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15. The Amplitude and the Period of Earthquake Motions. II.

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1. Introduction

It goes without saying that a seismogram is a record of wave motions at ground surface transmitted from the point of origin of the earthquake through the earth's crust as a vibration system and, in general, is recorded by means of the pendulum of a vibration system. That is to say, when $O(T)$, $C(T)$, $G(T)$, and $I(T)$ represent vibration characteristics of the point of origin, the earth's crust, the ground and the seismograph, respectively, the wave form of seismogram $R(T)$ can be written as follows:

$$R(T)=O(T).C(T).G(T).I(T)\,. \qquad (1)$$

In order to study analytically the mechanism of earthquake generation as well as the nature of the earth's crust, it is desirable to use a seismogram in which the effect of $G(T)$ is negligible, because of much complication of the geological conditions of the ground near the surface. The ground underneath Hitachi Mine (Ibaraki prefecture), where we have been making observations for more ten years, has a very good condition in this respect.

In the previous paper[1)], we obtained the relation between the amplitude and the period of earthquake motions, that is,

$$A \propto T^2\,, \qquad (2)$$

by using the initial motion of P-waves observed at Hitachi Mine, in consideration of the fact that the change degree of wave form on the way of propagation would be the smallest in the initial motion of P-waves. And, in the same paper, we entered into the discussion of the mechanism of earthquake generation by using the above formula.

1) K. KANAI, K. OSADA and S. YOSHIZAWA, "The Relation between the Amplitude and the Period of Earthquake Motion", *Bull. Earthq. Res. Inst.*, **31** (1953), 45.

In this paper, we are going to examine the relation between the amplitude and the period of earthquake motions using the results of spectral analysis of seismograms obtained 300 m underground in Hitachi Mine.

2. Spectral analysis of seismograms

The constants of the seismograph used in the present investigation are as follows: type=inverted pendulum, natural period of pendulum=1.0 sec, mechanical magnification=204∼440, damping ratio=13 : 1, recording system=scratching smoked rotating-paper, driving=6 watt motor for electric clock use, recording speed=2 mm per sec.

Table 2 shows the positions of the origin of the earthquakes treated herein which were obtained by the Japan Meteorological Agency, Tokyo,

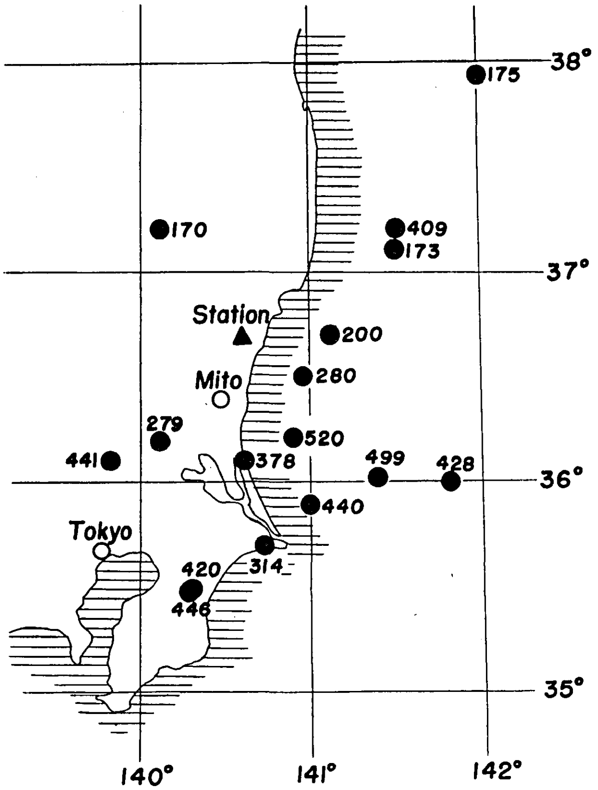

Fig. 1. The positions of the observing station and the epicenters.

and the duration of preliminary tremors observed at Hitachi Mine. The positions of the observing station and the epicenters written in the third column in Table 2 are shown in Fig. 1.

The displacement spectra of the earthquake motions obtained, from the photographs enlarged 3∼5 times the seismograms, by means of the response computer[2] are shown in Figs. 5∼20. The broken lines in Figs. 5∼20 represent the results which have been reduced to the results of spectral analysis for the seismograms with regard to vibration characteristics of the seismograph.

As we have seen from Figs. 5∼20 that there is a peak in every displacement-period relation, we try to find whether or not there exists a systematic relation between the maximum values of displacement spectra and the periods corresponding to the above mentioned values.

