velocity" is expected at the epicentre. Any way, propriety or impropriety of the assumption (5) is examined in the observation by (9). Concerning the underwater explosion,[2] it seems already proved experimentally that time-scaling factor is proportional to a cube-root of energy of explosives. On the other hand, for earthquakes, the following formula is presented by Gutenberg and Richter,[3]

$$\log_{10} T_0 = -1.1 + 0.1\,M \tag{10}$$

with

$$\log_{10} E = 11.4 + 1.5\,M$$

Though the meaning of T_0 in (10) may be somewhat different from that of T_0 in (9), a principle suggested by (9) is, nevertheless, discordant with (10). But, if we put in (4),

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[2] R. H. Cole, *Underwater Explosions* (Princeton Univ., 1948).

[3] B. Gutenberg and C. F. Richter, *Bull. Seism. Soc. Am.*, **46** (1956), 105-143.

$$\bar{\varepsilon} \propto E/T^n, \qquad n=18\sim 20, \tag{11}$$

instead of (3), a formula concordant with (10) may be easily obtained. If the formula (10) is the one empirically established, theoretical reasoning should be developed based on it; at least, n in (11) should be assumed $6\sim 10$ as a mean value of 3 and $18\sim 20$. However, as explained in the following, discordancies, too, are not negligible between the data from which (10) is derived and the observations of T_0 of, at least, several destructive earthquakes in Japan.

According to their data, the expected period of "Maximum amplitude" for an earthquake of the Magnitude of Shizuoka earthquake is 0.3 sec at the epicentre and 0.4 sec even at $\Delta=200$ km; for an earthquake of the Magnitude of Tottori earthquake in 1943, $M=7.3$, the expected T_0 is not more than 0.5 sec. On the other hand, in those two earthquakes, from the studies of earthquake damages in the epicentral areas, a lower limit of displacement amplitude and an upper limit of acceleration of the ground motion are concluded to be $10\sim 20$ cm and $300\sim 500$ cm/sec^2 respectively. From these figures, a reasonable lower limit of the period T_0 of the "Maximum amplitude" can be estimated; it turns out $1\sim 2$ sec at least, being supported also by the seismometrical observations at near stations. If, following after (10), we assume $T_0=0.5$ sec or less, for a displacement as large as 10 cm, acceleration thereof is 1600 cm/sec^2 or more; earthquake damages by a ground motion with such a large acceleration and a period of 0.5 sec would be decidedly more severe than those experienced in the two earthquakes.

To explain the reason of the discordancy between the data of T_0 in the two countries, various factors should be taken into account: definition of the period, mechanism of earthquake occurrence, characteristics of seismograph, geological conditions and so on. Difference in any of these points, that originally comes from the complicated natures of earthquake motions may confuse the discussions. At present, information on the difference on the various points concerned are not enough to make clear the nature of the discordancy between (9) and (10), still less to reject (9) for (10).

To examine the propriety of the way of thinking, $A\sim\Delta$ and $T\sim\Delta$ relations of two earthquakes of different Magnitude, Shizuoka earthquake in 1935 an Izu earthquake in 1930, are studied assuming the formula (5) in common to their source spectra. To begin with, however, there are some problems to be solved.

5. Assumptions for numerical computation

Though the exact response of seismograph used by J. M. A. in those days are not sure, we may assume its period $T_s=5$ sec and damping constant $h^2=0.35$. As to k or $1/Q$, in which $k=\pi/vQ$, various results of studies by many authors are obtained. $k=0.0096\,\text{km}^{-1}$ sec was assumed in the previous paper. Afterward, an interpretation of apparent $1/Q$ was studied, and a characteristic internal friction $1/Q_0=0.004$ was obtained[4]; it appears that the interpretation fits also exactly the results of recent studies of $1/Q$ by Aki[5]. Generally speaking, there are some basic problems yet unsolved to accept at once those results of studies of $1/Q$. However, it would be accepted to assume, putting $v=1.5\sim2$ km/sec, $k=0.007$ sec/km for the present studies. The last problem is concerned with the estimation of energy ratio between the two earthquakes.

Table 1.

