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17. *Regional Variations in Magnitude-Frequency Relation of Earthquakes.*

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1. Introduction

During the last decade a number of papers have been devoted to the magnitude-frequency relationship of earthquakes. The statistical formula proposed by Gutenberg and Richter[1] is:

$$\log n = a + b(8-M)\,. \tag{1}$$

The constant a depends on the periods of observation, the size of the investigated region, and the seismic activity, so it exhibits a significant variation from study to study. The coefficient b is more stable (about 1.0). Ishimoto and Iida[2] also obtained the following statistical formula:

$$n(a)da = n_0 a^{-m} da\,, \tag{2}$$

where a is the maximum trace amplitude of an earthquake, $n(a)da$ is the number of earthquakes having a maximum trace amplitude from a to $a+da$, and n_0 and m are constants. The high applicability of this formula has been established by many observations on relatively small earthquakes, such as aftershocks and earthquake swarms, in Japan[3,4]. Under some conditions, the formula (2) is equivalent to the formula (1), the relation between b and m being expressed by $m=b+1$.

Many observations have shown that the coefficient m or b is nearly similar in various cases and within a very wide range of earthquake magnitude. This constant b hypothesis has been supported by Suzuki[5], Isacks and Oliver[6], and other investigators.

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[1] B. GUTENBERG and C. F. RICHTER, *Seismicity of the earth and associated phenomena* (Princeton Univ. Press, 1954).

[2] M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 433-478.

[3] Z. SUZUKI, *Sci. Rep. Tôhoku Univ.*, [V], **5** (1953), 177-182; **6** (1955), 105-111; **10** (1958), 15-27; **11** (1959), 10-54.

[4] T. ASADA, *J. Phys. Earth*, **5** (1957), 83-113.

[5] Z. SUZUKI, *loc. cit.*, 3).

[6] B. ISACKS and J. OLIVER, *Bull. Seism. Soc. Amer.*, **54** (1964), 1941-1979.

On the other hand, an appreciable difference of the b value among three areas in and near Japan has been remarked by Tsuboi[7]. Further, Miyamura[8] discussed the question that on a world-wide area the b value of large shallow earthquakes varies appreciably among different tectonic areas. Kárník[9] and Duda[10] also obtained different b values in various regions in Europe and Pacific seismic regions, respectively. However, Isacks and Oliver[11] suggested that a large part of the variation of the b value from region to region can be attributed to statistical fluctuation or observational uncertainties. Katsumata[12], who studied recent seismic data in and near Japan, did not find any marked difference in the b value among several seismically active regions. On the other hand, Ichikawa[13] noted some difference of b values for different regions using recent materials. Notwithstanding numerous papers on the subject, conclusion on regional variations of the b value is left for future studies.

The physical significance of the regional variation in the magnitude-frequency relation is suggested from laboratory experiments on fracture, based on the fracture hypothesis of earthquakes. From this standpoint, elastic shocks accompanying local fractures of heterogeneous brittle solids have been measured by the acoustic method[14]. Their magnitude-frequency curves were quite similar to these of natural earthquakes. The $\log n - \log a$ relation is linear for heterogeneous solids having irregular structures, the constant m increasing as the degree of heterogeneity increases. Therefore, it may be expected that the b or m values of shallow earthquakes have a relation to structural states of the earth's crust. And yet it may not be so easy to detect the regional variations due to structural states, because the b or m value is not very sensitive to structure. Another approach is made from the shape of the magnitude-frequency curve[15] rather than the b value obtained from the rectilinear approximation of the curve. According to fracture experiments, the $\log n - \log a$ curve is often concave downward for solids having some regular structures. Particularly, in a block structure, the curve is composed of two straight lines intersecting at the amplitude, which is

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[7] C. TSUBOI, *J. Phys. Earth*, **1** (1952), 47-54.

[8] S. MIYAMURA, *Proc. Jap. Acad.*, **38** (1962), 27-30.