(i) First, the reduction of the amplitude of seismic waves according to hypocentral distance will be performed by the following formula:

$$A=\frac{A_0}{x}e^{-kx/T^2}, \qquad (3)$$

where

$$k=\frac{2\pi^2\xi}{\rho v^3}, \qquad (4)$$

in which, the following notations are employed, that is, x = hypocentral distance, A=amplitude at distance x, A_0=amplitude at origin, T=period of waves, ξ=viscous coefficient, ρ=density, v=velocity. If we adopt the value of damping factor $k=5\times10^{-8}$ C. G. S.[3], the amplitude of seismic waves at a spot 100 km away from the hypocenter may be written, based on equation (3), as follows:

$$A_{\Delta=100}=A\frac{x}{100}e^{(x-100)\times0.005/T^2}, \qquad (3')$$

in which, A is the amplitude at observation station and x is the hypocentral distance of the station in km.

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2) R. TAKAHASI, "A Response Computer, Preliminary Report", Proc. 3rd Japan National Cogr. *Applied Mech.*, (1953), 373.

3) K. SEZAWA and K. KANAI, "Viscosity Distribution within the Earth. Preliminary Notes", *Bull. Earthq. Res. Inst.*, **18** (1940), 169.

A. KUBOTERA, "Damping and Dispersion of Seismic Body-wave in the Earth's Crust", *Journ. Seism., Soc., Japan,* [ii], **5**, No. 3 (1952), 9, (in Japanese).

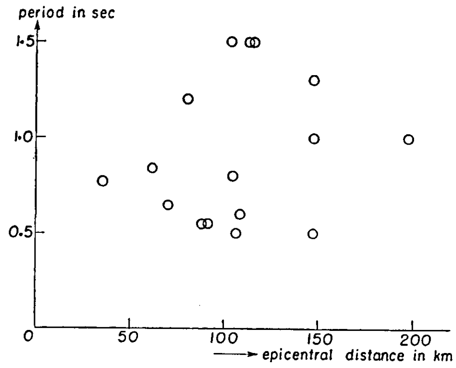

Fig. 2. Variations of the periods corresponding to the maximum amplitudes in Figs. 5~20 with the epicentral distances.

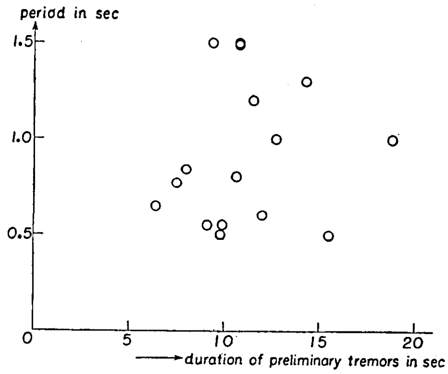

Fig. 3. Variations of the periods corresponding to the maximum amplitudes in Figs. 5~20 with the duration of preliminary tremors observed at Hitachi Mine.

(ii) Figs. 2 and 3 show the variations of the periods corresponding to the maximum amplitudes in Figs. 5~20 with the epicentral distances and the duration of preliminary tremors observed at Hitachi Mine. The only conclusion drawn from Figs. 2 and 3 is, that there is a scarcely recognizable relationship between the period of waves and epicentral distance and that is the same as the other results[4]. Therefore, in the present investigation, the reduction of the period of seismic waves according to hypocentral distance will be left out of consideration.

(iii) As the observations were made concerning only one component of seismic waves as shown in Table 1, the resultant amplitude will be calculated by assuming them to be S-waves.

The relation between the maximum values of displacement which are given by three kinds of reduction mentioned above to the results of spectral analysis shown in Figs. 5~20, and the periods corresponding to them is shown in Fig. 4. From Fig. 4, the empirical formula concerning the relation of maximum displacement to the period of seismic waves may be written as follows:

$$A_{m\cdot s}=53T_m^{2.56} \qquad (5)$$

in which, units of $A_{m\cdot s}$ and T_m are micron and sec, respectively.

4) B. GUTENBERG and C. F. RICHTER, "Earthquake Magnitude, Intensity, Energy, and Acceleration", *Bull. Seism. Soc. Amer.*, **32** (1942), 163.

