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# 16. *An Empirical Formula for the Spectrum of Strong Earthquake Motions. II.*

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From the previous investigations, it has been known that not only the spectra of earthquake motions including destructive ones have similar shapes at respective places[1] but also that the predominant periods of earthquake motions in which values of magnitude are larger than about five take the constant values at each place.[2] It has also been ascertained that the maximum values of the strong earthquake motion records obtained in the U. S. A. coincide well with the calculated values by means of an empirical formula for the spectrum of earthquake motion at ground surface.[3]

In this paper, the seismic characteristics of ground will be investigated by comparing the maximum amplitudes of the strong earthquake motion records to the calculated values of the amplitudes at bed rock by using an empirical formula.[4]

The comparisons of the maximum values of recorded acceleration of the earthquakes and the calculated values of acceleration at bed rock by substituting the values of magnitude, hypocentral distance and period corresponding to the actual maximum acceleration of each earthquake in equation (1)[5] are represented in Tables 2 and 3.

$$a_0=\frac{1}{T}10^{0.61M-1.73\log x+0.13} \tag{1}$$

in which $a_0$, $T$, $x$ and $M$ represent the acceleration amplitude in cm/sec$^2$ at bed rock, the period of waves in sec, the hypocentral distance in

1) K. KANAI, "On the Spectrum of Strong Earthquake Motions," *Bull. Earthq. Res. Inst.*, **40** (1962), 71-90.

2) K. KANAI, "On the Predominant Period of Earthquake Motions," *Bull. Earthq. Res. Inst.*, **40** (1962), 855-860.

3) K. KANAI, "An Empirical Formula for the Spectrum of Strong Earthquake Motions," *Bull. Earthq. Res. Inst.*, **39** (1961), 85-95.

4) *ditto*, 3).

5) See appendix.

km and the magnitude of an earthquake, respectively.

It will be seen in Tables 2 and 3 that the ratios of the observed values on ground surface to the calculated values on bed rock at each place, that is, Kushiro and Earthquake Research Institute in Japan and Vernon, Los Angeles Subway Terminal, El Centro and Ferndale, California in the U. S. A. have equal values without regard to intensity of earthquake motions. Therefore, it may be considered that the ratios mentioned above correspond to the magnification constants of earthquake motions in the ground at each place.

The values of the ratio are 2–5 at the ordinary places and more than ten at particular places like Kushiro.

The value of the maximum acceleration of the Hiroo-Oki earthquake of April 23, 1962 observed at Kushiro Meteorological Observatory is the largest one among the earthquake motions which had already been recorded

Table 1. Damage caused by the Hiroo-Oki earthquake in Kushiro area.

| | | |
|---|---|---|
| Only one lightly wounded man. | | |
| No. of damaged houses | half destroyed | 2 |
| | inclined | 2 |
| | plaster dropped or cracked | 41 |

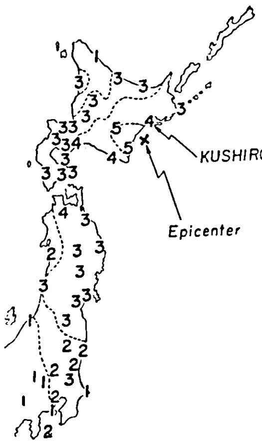


Fig. 1. Isoseismal map of the Hiroo-Oki earthquake of April 23, 1962.

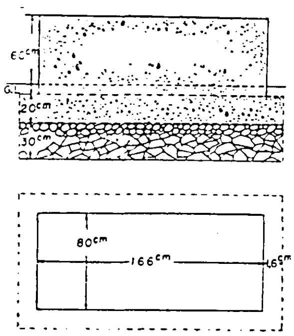


Fig. 2. Foundation of a SMAC seismograph installed at Kushiro Meteorological Observatory.

in the world. (See Fig. 3.[6]) Nevertheless, the damage to the structures caused by the earthquake had been slight as shown in Table 1. Such a phenomenon can be explained qualitatively from the peculiarity of the seismic characteristics of ground. On the ground where the magnification constant is particularly large, vibrations of large acceleration are found only within a narrow range of period because the formation of ground in elasticity is very simple and also the impedance ratio of the ground to the bed rock is very small. Consequently the period of a house will become longer at large amplitudes of earthquake motion and the amplitude of the house stops increasing even if the houses have the natural period which coincides with the predominant period of earthquake motion.

Anyway, the reason of the relation between the large acceleration and the slight damage remains to be treated as one of the most important problem on earthquake engineering in future.

Next, the results of some investigations concerning this problem will be described.