[9] V. KÁRNÍK, *Bull. Inter. Inst. Seism. Earthq. Eng.*, **1** (1964), 9-32.

[10] S. J. DUDA, *Tectonophysics*, **2** (1965), 409-452.

[11] R. ISACKS and J. OLIVER, *loc. cit.*, 6).

[12] M. KATSUMATA, *J. Seism. Soc. Japan* (*Zisin*), [ii], **18** (1965), 219-234; **19** (1966), 1-10.

[13] M. ICHIKAWA, *Papers in Meteor. and Geophys.*, **16** (1965), 104-156.

[14] K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 125-173; 831-853.

[15] K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 831-853.

related to the unit dimension of the block structure (Fig. 6). These results suggest a possibility that the structural states in seismic regions are also deduced from the curve shape. In this paper, the regional variations in the magnitude-frequency relation are discussed by observing the shape of recurrence diagram.

2. Procedure of investigation

From the above-mentioned point of view, the magnitude-frequency relations of shallow earthquakes in and near Japan were investigated. Japan and its neighboring area contains tectonically different regions, and numerous seismic data are available. The data were principally adopted from "Catalogue of major earthquakes which occurred in and near Japan (1926–1956)" published by JMA or Japan Meteorological Agency. In addition to this, data on the largest earthquakes were adopted from "Rikanenpyô".[16] Earthquakes deeper than 60 km were excluded in the present discussion. The principal procedure of investigation is described below.

(1) *Division of the area*

Japan and its neighboring area was divided into eight areas (Fig. 1).

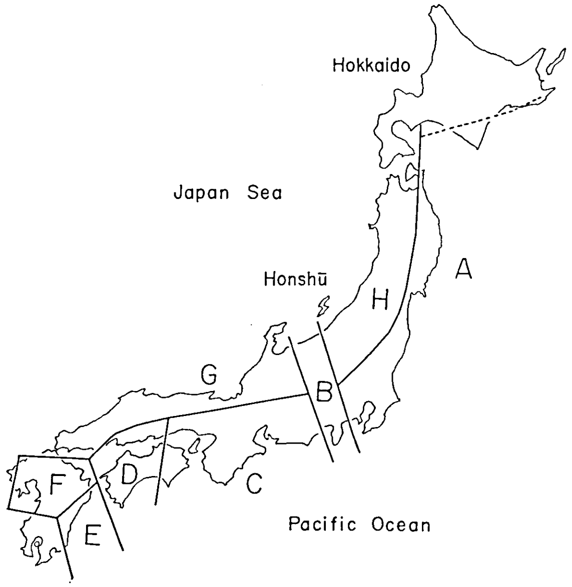

Fig. 1. Division of Japan and its neighboring area.

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[16] Rikanenpyô (Science Calendar, Tokyo Astronomical Observatory) ed. by T. Hagiwara (Maruzen, 1967).

To discover significant variations in the magnitude-frequency relation from region to region, suitable division of the area is essentially important. In a previous papar[17], the present author divided the area by the degree of fracturing of the earth's crust estimated from the occurrence of different types of earthquake sequences. These structural features of the earth's crust are probably one of the most important factors, effective to the magnitude-frequency relation of shallow earthquakes, so that this division was also adopted in this study with a slight modification.