Station	Earthq.	Δ km	I	A(N)	A(E)	T(N)	T(E)	α(N)	α(E)
Kanazawa	S.	236	0	+1600	−1370	—	—	—	—
	K.	242	2	+1820	−1300	3.0	2.6	0.8	0.7
Nagoya	S.	135	3	+3600	−3150	2.5	2.5	2.3	2.0
	K.	137	2	+4000	+3600	1.9	1.9	4.4	3.9
Kyoto	S.	245	0	+1250	+ 544	2.5	2.4	0.8	0.3
	K.	55	5	−3600	+2150	0.7	—	29.3	—
Iida	S.	80	3	+1350	+1180	0.9	1.1	6.6	3.9
	K.	226	2	—	−1910	—	1.5	—	3.3
Takayama	S.	167	2	+ 820	−1280	3.0	3.0	0.3	0.5
	K.	230	2	−1200	+ 840	2.9	2.9	0.5	0.3
Kochi	S.	476	0	160	180	3.0	3.0	0.0	0.0
	K.	223	0	950	400	2.4	2.4	0.6	0.3
Choshi	S.	232	0	− 550	− 520	2.3	2.1	0.4	0.4
	K.	488	0	− 117	− 98	2.8	3.2	0.0	0.0

(*from* "Kisyō-Yōran" by J. M. A.)

S.: obs. for Shizuoka earthquake.
K.: obs. for Kawachi-Yamato earthquake.
I: Intensity. A: in μ. T: in sec. α: in gal.

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[4] R. Yoshiyama, *Bull. Earthq. Res. Inst.*, **38** (1960), 361-368.

[5] K. Aki, *Bull. Earthq. Res. Inst.*, **44** (1966), 23-72.

Magnitude of Shizuoka earthquake is as already stated 6.3 by J.M.A. Gutenberg does not give his estimation, but he gives an estimation as large as 6 to Kawachi-Yamato earthquake in 1936, whose Magnitude by J.M.A. is 6.4. To deduce the Magnitude of Shizuoka earthquake from that of Kawachi-Yamato earthquake, Table 1 is given. The table represents the observations of these two earthquakes by J.M.A., selected from its "Kisyō-Yōran". Intensity I in J.M.A. scale, "Maximum amplitude" A and its period T are the observations by J.M.A., while acceleration α_N and α_E are calculated by the formula $4\pi^2 A/T^2$. Though numerical reliability of these observations is doubtful, and, moreover, calculated acceleration may not be the maximum acceleration, Kawachi-Yamato earthquake is slightly but clearly bigger than Shizuoka earthquake, and the radius of the boundary where acceleration is 1 gal is 240 km for the former and 230 km for the latter. If we put these radii into the following formula of the relation between Magnitude and radius of perceptibility, r in km,

$$M = -3 + 3.8 \log_{10} r, \tag{12}$$

by Gutenberg and Richter, we obtain $M=6.0$ for both earthquakes: even if $r=250$ km, $M=6.1$ at most. Therefore, $M=6.0$ for Shizuoka earthquake seems one of reasonable estimation. On the other hand, Magnitude of Izu earthquake estimated by J.M.A. is 7.0, while Magni-

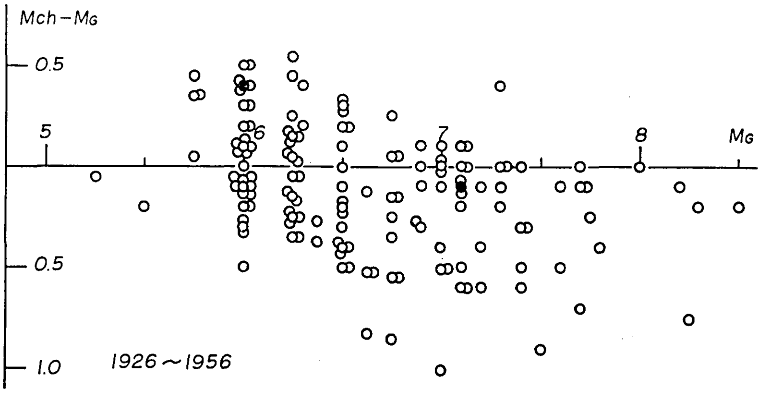

Fig. 3. Systematic deviations of Magnitude by J.M.A. from that by Gutenberg, calculated according to the "Catalogue of Major Earthquakes in and near Japan, 1926-1956" (1958) by J.M.A.