The isoseismal map as well as the epicenter of the earthquake are shown in Fig. 1. A SMAC seismograph is installed on the concrete block as shown in Fig. 2 in the basement floor of the Kushiro Meteorological Observatory of a three-storied mortary wooden-framed house.

The period distribution curves of microtremors observed on the bed of the SMAC seismograph and the ground near the house are shown in Figs. 4 and 5.

The period distribution curves of three earthquakes, that is, those of February 21, April 23 and July 18, 1962, observed here are also shown in Figs. 6–8.

Figs. 4–8 tell us that the predominant periods of all of these three earthquakes and microtremors on the ground are the same values of about 0.3 sec and those of microtremors on the bed have different values of about 0.25 sec.

The periods of the maximum amplitudes of the results of Fourier's analysis of February 21 and April 23 earthquakes are about 0.25 sec and 0.3 sec, respectively.[7] At any rate, it should be born in mind that not only the periods of the so-called predominant period of many of

6) Strong Motion Earthquake Records in Japan, Special Publication Vol. 1, Strong-Motion Earthq. Obs. Comm., Nov., 1962.

7) H. KAWASUMI and E. SHIMA, "Standard Strong Earthquake Motion for the Use in Antiseismic Designing", *Proc. Japan National Symposium Earthq. Engg.*, Tokyo (1962), 18, Figs. 7 and 10.

the earthquake motions but also the period of the maximum amplitude of the Fourier's spectrum of the largest earthquake coincide well with the natural period of the ground.

From the present investigation, it has been ascertained that the feature of destructive earthquake motions depends mostly on the seismic characteristics of ground.

In conclusion, we wish to express our sincere thanks to Mr. S. Amamiya of the Kushiro Meteorological Observatory and Mr. M. Saito of Akashi Seisakusho Ltd. for their contributions to the present investigation.

Table 2. The comparision of the maximum values of the recorded accelerations of the earthquakes in Japan and the calculated values of acceleration at the bed rocks.

| Station | Earthquake | | | | | | Acceleration (gal) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Date | Hypocenter N | Hypocenter E | Hypocenter h (km) | $M$ | Period (sec) | $A$. Calculated (bed rock) | Orientation | $B$. Observed (surface) | $B/A$ |
| Kushiro | II 21, '62 | 42.7 | 145.5 | 60 | 6.2 | 0.27 | 9 | EW | 104 | **11.5** |
| | | | | | | | | NS | 83 | 9.0 |
| | IV 23, '62 | 42.4 | 143.8 | 40 | 7.0 | 0.35 | 31 | EW | 340 | **11.0** |
| | | | | | | | | NS | 190 | 6.0 |
| | VII 18, '62 | 42.8 | 145.2 | 40 | 5.6 | 0.30 | 6 | NS | 68 | **11.0** |
| | | | | | | | | EW | 48 | 8.0 |
| Sendai | IV 30, '62 | 38.8 | 140.9 | 10 | 6.4 | 0.30 | 30 | NS | 42 | **1.5** |
| | | | | | | | | EW | 40 | 1.5 |
| Earthq. Res. Inst. | II 14, '56 | 35.7 | 139.9 | 50 | 6.0 | 0.30 | 23 | NS | 51 | **2.0** |
| | | | | | | | | EW | 50 | 2.0 |
| | IX 30, '56 | 35.5 | 140.2 | 70 | 6.75 | 0.40 | 21 | NS | 37 | **2.0** |
| | | | | | | | | EW | 35 | 1.5 |
| D. Bldg. in Tokyo | 〃 | 〃 | 〃 | 〃 | 〃 | 0.63 | 13 | NS | 44 | **3.5** |
| | | | | | | | | EW | 38 | 3.0 |

