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22. *Propagation of Surface Waves and Internal Friction.*

By Ryoichi YOSHIYAMA,

Earthquake Research Institute.

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1. Introduction.

Ewing and Press[1] studied the attenuation constant of the "Mantle Rayleigh waves" from the observations of its "earlier" and "later" orders, and found out the magnitude of internal friction of the earth's interior, which seems reasonable from a laboratory experiment. Their studies were followed by Satô's[2] of the "Mantle Love waves" or "G waves." His method of mathematical treatment seems more rigorous than that adopted by Ewing and Press; in his studies, the waves are analyzed into a complete set of monochromatic waves by Fourier's transforms, and applications of a complicated theory of waves of group are avoided. The results seem to give excellent data, perhaps incomparable for the present at least, for theoretical study of the wave, and yet there is something incomprehensible to the present writer. Because, the quantity $1/Q$ which is presumed constant for some range of period of waves turned out in his studies to vary beyond measure with the period of waves, moreover, in quite a different way from that noticed by Ewing and Press. And, it seems the way to interpret such a suspected discordant point reasonably has an important effect on the theoretical study of wave propagation as well as on the estimate of the amplitude of the wave at the hypocentral or epicentral region.

Theoretically rigorous studies of the problem are clearly difficult by any means, and, in this paper, it is hoped to get an empirical formula for theoretical studies in future.

2. Discordant point revealed.

Regarding an infinite train of monochromatic surface wave over an

1) M. EWING and F. PRESS, "An Investigation of Mantle Rayleigh Waves." *B.S.S.A.*, **44** (1954) 127-147; "Mantle Rayleigh Waves from the Kamchatka Earthquake of November 4, 1952." *B.S.S.A.*, **44** (1954) 471-479.

2) Y. SATÔ, "Attenuation, Dispersion, and the Wave Guide of the G Wave." *B.S.S.A.* **48** (1958) 231-251.

idealized spherical earth, the theoretically expected relation between its amplitude A and its travel or epicentral distance is given, under a certain condition, by

$$A \propto (1/\sqrt{|\sin\theta|}) \exp(-k\Delta) \cdots\cdots\cdots\cdots\cdots (1)$$

where θ is epicentral angular distance, $\Delta = R\theta$ travel distance over the earth's surface, R radius of the earth. Reduction of the observations of surface waves has been generally based on this formula (1). And, according to Born, Birch and other senior authors, most laboratory experiments and field experiments by small explosions indicate that k is inversely proportional to the period of the wave. Then, putting

$$k = \pi / QVT \cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where V is the phase velocity of the wave and T the period, $1/Q$ is a dimensionless constant of the medium, named internal friction; laboratory experiment shows, also, that $1/Q$ of the same material decreases as temperature or pressure increases. Therefore, concerning seismic surface waves, because of changes of materials or state of materials towards the interior of the earth and because of the effect of penetration of the wave energy to the depth, apparent $1/Q$ is not necessarily constant, and, for a wave of long period, may be small as already pointed out by Ewing and Press in their studies, though numerical precision of their results will not yet be free from some questions.

On the other hand, as reproduced in Table 1, the results of Satô's study of $1/Q$ of New Guinea Earthquake, on Feb. 1, 1938, and Kamchatka Earthquake, on Nov. 4, 1952, are clearly discordant with the above explained presumption; $1/Q$ from his results remarkably increases with T, the period of the wave. There are two ways to study the discordancy, one is to put aside the presumption and assume a different dependency of the friction on the period T; the other is to find out some reasonable unknown factors which disturb the generally used formula (1). In this paper, following the above-mentioned results of laboratory experiments, the latter is studied. Because, studying Satô's paper in detail, systematic differences are observed not only between the