(2) *New expression of the magnitude-frequency relation of earthquakes*

For emphasizing differences among the recurrence curves, a new

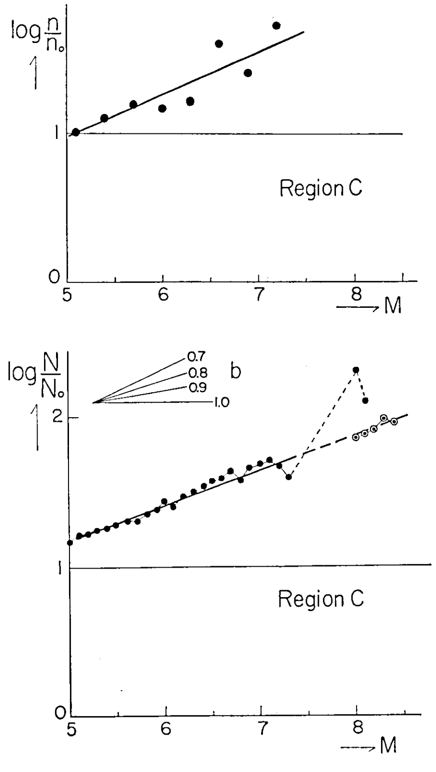

Fig. 2. New expression for magnitude-frequency relation. n: earthquake frequency, n_0: earthquake frequency in the standard case (b=1). N and N_0: accumulated frequencies corresponding to n and n_0, respectively. closed circle: from the Catalogue by J.M.A. double circle: from historical data.

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[17] K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 615-658.

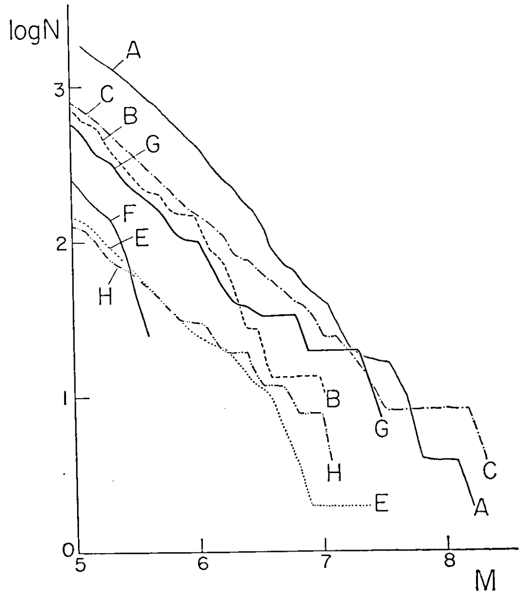

Fig. 3. Current frequency diagrams for different regions in and near Japan.

expression of the magnitude-frequency relation was used. As mentioned above, since most curves of $\log n$ vs. M are approximated by a straight line with a constant slope ($b=1$), this curve was taken as the standard curve. Then, the deviation of each curve from the standard curve was shown by plotting the ratio of the observed frequency n to the frequency n_0 for the standard curve in logarithmic scale against the magnitude. In the present study, to reduce a statistical fluctuation of earthquake frequency, the following accumulated frequency N is used instead of the frequency n:

$$N(M)=\int_{M}^{\infty} n(M)dM\,. \qquad (3)$$

The ratio $N(M)/N_0(M)$ in logarithmic scale is plotted against M, where $N_0(M)$ is the accumulated frequency for the standard curve (Fig. 2). This diagram is more advantageous for expressing more explicitly the regional variations in the magnitude-frequency relation than the current diagram (Fig. 3).

(3) *Use of historical data*

As shown by the formulas (1) and (2), the frequency of earthquakes markedly decreases as their magnitude increases. Therefore, the fre-