A. E. JONES, "Empirical Studies of Some of the Seismic Phenomena of Hawaii", *ditto*, **28** (1938), 313.

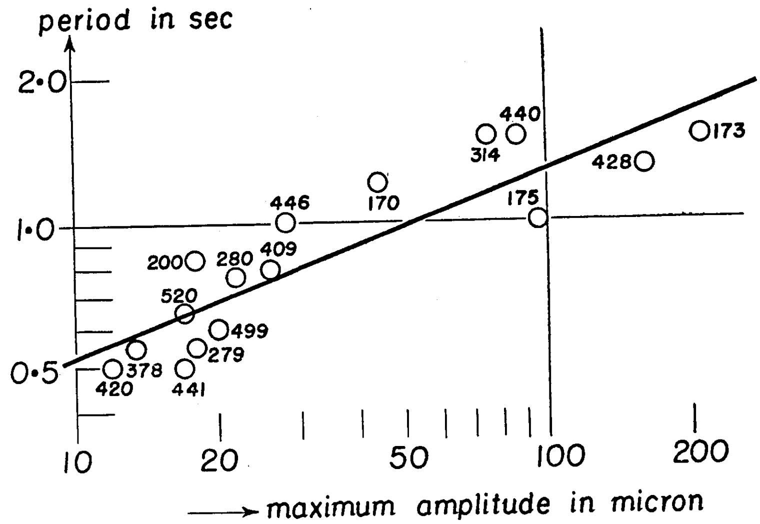

Fig. 4. Relation between the maximum values of displacement which are given by three kinds of reduction to the results of spectral analysis shown in Figs. 5~20, and the periods corresponding to them.

3. Discussion

If we express the relation of the amplitude to the period of seismic waves as follows:

$$A \propto T^n, \tag{6}$$

the values of n which have already been obtained by many authors can be tabulated as in Table 1. In Table 1, the results of magnitde-period relation of earthquakes are converted into the amplitude-period relation, using the original relation in which magnitude is proportional to the logarithm of amplitude.

It will be seen in Table 1 that the values of n in the first seven cases are considerably different from those of the last two cases, that is, n of the former takes 2~3, while that of the latter becomes twice their value.

Now, we try to find the reason of such a large difference.

(a) The periods in the first seven cases were obtained from seismograms directly or after having made some analyses on seismograms, but in the last two cases, the periods were obtained after having made complicated calculations including the assumption $\alpha=(2\pi/T)^2A$, in which α

Table 1. Values of n in $A \propto T^n$.

Authors	Observation places	Kinds of waves	n
Jones[5]	Hawaii	P-waves	2.94
		S-waves	2.93～3.02
Honda[6]	Japan	deep earthquake, P-waves, initial	2
Kanai[7]	Hitachi, −300 m	P-waves, initial	2
		S-waves, initial	2
Kanai	Hitachi, −300 m	maximum	2.56
Aki[8]	Japan	maximum	3.3
Gutenb erg[9]	California	maximum	4.5
Asada[10]	Tokyo	maximum	5.9

and A represent the maximum accerelation and displacement of an earthquake motion.

(b) In the first seven cases either the seismographs of comparatively long period have been used or the reduction of the vibration characteristics of seismographs has been carried out ; on the other hand, in the last two cases, the results obtained by using the seismographs of comparatively short period have been utilized.

At any rate, from the above considerations only, we can scarcely get the satisfactory answer to the present problem. But, these considerations will serve in the investigations of the mechanism of wave generation at the point of origin of an earthquake, in future.

Next, velocities and accelerations have been calculated from the displacements shown in Figs. 5～20, on the assumption of simple harmonic motion. These velocity and acceleration spectra are presented in Figs.

5) A. E. JONES, *loc. cit.*, 4).

6) H. HONDA and H. ITO, "On the Period of the P-waves and the Magnitude of the Earthquake", *Geophys. Mag.*, **13** (1939), 155.

7) K. KANAI, K. OSADA and S. YOSHIZAWA, *loc. cit.*, 1).

8) K. AKI, "Correlogram Analysis of Seismograms", *Journ. Seism. Soc., Japan*, [ii], **8**, No. 2 (1955), 99, (in Japanese).

9) B. GUTENBERG and C. F. RICHTER, *loc. cit.*, 4).

10) T. ASADA, "On the Relation between the Predominant Period and the Maximum Amplitude of Earthquake-Motions", *Journ. Seism. Soc., Japan*, [ii], **6**, No. 2 (1953), 1, (in Japanese).