●: Shizuoka earthquake and Izu earthquake.

tude by Gutenberg is 7.1 and exceeds the estimation by J.M.A. contrarily to the case of Shizuoka and Kawachi-Yamato earthquakes. As it is shown in Fig. 3, there is a systematic difference between the Magnitude by J. M. A. and that by Gutenberg. Magnitude-estimation by J. M. A. being carried out on the Tsuboi's formula which was introduced to give none but the Magnitude by Gutenberg, the result that shows such a systematic difference seems something self-contradictory; it is perhaps due to a theoretical defect of the formula. And it seems in Fig. 3 that the differences of Magnitude between J. M. A. and Gutenberg in the two earthquakes, Shizuoka earthquake and Izu earthquake, is also nothing but systematic ones. Therefore, in spite of the estimation by J. M. A., it seems reasonable to adopt $M=6.0$ for Shizuoka earthquake and $M=7.1$ for Izu earthquake, especially for our studies, because the difference of the Magnitude or the ratio of energies of the two earthquakes are now concerned, and, moreover, because we use the Magnitude-energy formula originally suggested by Gutenberg and Richter. To get $M=7.1$ for Izu earthquake by the formula (12), relation between Magnitude and radius of perceptibility, we have to put $r=460\,\mathrm{km}\pm10\,\mathrm{km}$, while we put $r=230\,\mathrm{km}$ in Shizuoka earthquake and obtained $M=6$. From the observations in "Kisyō-Yōran" by J. M. A., it is difficult to find out a plausible reason to assume $r=460\,\mathrm{km}$. It is partly because of the unfavourable circumstances of observation in those days, but, chiefly, because of the complicated nature of a large earthquake, which makes the estimation of acceleration or velocity of the earthquake motion difficult.

All considered, we may conclude that the difference of Magnitude dM between Izu earthquake and Shizuoka earthquake is 0.7 by J.M.A. or 1.1 by Gutenberg. From the difference of Magnitude, ratio of the energies of the two earthquakes is calculated. There are various improved formulas of Magnitude-energy relation:

$$\log_{10} E=12+1.8M \tag{13}$$

$$\log_{10} E=9.4+2.14M-0.054M^2 \tag{14}$$

$$\log_{10} E=11.8+1.5M \tag{15}$$

$$\log_{10} E=7.2+2.0M \tag{16}$$

and it seems (15) is widely used in Japan. So that the minimum value of the ratio is 11 : 1 by J. M. A.; the largest estimation, 160 : 1, is obtained by (16) with $dM=1.1$. Sagisaka[6] estimated in 1940, at J. M. A., the energy of Izu earthquake $4.0 \cdot 10^{21}$ erg and the energy of Shizuoka earthquake $6.3 \cdot 10^{18}$ erg, from which the ratio turns out at about 630 : 1, decidedly bigger than any other estimation. Such a large difference even among the relative estimations of the two earthquakes, studied macroseismically, too, in detail by many seismologists indicates an unusual difficulty of the estimation of energies of earthquakes. In this paper, the ratio is assumed 50 : 1, a little bigger than that estimated, with $dM=1.1$, from (14) or (15), but much less than that from (13).

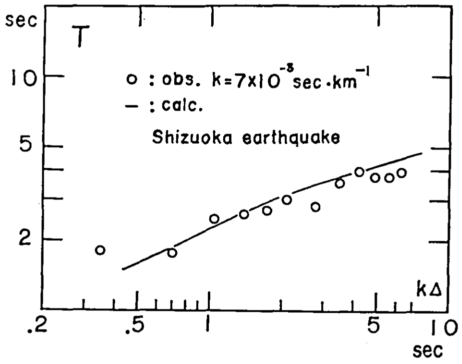

Fig. 4. Fitting to the observations of $T \sim \varDelta$ relation in Shizuoka earthquake to determine α and β.