Table. 1. $1/Q$ from Satô's paper

Period T	360	210	108	72	54	43.2	sec.
New Guinea Earthq.	$13\frac{1}{2}$	$8\frac{1}{2}$	9	8	6	4	$\times 10^{-3}$
Kamchatka Earthq.	19	12	11	8			$\times 10^{-3}$

two series of $1/Q$-values in Table 1 of the two earthquakes, but also between two series of k-values, or γ-value in his notation, computed for each earthquake from two combinations respectively of different orders, i.e. G1N–G3N, or G1K–G3K, and G2N–G4N, or G2K–G4K. k-value from the former combination, G1–G3, is always larger than that from the latter, G2–G4, regardless of the period of the wave, and a similar tendency is observed also in the figures given by Ewing and Press in their studies. Those two series of k-values for each earthquake, four series of k-values in all, are presented in his paper graphically, though not numerically, and are expected to give as many series of $1/Q$-values. But, it seems mean values of each two are used respectively in his computation of $1/Q$ for each earthquake. The present writer tries to deduce a characteristic internal friction $1/Q_0$ from the four series of apparent $1/Q$-values. The studies are naturally directed to the re-examination of the factor $1/\sqrt{|\sin\theta|}$ of the formula (1).

3. Preliminary reasoning and formula obtained.

Wave front between two wave rays which make an angle $\delta\varphi$ at the epicentre spreads as $\delta\varphi R|\sin\theta|$. And, as far as the factor $1/\sqrt{|\sin\theta|}$ is concerned, the formula (1) implies a perfect and repeated convergence of wave front to a point at the epicentre and its antipode, and implies a conservation of wave energy along the wave ray over the spherical and homogeneous surface of the idealized earth. However, it is rather unreasonable to expect such a perfect convergence of wave front and conservation of wave energy along the wave ray over the surface of the earth. Because of geographical as well as geological irregularities of its surface, phase relation between the waves that rules boundary conditions of the waves propagated along contiguous wave rays, must undergo a change every moment on the way of propagation, and, accordingly, wave energies are partly refracted out of the proper wave ray resulting in a noise.

Based on the above-mentioned idea of wave propagation, a function of epicentral distance $f(\Delta)$ is introduced in place of $|\sin\theta|$. Since we have no theoretical basis at present to determine the mathematical expression of $f(\Delta)$, the following is expediently assumed for its empirical formula,

$$f(\Delta)=\alpha\{R|\sin\theta|+\beta\Delta^m\} \quad \cdots\cdots\cdots\cdots\cdots (3)$$

By changing α-and β-value, the formula (3) is likely adaptable to various presumable features of diffusing wave front as follows:

i) If $\beta=0$ and $\alpha=1, f(\Delta)=R|\sin\theta|$; this is the idealized case.

ii) If $\beta\rightarrow\infty, \alpha\rightarrow 0, \alpha\beta=1$ and $m=1, f(\Delta)=\Delta$; this is a case that even a slight convergence of wave front does not occur.

iii) An intermediate case of i) and ii) is given by certain α- and β-value, some convergence of wave front being implied.

Then we have, instead of (1), as an amplitude-distance relation,

$$A \propto \frac{1}{\sqrt{f(\Delta)}} \exp(-k_0\Delta) \cdots\cdots\cdots\cdots\cdots (4)$$

or, by a simple mathematical transformation,

$$A \propto \exp\left\{-k_0\Delta - \frac{1}{2}\log f(\Delta)\right\}. \cdots\cdots\cdots\cdots (5)$$

If A_1 and A_2 are respectively the amplitudes of the wave at two stations where travel distances of the wave are Δ_1 and Δ_2, while epicentral angular distances are θ_1 and θ_2, we get from (5),

$$\log\frac{A_1}{A_2} = (\Delta_2-\Delta_1)\left\{k_0 + \frac{1}{2}\frac{1}{\Delta_2-\Delta_1}\log\frac{f(\Delta_2)}{f(\Delta_1)}\right\} \cdots\cdots\cdots (6)$$