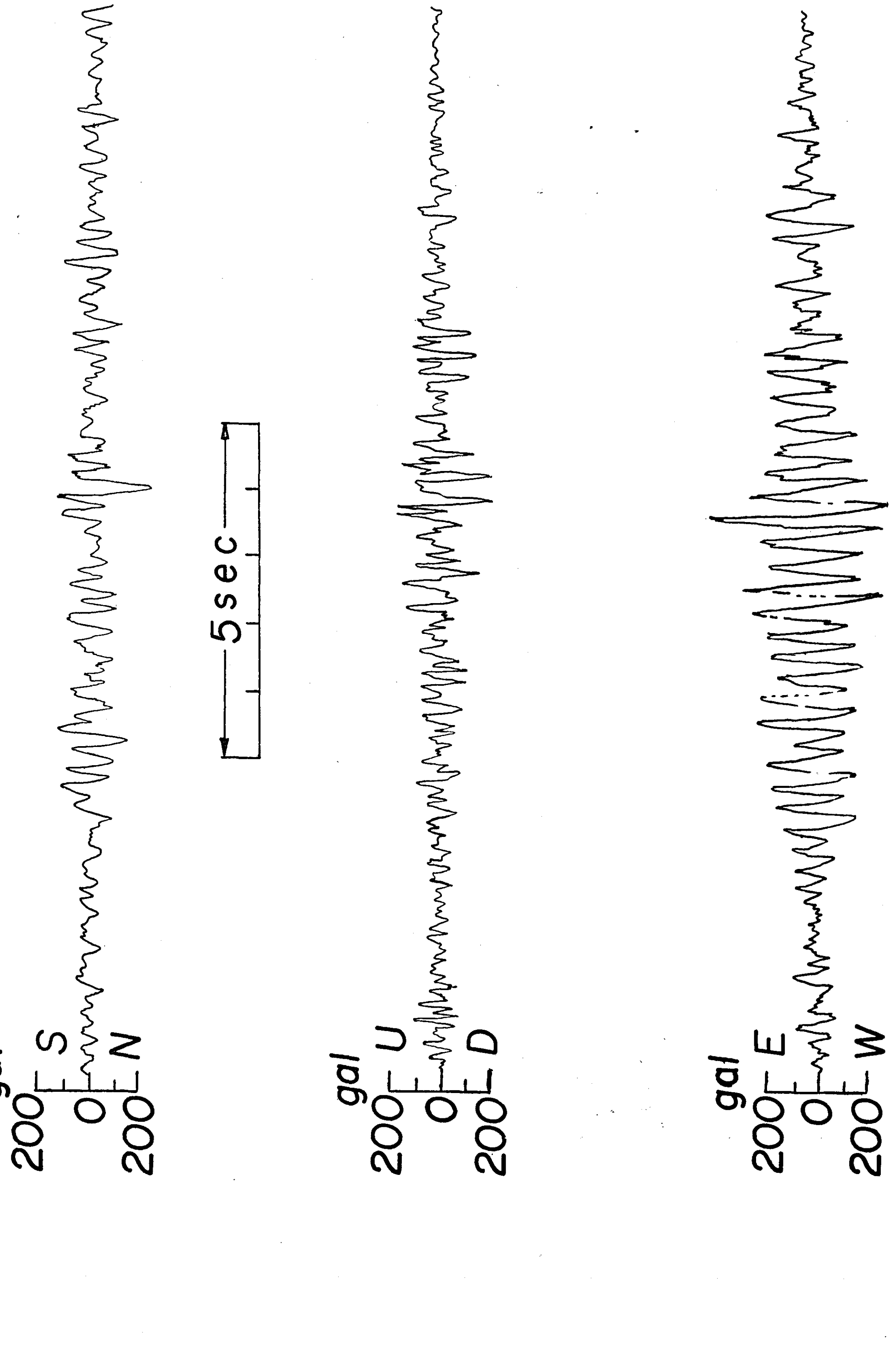


Fig. 3. The records of the Hiroo-Oki earthquake of April 23, 1962 obtained at Kushiro Meteorological Observatory.

Table 3. The comparison of the maximum values of the recorded accelerations of the earthquakes in the U. S. A. and the calculated values of acceleration at the bed rocks.

| Station | Earthquake Date | Earthquake M | Period (sec) | A. Calculated (bed rock) (gal) | Orientation | B. Observed (surface) (gal) | B/A |
|---|---|---|---|---|---|---|---|
| Vernon | III 10, '33 | 6.3 | 0.3 | 43 | N98E | 121 | **3.0** |
| | | | | | N08E | 102 | 2.5 |
| | X 2, '33 | 5.3 | 0.3 | 26 | N98E | 83 | **3.0** |
| | | | | | N08E | 64 | 2.5 |
| Los Angeles Subway Terminal | III 10, '33 | 6.3 | 0.7 | 14 | N39E | 42 | **3.0** |
| | | | | | N129E | 32 | 2.5 |
| | X 2, '33 | 5.3 | 0.6 | 8 | N39E | 31 | **4.0** |
| | | | | | N129E | 23 | 3.0 |
| El Centro | XII 30, '34 | 6.5 | 0.5 | 24 | NS | 141 | **6.0** |
| | | | | | EW | 116 | 5.0 |
| | V 18, '40 | 7.0 | 0.5 | 62 | NS | 278 | **4.5** |
| | | | | | EW | 162 | 2.5 |
| Helena | X 31, '35 | 6.0 | 0.35 | 96 | EW | 122 | **1.5** |
| | | | | | NS | 104 | 1.0 |
| Ferndale | IX 11, '38 | 5.5 | 0.2 | 15 | N45E | 73 | **5.0** |
| | | | | | N135E | 57 | 4.0 |
| | II 9, '41 | 6.6 | 0.3 | 12 | N45E | 46 | **4.0** |
| | | | | | N135E | 31 | 2.5 |
| | X 3, '41 | 6.4 | 0.3 | 18 | N45E | 70 | **4.0** |
| | | | | | N135E | 70 | 4.0 |
| Santa Barbara | VI 30, '40 | 5.9 | 0.3 | 73 | N42E | 158 | **2.0** |
| | | | | | N132E | 121 | 1.5 |
| Hollister | III 9, '49 | 5.3 | 0.35 | 55 | N91E | 140 | **2.5** |
| | | | | | N01E | 93 | 1.5 |