6. $A \sim \varDelta$ and $T \sim \varDelta$ relations of the two earthquakes

From the previous paper, observed $T \sim \varDelta$ relation of Shizuoka earthquake is reproduced in Fig. 4, taking, however, $k\varDelta$ instead of $\varDelta$ in the abscissa. Putting T_0, period of the " Maximum amplitude " at the epicentre, equal to 1 sec, we get $\alpha=2$ sec^{-3}. From the observations of T at the distances $\varDelta=100$ km$\sim$500 km, $\beta=0.01$ sec^{-2} is assumed; the calculated $T \sim k\varDelta$ relation is given by a curve in the figure. On the other hand, observations of $\sqrt{\varDelta}A \sim k\varDelta$ relation is given by Fig. 5, being calculated from Fig.4

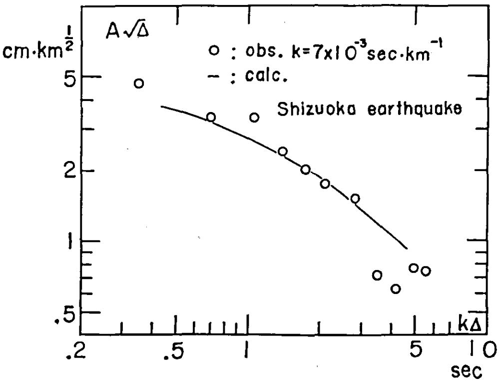

Fig. 5. Using α, β, that were determined from $T \sim \varDelta$ relation in Fig. 4, and γ, determined to fit the observation at $\varDelta=250$ km or $k\varDelta=1.75$, calculated $\sqrt{\varDelta}A \sim k\varDelta$ relation is compared with the observations.

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[6] K. SAGISAKA, *Kenshin-zihō* (*Quart. Jour. Seism.*), J. M. A., **10** (1940), 385-448. J. M. A. was called C. M. O. in those days.

of the previous paper. Using the above obtained $\alpha=2\,\mathrm{sec}^{-3}$ and $\beta=0.01\,\mathrm{sec}^{-2}$, and putting, to fit the observations at a mean distance of above stated 100 km and 500 km, $\gamma=9.1\,\mathrm{cm}\cdot\mathrm{sec}^{-1}\cdot\mathrm{km}^{1/2}$, $\sqrt{\Delta}A\sim k\Delta$ relation is calculated and is given by a curve in the figure. That the curve in Fig. 5 almost agrees, as it seems, to the observations for a certain range of distances may give a support to the introduced basic assumptions; we may have a better fitting by a slight improvement of α, β or formula $\xi(\bar{\varepsilon})$.

Now the problem is to deduce from these results the theoretical $A\sim\Delta$ and $T\sim\Delta$ relations of Izu earthquake to compare with the observations.

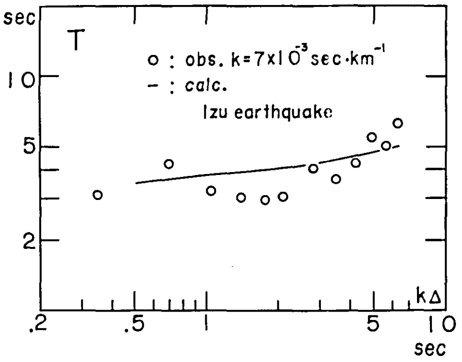

Fig. 6. Using α and β, determined for Izu earthquake from those for Shizuoka earthquake, calculated $T\sim\Delta$ relation are compared with the observations.

If we assume 50 : 1 as the energy ratio of the two earthquakes, $\alpha=0.04\,\mathrm{sec}^{-3}$ and $\beta=0.0002\,\mathrm{sec}^{-2}$ for Izu earthquake, while γ being expected to remain unchanged is $9.1\,\mathrm{cm}\cdot\mathrm{sec}^{-1}\cdot\mathrm{km}^{1/2}$. The results of calculations are shown in Fig. 6 and Fig. 7 together with the observations, being reproduced or calculated from Fig. 4 and Fig. 12 in the previous paper.

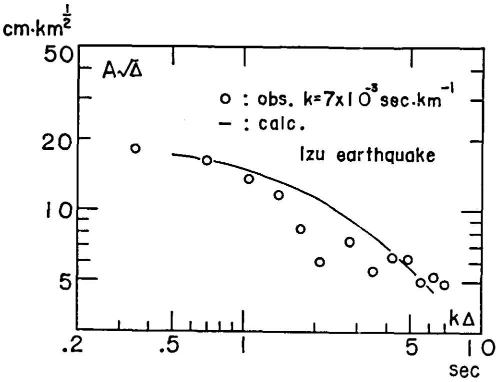

Fig. 7. $\sqrt{\Delta}A\sim k\Delta$ relation is calculated and is compared with the observations for Izu earthquake.