So that k-value, if calculated by the generally used formula (1), must be related to k_0, Δ_1, Δ_2, θ_1 and θ_2 by the following formula,

$$k = k_0 + \frac{1}{2}\frac{1}{\Delta_2-\Delta_1}\log\frac{f(\Delta_2)\,|\sin\theta_1|}{f(\Delta_1)\,|\sin\theta_2|}$$

and $1/Q$ by,

$$\frac{1}{Q} = \frac{1}{Q_0} + \frac{TV}{2\pi}\frac{1}{\Delta_2-\Delta_1}\log\frac{f(\Delta_2)\,|\sin\theta_1|}{f(\Delta_1)\,|\sin\theta_2|} \cdots\cdots (7)$$

where $k_0=\pi/Q_0TV$, and $1/Q_0$ is the expected characteristic internal friction. It is evident from (7) that, even when $1/Q_0$ is a constant, apparent $1/Q$ is a function of T; and, roughly speaking, as the phase velocity V changes but slightly for some range of T, it is nearly a linear function of T, and its gradient with respect to T depends on the epicentral distances of the stations concerned; and also, in the idealized case, since $f(\Delta)=R\,|\sin\theta|$, the gradient term vanishes.

If we take for A_1 and A_2 the amplitudes of the wave of every other order at a station, $\sin\theta_1=\sin\theta_2$, $\Delta_2-\Delta_1=40{,}000$km and $f(\Delta_2)>f(\Delta_1)$ except the idealized case. And, the formula (7) will be accepted to give $1/Q$ of a tendency consistent with that in question of Table 1.

4. Further numerical computation based on Satô's results.

According to Satô, the epicentral angular distances of the station are $\theta=109.^\circ 6$ for the New Guinea earthquake, and $\theta=58.^\circ 6$ for the Kamchatka earthquake. So that the travel distances for respective G-waves are given by Table 2.

Table 2.

	G1	G2	O3	G4	
New Guinea Earthq.	120	280	520	680	10^2 km
Kamchatka Earthq.	64	336	464	736	10^2 km

Satô also gives a formula of the phase velocity as follows;

$$V=V_0/(1-\kappa T), \qquad V_0=4.4\ \text{km/sec}, \qquad \kappa=5.3\times 10^{-4}\ \text{sec}^{-1}.$$

Substituting those together with $\Delta_2-\Delta_1=40{,}000$ km into (7), we get

$$\frac{1}{Q}=\frac{1}{Q_0}+0.175\cdot 10^{-4}\frac{T}{1-\kappa T}\log\frac{f(\Delta_2)}{f(\Delta_1)}\ \cdots\cdots\cdots\cdots(8)$$

From the observations of sufficient accuracy, it will be possible in future

Table 3. $0.175\cdot 10^{-4}\log\dfrac{f(\Delta_2)}{f(\Delta_1)}$ in $10^{-5}\ \text{sec}^{-1}$

m	β $(\text{km})^{1-m}$	New Guinea Earthq.			Kamchatka Earthq.		
		G1−G3	G2−G4	mean	G1−G3	G2−G4	mean
3	∞	7.70	4.66	6.18	10.40	4.11	7.25
	$3\cdot 10^{-10}$	3.55	4.38	3.96	3.21	3.52	3.36
	$1\cdot 10^{-10}$	2.12	4.23	3.17	1.77	2.80	2.28
2	∞	5.12	3.10	4.11	6.93	2.75	4.84
	$10\cdot 10^{-5}$	4.55	2.99	3.77	5.51	2.68	4.09
	6.10^{-5}	4.27	2.94	3.60	4.98	2.64	3.81
	$2\cdot 10^{-5}$	3.33	2.65	2.99	3.60	2.45	3.02
3/2	∞	3.85	2.33	3.09	5.20	2.06	3.63
	$5\cdot 10^{-2}$	3.71	2.29	3.00	4.88	2.04	3.46
	$1\cdot 10^{-2}$	3.18	2.17	2.67	4.04	1.95	2.99
1	∞	2.57	1.55	2.06	3.46	1.37	2.41
	10	2.50	1.53	2.01	3.34	1.32	2.35
	1	2.11	1.36	1.73	2.67	1.24	1.95

to obtain the expression of the function $f(\Delta)$ empirically. In this paper, assuming (3), β and m are determined from the data given by Satô. Since κ is small, i.e. phase velocity V of the wave changes but slightly for some range of the period T, $(\partial/\partial T)(1/Q) \doteqdot 0.175 \cdot 10^{-4} \log f(\Delta_2)/f(\Delta_1)$, which is given in Table 3 with several numerical examples of β and m for the two earthquakes.