(to be continued.)

(Table 3. continued)

| Station | Earthquake | | Period (sec) | Acceleration (gal) | | | |
|---|---|---|---|---|---|---|---|
| | Date | *M* | | ***A.*** Calculated (bed rock) | Orientation | ***B.*** Observed (surface) | ***B/A*** |
| Olympia | IV 13, '49 | 7.1 | 0.35 | 51 | N80E | 175 | **3.5** |
| | | | | | N170E | 148 | 3.0 |
| Seattle | 〃 | 〃 | 0.9 | 14 | N02E | 51 | **3.5** |
| | | | | | N92E | 46 | 3.5 |
| Alexander Bldg. | III 22, '57 | 5.3 | 0.35 | 37 | N171E | 47 | **1.5** |
| | | | | | N81E | 46 | 1.5 |
| S. Pacific Bldg. | 〃 | 〃 | 0.3 | 30 | N45E | 38 | **1.5** |
| | | | | | N155E | 36 | 1.0 |
| State Bldg. | 〃 | 〃 | 0.3 | 35 | N171E | 85 | **2.5** |
| | | | | | N81E | 54 | 1.5 |
| Golden Gate Park | 〃 | 〃 | 0.3 | 47 | N100E | 107 | **2.5** |
| | | | | | N10E | 79 | 1.5 |
| Oakland | 〃 | 〃 | 0.3 | 21 | N26E | 41 | **2.0** |
| | | | | | N116E | 21 | 1.0 |

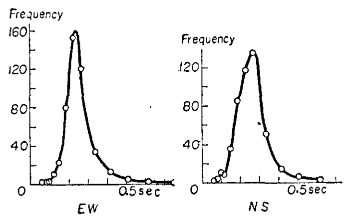


Fig. 4. Period distribution curves of microtremors on the foundation of a SMAC seismograph at Kushiro Meteorological Observatory.

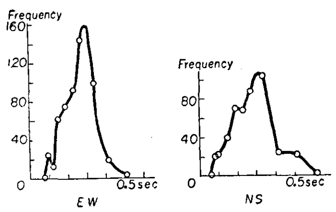


Fig. 5. Period distribution curves ot microtremors on the ground at Kushiro Meteorological Observatory site.

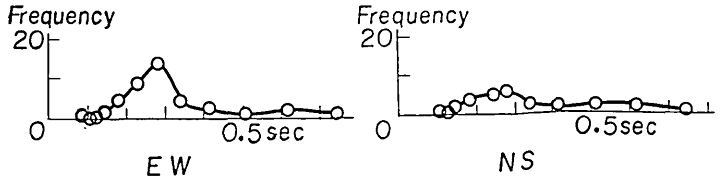


Fig. 6. Period distribution curves of the SMAC seismograph records of the Kushiro-Oki earthquake of February 21, 1962 obtained at Kushiro Meteorological Observatory.

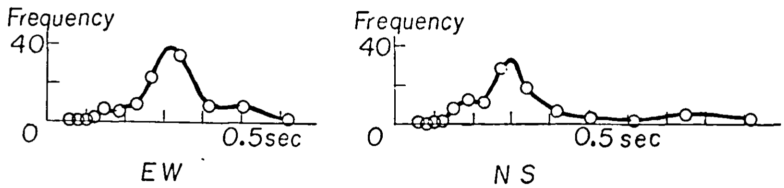


Fig. 7. Period distribution curves of the SMAC seismograph records of the Hiroo-Oki earthquake of April 23, 1962 obtained at Kushiro Meteorological Observatory.