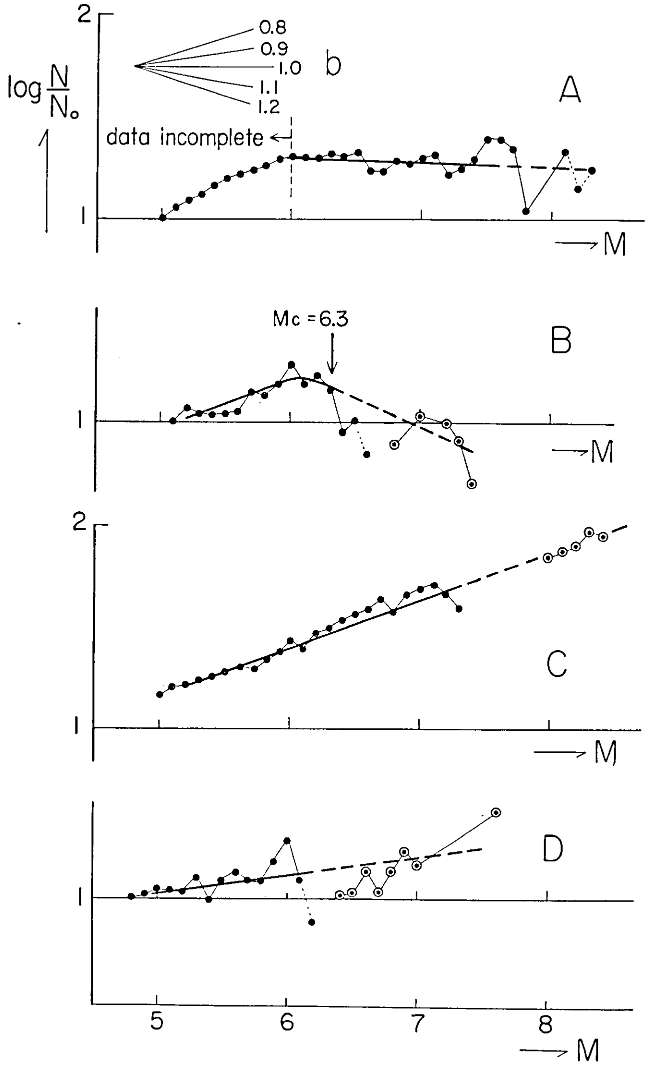

Fig. 4 (a). Log $N(M)/N_0(M)$ vs. M curves for eight regions in and near Japan. Arrows indicate breaking points of these curves, and numerals show the critical magnitude. closed circle: from the Catalogue by J.M.A. double circle: from historical data.

quency of larger earthquakes is accompanied by a large statistical fluctuation. To reduce the fluctuation, historical data on largest earthquakes over a long period are useful under the assumption that earthquake occurrence is nearly stationary during the period. Since data

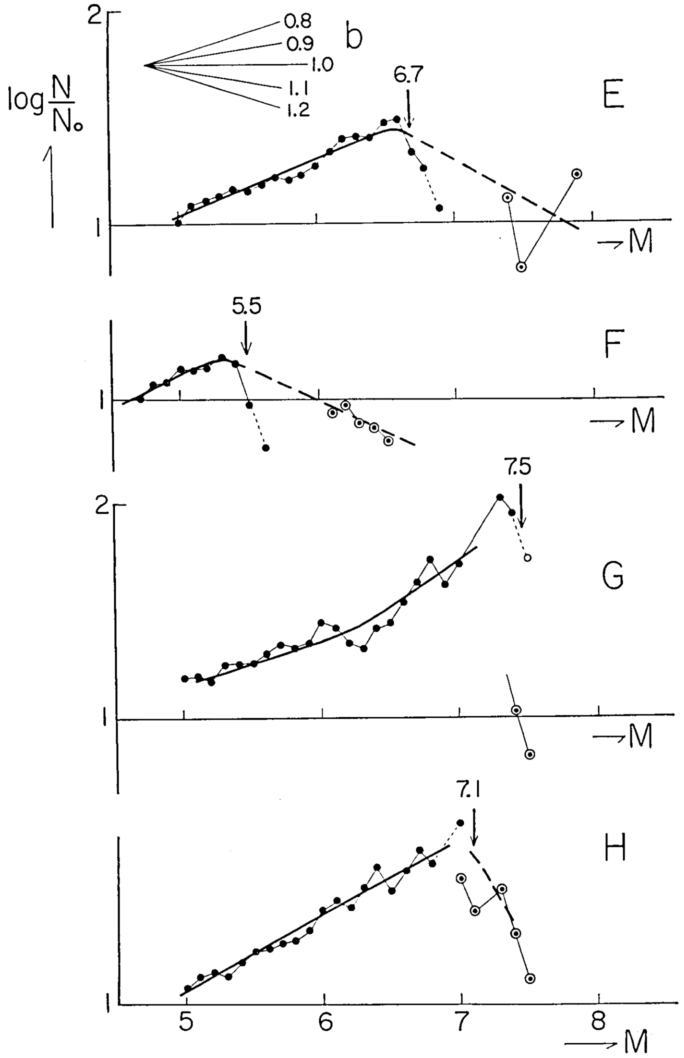

Fig. 4 (b). Log $N(M)/N_0(M)$ vs. M curves for eight regions in and near Japan. Arrows indicate breaking points of these curves, and numerals show the critical magnitude. closed circle: from the Catalogue by J.M.A. double circle: from historical data.