Remarkable points that we learn from Satô's results, though he himself does not point them out, are as follows; i) apparent $1/Q$ from the Kamchatka earthquake seems a little greater than that from the New Guinea earthquake, ii) apparent $1/Q$ from G1–G3 seems a little greater than that from G2–G4, and the difference between the two seems more remarkable in case of Kamchatka earthquake than in case of New Guinea earthquake, iii) if we assume, from Table 1, $1/Q = 4 \times 10^{-3}$ for $T = 43.2$ sec and $1/Q = 19 \times 10^{-3}$ for $T = 360$ sec, mean gradient of $1/Q$ is not greater than $5 \times 10^{-5} \text{sec}^{-1}$ at most, but, on the other hand, its lower limit seems about $2 \times 10^{-5} \text{sec}^{-1}$. Therefore, perhaps we can find the most appropriate values of m and β when $1 \leqq m \leqq 2$; if we take $m = 2$, $\beta = 2 \sim 10 \times 10^{-5} \text{ km}^{-1}$, or, if we take $m = 1$, $\beta > 1 \text{ km}^0$ seems plausible.

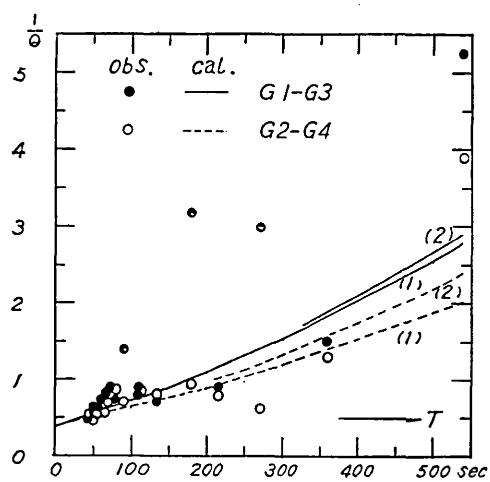

Fig. 1. Obs. and Cal. relations between $1/Q$ and period of wave of the New Guinea Earthq.
(1): $m=3/2$ $\beta=10^{-2}\text{km}^{-1/2}$
(2): $m=2$ $\beta=2\times10^{-5}\text{km}^{-1}$
Obs.: Computed from Satô's paper.
$1/Q$ in 10^{-2}.

The four series of $1/Q$ value are calculated by the writer, though considerable errors from misreading might have been introduced, from Satô's figures of spectrum of the wave and γ-values, Figs. 3, 6, 9 and 11 in his paper, and are given in the Figs. 1 and 2 together with the calculated $1/Q$ when 1) $m = 3/2$, $\beta = 1 \times 10^{-2} \text{ km}^{-1/2}$, $1/Q_0 = 4 \times 10^{-3}$, and 2) $m = 2$, $\beta = 2 \times 10^{-5} \text{ km}^{-1}$, $1/Q_0 = 4 \times 10^{-3}$ respectively. These two examples of m and β are adopted from various considerations of Satô's results; to pick up from Table 3 a pair of m and β which satisfies the four series of $1/Q$-values at the same time is not so easy. This fact indicates namely that m and β will be obtained uniquely, or, at least, in a limited range of magnitudes. So far as these two earthquakes are concerned,

$m=3/2$ and $\beta=10^{-2}$ km$^{-1/2}$ seem more suitable than $m=2$ and $\beta=2\times10^{-5}$ km^{-1}, and, from these figures, characteristic internal friction $1/Q_0$ is estimated about 4×10^{-3}. Of course, it is absolutely difficult to arrive at a conclusion from these figures alone, but certainly we can expect in future study on the observations of high accuracy. In future study, our interest is also in a re-examination of observations concerning remarkably large $1/Q$, apparent in the figures for the wave of period of 60–100 sec: it may be attributed to selective absorption of wave energy by some mechanism of propagation of wave through the earth's crust, or, to a branch for $T<100$ sec expressed by large m and β. It is evident from the figures that observations and study of $1/Q$-value for the wave whose period $T<50$ sec have an important effect on the interpretation.