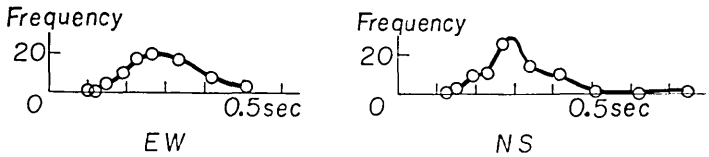


Fig. 8. Period distribution curves of the SMAC seismograph records of the Kushiro-Oki earthquake of July 18, 1962 obtained at Kushiro Meteorological Observatory.

## Appendix

As we have seen from the previous investigations[8] that there is a peak in every displacement-period curve of the seismic waves at a deep depth and the velocity-period relation of them satisfies the nature of energy equipartition, excepting considerably short periods and long periods of more than the peak period mentioned above, we try to find the latter relation by a rather systematic way. The results, by making the following reduction to the velocity spectra of seismic waves obtained at 300 m depth in Hitachi Mine in Ibaraki prefecture about 120 km northwest of Tokyo which have been represented in the previous paper,

8) K. KANAI and S. YOSHIZAWA, "The Amplitude and the Period of Earthquake Motions. II," *Bull. Earthq. Res. Inst.*, **36** (1958), 275-293.

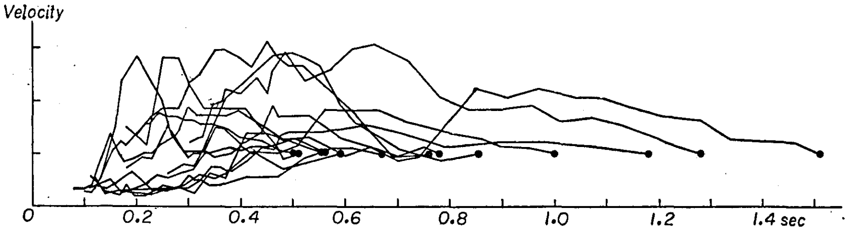


Fig. 9. Velocity spectra of the seismic waves of thirteen earthquakes observed at 300 m depth in Hitachi Mine.

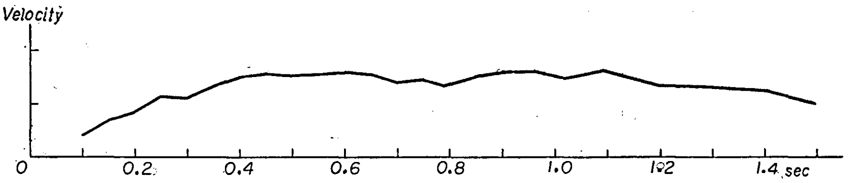


Fig. 10. Average velocity spectrum of seismic waves at 300 m depth in Hitachi Mine.

are shown in Fig. 9. That is, the values of velocity corresponding to the peak periods of the displacement-period curves in each of thirteen earthquakes are taken as constant and are represented by the black circles in Fig. 9 but the velocity-period curves beyond the periods just mentioned are neglected. A curve represented in Fig. 10 is the average of thirteen curves of Fig. 9. It can be seen clearly in Fig. 10 that the velocity spectrum takes a considerably flat form. Therefore, it may be said still more conclusively that the seismic waves of considerably wide range of period in bed rock satisfy the nature of energy equipartition. That is to say, the amplitude-period relation of seismic waves in bed rock can be assumed as follows:

$$\frac{A}{T}=\text{constant}, \ [(0.1-0.2\,\text{sec})<T<T_m]$$

in which $A$, $T$ and $T_m$, respectively, represent each amplitude and period of seismic waves and the period corresponding to the peak of displacement-period curve which is a function of magnitude.

# 16. 強震動スペクトルの実験式 第2報

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日本とアメリカで得られた強震計記録上の最大加速度と，それらの地震の常数を基盤における地震動に関する実験式に代入して求めた加速度とを比較して，両者の比は，地震動の強弱に関係なく同一場所では一定であることがわかつた．このことは，一面では，この実験式の利用価値を裏付けるものであり，他面では，地震動の大きさが，地盤特性に大きく支配されることを意味するものである．

基盤の加速度に対する地表面のそれの比は，普通2～5倍であるが，釧路のような特別な場所では10倍以上にも達する．1962年4月23日の広尾沖地震の釧路気象台のSMAC強震計の最大値が340 galにも達したのにかかわらず，被害が極めて小さかつたことに対する解釈はいろいろあろうが，地盤の弾性的にみた構成が非常に単純な上に，基盤とのインピーダンス比が非常に小さい場合には，地盤中での最大増巾度が非常に大きいがその周期範囲が極めて狭いために，構造物の破壊をある程度以上に進めないのではないかということも，1つの解釈であろう．

附録として，基盤における速度スペクトルが普通構造物の耐震問題に関係する周期範囲では，ほぼ一定であることを示した．