during the last two hundred years seem to be nearly homogeneous in the land area in Japan[18], the annual number of larger earthquakes was estimated from data during the period from 1700 to 1966 (Fig. 2).

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[18] Rikanenpyô, *loc. cit.*, 16).

3. Results

The logarithm of $N(M)/N_0(M)$ vs. M diagrams for eight areas are shown in Figs. 4(a) and (b). Solid circles are obtained from the "Catalogue of major earthquakes" and double circles are obtained by adding the historical data on large earthquakes (1700–1926). From these figures, the following results are obtained.

(1) In Region **A**, the curve can be approximated by a straight line for the magnitude 6 and larger. The b value of this region is about 1.0.

(2) The curves for Regions **C** and **D** are also approximated by a

Region	Mc	D.F.
F	5.5	1
B	6.3	1
E	6.7	2
G	7.5	2
H	7.1	2
A	>8	$3\sim4$
C	>8	$3\sim4$
D	>7	4

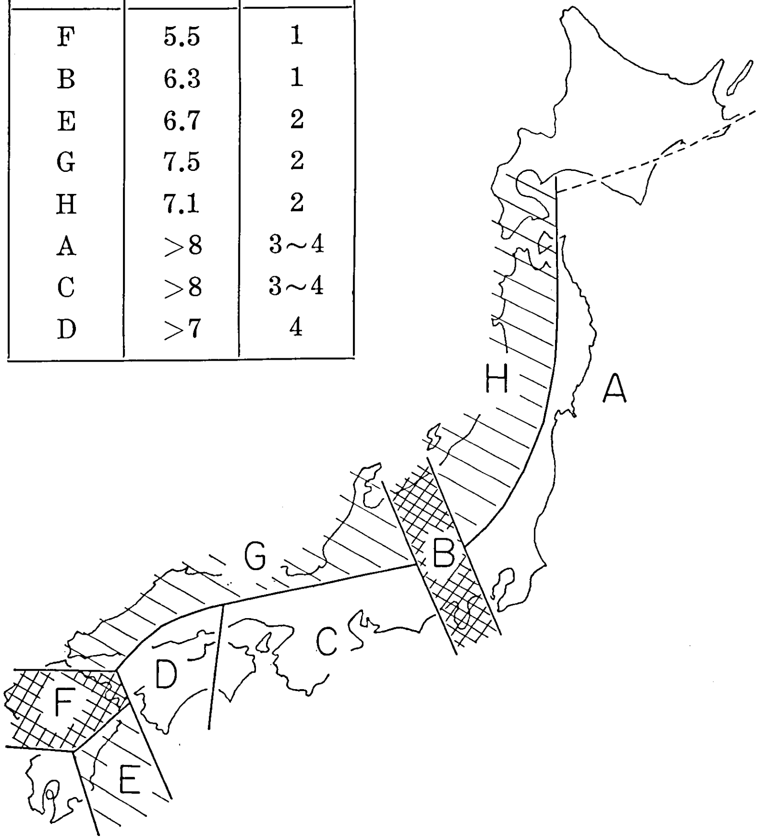

Fig. 5. Relation between the degree of fracturing ($D.F.$) deduced from occurrence of various earthquake sequences, which are schematically shown in the map, and the critical magnitude (Mc) indicated in frequency diagrams.

straight line including historical data. The b value in Region C is appreciably smaller than 1.0.