Concerning the amplitudes at short epicentral distances in a destructive earthquake such that $\Delta<100$ km, data from seismometrical observations are few in spite of their importance; data available are generally obtained from macroseismic studies in the epicentral region, which may suggest seismic intensities more severe than those extrapolated from the theoretical curves at large distances. Theoretical curve may be much improved for short distances to fit those observations, if we assume another system of spectrum with large γ-value, using, however, propagation-exponent of bodily wave $n_0=1$ instead of $n_0=1/2$.

Taking the incompleteness of observation into account, it would be too hasty to discuss in detail at present the concordancy or discordancy between the observations and calculations at short epicentral distances. Moreover, strictly speaking, the problem in the epicentral region cannot be solved by mere propagation of waves.

7. Concluding remarks

In spite of various kinds of formulas, absolute estimation of energy of an earthquake is difficult. If we assume the energy of Shizuoka earthquake $E=10^{20}$ erg and $v=1.5$ km/sec, $\alpha'=7\cdot10^{-5}\,\mathrm{cm}^3\cdot\mathrm{erg}^{-1}$, $\beta'=70\,\mathrm{cm}^{-3}\cdot\mathrm{erg}\cdot\mathrm{sec}$ and $\gamma'=150\,\mathrm{cm}^3\cdot\mathrm{sec}^{-1}\cdot\mathrm{erg}^{-1/2}$: β' is proportional, while α' is reciprocally proportional, to the assumed energy of Shizuoka earthquake, and γ' is reciprocally proportional to the square root of the assumed energy. Putting these constants into (4), we have

$$\dot{u}(T)=150\sqrt{\frac{\bar{\varepsilon}-70/T}{1+7\cdot10^{-5}\bar{\varepsilon}}}\,\mathrm{cm}^{3/2}\cdot\mathrm{sec}^{-1}\,, \qquad \bar{\varepsilon};\ \text{in erg}\cdot\mathrm{cm}^{-3}\,,$$

that may be examined by a theoretical reasoning and by a laboratory experiment, and the basic assumptions may be improved.

Though the problem is not yet solved, the writer hopes that the way of thinking in this paper may throw some light on a theoretical study of source spectrum of an earthquake and on a study of presumable spectrum of earthquake motion against earthquake damages.

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2. 最大振幅と震央距離との関係について（第2報）

—震源スペクトルの考察—

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前報告では所謂破壊的地震について $A\sim\Delta$, $T\sim\Delta$ 関係をしらべ，その代表的な2つの地震について震源スペクトルとも云うべきものを出した．ここに再び同じ問題をとり上げる．

前回はそれぞれの地震について独立に考えたが今回は推理をすすめて共通の基本的仮定に立って処理することが出来るかどうかを見るのが目的である．それが可能ならば大地震に予想されるスペクトルを理論的に決定することが出来る．調査対象の地震としては，震央が陸上にあって，近距離に最大振幅並びに周期の観測があり，又被害調査からの資料も豊富で，エネルギーの少くとも相対的な推定値に確度の高いこと，しかも大いさが出来るだけ違うことが必要である．之等の観点から1つは前回の調査にもある静岡地震，他は昭和5年の伊豆地震をえらんだ．伊豆地震の頃は地震計に不備な点が多かったが，時期が静岡地震と接近し，それとの比較調査に都合がよい．又被害調査も多くの人々によってなされ，エネルギーの過大評価，過小評価の危険が少くない様に思はれる．

基本的に設定した仮定は考えの進みに従って4段階に示される．それにはまだ未熟な点もあるが，結果について云えば形式的には一応成功した様に思はれる．観測を支配する外部要素にまだ研究の十分でないものも数多く，従って所謂マグニチユードの推定にすら種々の困難があり，ましてエネルギー推定の方法も確立されていないのであるから，この種の研究には或る程度循環論的なあいまいさがつきまとうことは止むを得ない．

今後更にここに設定した仮定の物理学的意義を見出すためには理論的な検討が必要であるが，又観測値自体にも物理学的な意義が考慮された精度の高い資料が先づ必要であることは云うまでもない．