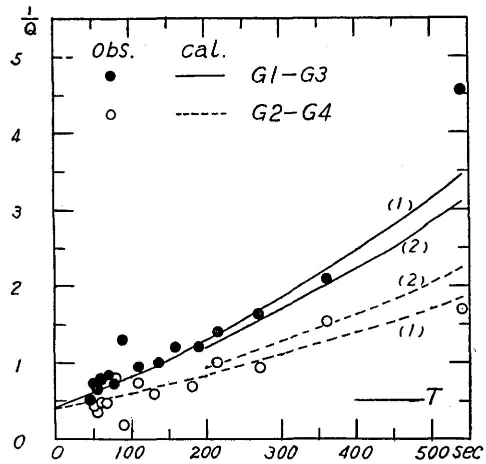

Fig. 2. Obs. and Cal. relation between $1/Q$ and period of wave of the Kamchatka Earthq.
(1): $m=3/2$ $\beta=10^{-2}$km$^{-1/2}$
(2): $m=2$ $\beta=2\times10^{-5}$km^{-1}
Obs.: Computed from Satô's paper. $1/Q$ in 10^{-2}.

5. Concluding remarks and acknowledgement

Before anything, the author expresses his respect to Dr. Y. Satô for his works that present such excellent data. His results of $1/Q$ alone, ingeniously and carefully obtained for so wide a range of period of wave from each earthquake, enable us to study the characteristic constant of friction. Although the systematic slip out of the observations from the expected linearity apparent in Fig. 10 and Fig. 3 of the two papers respectively by Ewing and Press will be well explained by the method in this paper, their method of analysis will give $1/Q$ of but a limited range of period from each earthquake, and, therefore, statistical studies of observations of many earthquakes are necessary to get sufficient numbers of data of $1/Q$ for the study of the "characteristic constant." In that respect, Satô's method, not to mention its theoretical lucidness, is extremely convenient giving sufficient number of data even

from single earthquake. However, careful arrangement of observations and equally careful operation of the computor are strictly demanded in his method; otherwise, even a slight misreading of the seismogram will cause a false component wave which may be conveniently reduced in a statistically computed data.

Since, at present, physical interpretation of anelastic property of the earth materials is not yet fully discussed, there is something ambiguous in the definition of "internal friction of the earth." The physical interpretation will be advanced by future study of attenuation constant of seismic waves, and the ambiguous points will be cleared at the same time. Though there remains such ambiguous point in its definition, internal friction of the earth came out, in this paper, $4\cdot10^{-3}$, a little smaller than expected from a study of "Maximum Amplitude and Epicentral Distance" by the present writer.

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22. 表面波と地球の内部摩擦

地震研究所　吉　山　良　一

地震波の走距離による減衰を調査して地球の内部摩擦係数 $1/Q$ を計算することは Gutenberg 等により取上げられてきた研究課題であるが，その最も困難な点は各観測所の観測状況が同一でないことと，それら観測所に達する地震波の energy が震源をでる時すでに必しも同一でないことであつた．Ewing と Press は同一地点に於ける Mantle Rayleigh wave の観測を使うことによつてこの困難を除去して成果をあげた．それにつづいて Satô は G-wave の観測を使つて更に精度の高い数値をだした．しかしこれら三氏が $1/Q$ としている値は純粋な内部摩擦 $1/Q_0$ の上に波面の拡散ともいうべき現象による影響があるらしく思われる．現在の資料ではそれを定量的にはもちろんのこと定性的にすら断言するには早すぎるかも知れないが，地震波の理論的な見通しが観測の整理方針延いては資料の精度に与える影響も無視し得ないと思われるので一応の計算結果を示した次第である．