(3) The curves for other regions (**B**, **E**, **F**, **G**, and **H**) cannot be expressed by a straight line, but roughly by two branches with different slopes. The gradient of the right branch, which is clearly smaller than that of the left branch, is usually negative. The arrows in these figures indicate the intersecting point of two branches. The critical magnitude Mc corresponding to the intersecting point is different from curve to curve. The Mc value in each region seems to have some relation to the structural states of these regions estimated previously from earthquake sequences[19] (Fig. 5). The magnitude Mc is smallest in the highly fractured regions (**B** and **F**), intermediate in the next fractured regions (**E**, **G**, and **H**), and largest in the homogeneous regions (**A**, C, and **D**). This noticeable correlation between frequency diagrams and structural states of the earth's crust will be discussed below.

4. Discussion

Frequency diagrams in these areas are often composed of two branches with different slopes. Kárník[20] and Duda[21] also reported similar non-linear diagrams. As mentioned above, the physical meaning of such non-linear diagrams is suggested from fracture experiments in the laboratory[22]. According to laboratory experiments, the curve composed of two branches with different slopes is obtained in materials having highly regular structures (Fig. 6). One of the probable regular structures in the earth's crust may be a block structure of which boundaries correspond to structural discontinuities, such as faults. If the earth's crust in each region has a simple block structure, the critical magnitude Mc in the diagram may correspond to the magnitude of the earthquake of which the energy is equal to the maximum strain energy accumulated in the unit block. The linear dimension (L) of the unit block corresponding to the critical magnitude (Mc) may be estimated from the following statistical relation[23] between the magnitude (M) of an earthquake and the linear dimension (L) of its aftershock area, under the assumption that the energy of the earthquake is accumulated as strain energy in the aftershock region (Fig. 7):

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19) K. Mogi, *loc. cit.*, 15).

20) V. Kárník, *loc. cit.*, 9).

21) S. J. Duda, *Ann. Geofis.* (Rome), **18** (1965), 365-397.

22) K. Mogi, *loc. cit.*, 14).

23) T. Utsu, *Geophys. Mag.*, **30** (1961), 521-605.

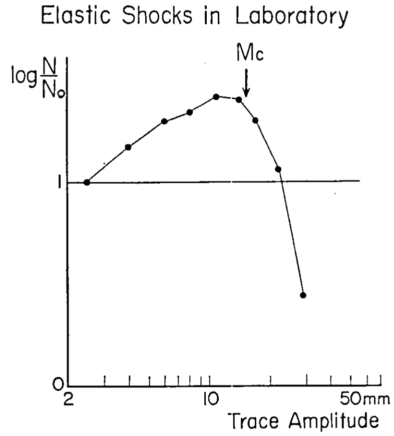

Fig. 6. Log N/N_0 vs. trace amplitude for elastic shocks accompanying local fractures of a material having a block structure. The magnitude *Mc* corresponding to the critical amplitude is a function of the linear dimension of the unit block.

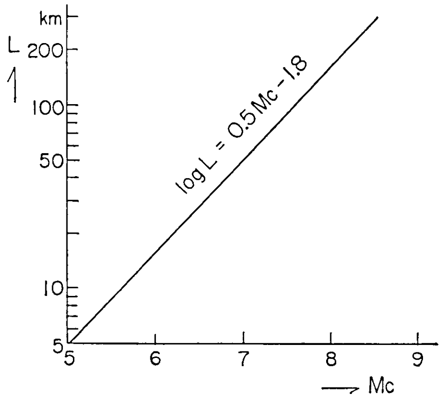

Fig. 7. Linear dimension (L) of a unit block in a regular structure vs. the critical magnitude (Mc) in the corresponding frequency diagram.

$$\log L = 0.5M - 1.8. \tag{4}$$

The computed unit dimension was about 9 km for Region **F**, about 22 km for Region **B**, and larger than 150 km for Regions **A** and **C**. The areal density of boundaries of blocks is inversely proportional to the linear dimension (L) of the unit block. If the structural discontinuities correspond to faults, this density shows the degree of fracturing in these regions. This result is schematically shown in Fig. 8. This is roughly consistent with the structural states[24] in Fig. 5. Thus, the high correlation between *Mc* and the degree of fracturing, deduced from the occurrence of earthquake sequences, can be reasonably explained.

The relation between $\log N(M)/N_0(M)$ for the whole area in and near Japan are shown in Fig. 9. This curve is quite similar to that of Region *A*, shown in Fig. 4(a). This derives from the fact that a large number of earthqnakes in the whole area occurred in Region *A*. The curve for total earthquakes does not correspond to the average structural state of the area, but that of the most active region (A). Thus, a suitable division of an area is very important in discussing regional variations in recurrence curves.

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[24] K. Mogi, *loc. cit.*, 15).

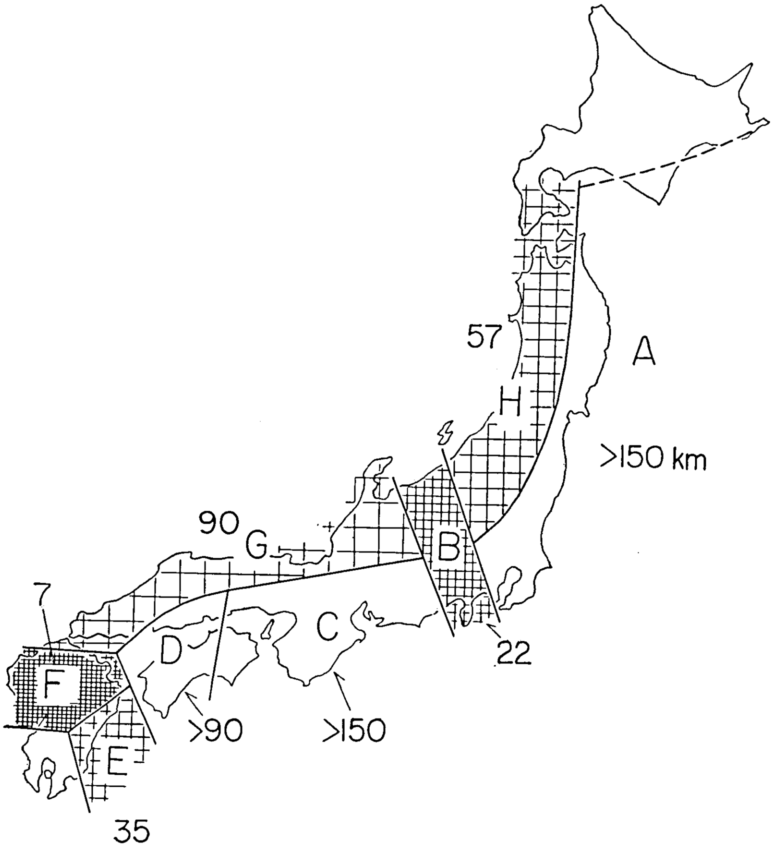

Fig. 8. The degree of fracturing of the earth's crust in and near Japan, deduced from the magnitude-frequency diagram of earthquakes. Numerals show the linear dimension of the unit block in regular structures.

Two other examples of the frequency diagrams obtained by the present expression are also represented in Fig. 9. The curve for *southern California* (1912-1963)[25] can be approximated by a straight line, the *b* value being about 1.0. Allen and others[26] studied the current diagrams separately for six areas in this region. According to their results, the *b* value is nearly the same (1.0) in these areas except for the Los Angeles basin. This suggests mechanical homogeneity throughout the region, as mentioned by Allen and others.

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[25] C. R. ALLEN, P. St. AMAND, C. F. RICHTER, and J. M. NORDQUIST, *Bull. Seism. Soc. Amer.*, **55** (1965), 753-797.

[26] C. R. ALLEN, P. St. AMAND, C. F. RICHTER, and J. M. NORDQUIST, *loc. cit.*, 25).

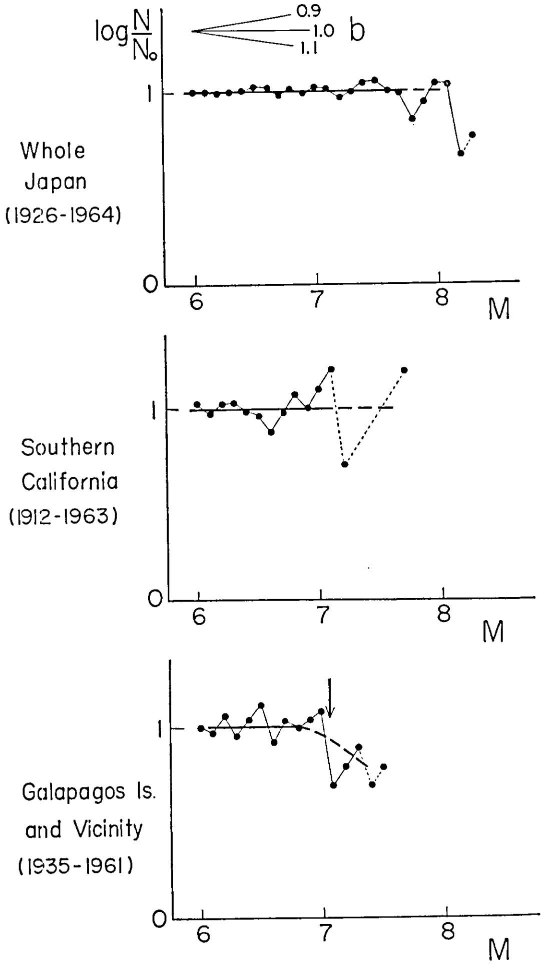

Fig. 9. Log $N(M)/N_0(M)$ vs. M curves for the whole Japan (1926-1964), southern California (1912-1963), and Galapagos Is. and vicinity (1935-1961).

The curve for *Galapagos Is. and vicinity*[27], including oceanic rises, cannot be approximated by a single straight line. The critical magnitude is about 7. This result suggests a fractured structure, which is supported by geological evidences[28].

In conclusion it can be stated that the shape of the recurrence diagrams seems to have a close relation to structural states of the earth's crust.

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[27] H. K. Acharya, *Bull. Seism. Soc. Amer.*, **55** (1965), 609-617.

[28] H. W. Menard, *Marine geology of the Pacific* (McGraw Hill, 1964).

17. 地震の規模別頻度分布の地域性

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地震の規模別頻度分布の地域性については，これまで多くの研究がなされている．これらの研究は，$\log n-M$ 曲線が一つの直線であらわされるとして，その係数 b について論じたものであるが，まだはつきりした結論に達していない．本論では，$\log n-M$ 曲線が必ずしも一つの直線であらわされないことに注目して，その形の地域的差異を論じた．主に，日本およびその周辺を取扱つたが，その結果は次の通りである．

（1） 日本を8地区に分割して，$\log n-M$ 曲線を作成した．

（2） 地域的差異を明瞭に表わすために，$b=1$ の場合の直線を標準曲線として，これからのくいちがいを表わす新しい表現法を用いた．

（3） 頻度曲線の直線性からのずれは，M の大きい範囲であらわれるが，この範囲では，頻度のバラツキが大きい．これを減らすために，大きい地震については1700年以来の地震の歴史的記録をも用いた．

（4） 頻度曲線は，一つの直線で表わされる場合もあるが，途中で急に折れ曲がる場合が少なくない．この折れ曲がり点の M の値に着目すると，これらの地域の構造状態と明瞭な相関を示した．

（5） 破壊実験からの類推により，頻度曲線の折れ曲がりから，地下の構造が推定されたが，他の地震資料から得られている結果とかなり符号する．

以上，頻度曲線の形が地域の構造状態と共に系統的にかわることが見出された．