21~36, and 37~52, respectively. From Figs. 21~36, it can be seen that the velocity spectra at 300 m underground, excepting a few cases, take a considerably flat form. That is to say, Figs. 21~36 tell us that the amplitude-period relation of seismic waves, excepting considerably short and long periods, can be assumed as follows:

$$\frac{2\pi A}{T}(\equiv \text{velocity}) = \text{constant} \qquad (7)$$

in which A and T represent each amplitude and period of seismic waves, respectively. It means that, seismic waves of considerably wide range of period satisfy the nature of energy equipartition.

4. Acknowledgement

In conclusion, we wish to express our thanks to the Science Section of the Educational Ministry, for the financial aid (Research Funds) granted us. Also thanks are due to the members of Motoyama Office, Hitachi Mine, Prof. R. Takahasi and Mr. I. Aida for their help, and to Messrs. T. Tanaka, T, Suzuki and K. Osada for their cooperation, in the course of these investigations.

Table 2.

Earth-quake No.	Date	Origin			Duration of preliminary tremor in sec	Observation direction
		°N	°E	Depth (km)		
170	1951 I 26	37.2	140.1	shallow	11.5	EW
173	〃 II 3	37.1	141.5	40~50	10.8	〃
175	〃 IV 5	37.9	142.0	70	18.8	〃
200	1952 III 10	36.7	141.1	40	8.0	〃
279	〃 VII 28	36.2	140.1	55	9.1	〃
280	〃 VIII 9	36.5	140.9	0~10	7.5	〃
314	〃 IX 14	35.7	140.7	30	10.8	〃
378	〃 XI 18	36.1	140.6	60	9.9	NS
409	〃 XII 11	37.2	141.5	30	10.6	EW
420	〃 〃 25	35.5	140.3	50	15.5	〃
428	1954 I 14	36.0	141.8	60	14.3	〃
440	〃 〃 22	35.9	141.0	60	9.4	NS
441	〃 〃 26	36.1	139.9	40	9.8	EW
446	〃 II 3	35.5	140.3	50	12.7	〃
499	〃 IV 10	36.0	141.4	20	12.0	NS
520	〃 V 20	36.2	140.9	40~30	6.4	EW

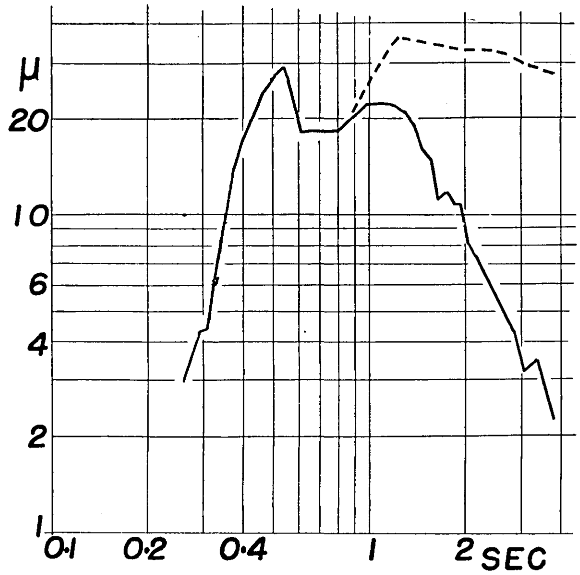

Fig. 5. Displacement spectrum for No. 170 earthquake.

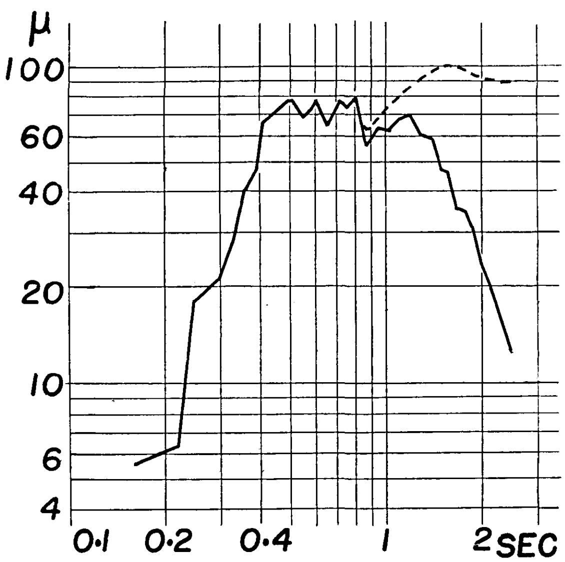

Fig. 6. Displacement spectrum for No. 173 earthquake.

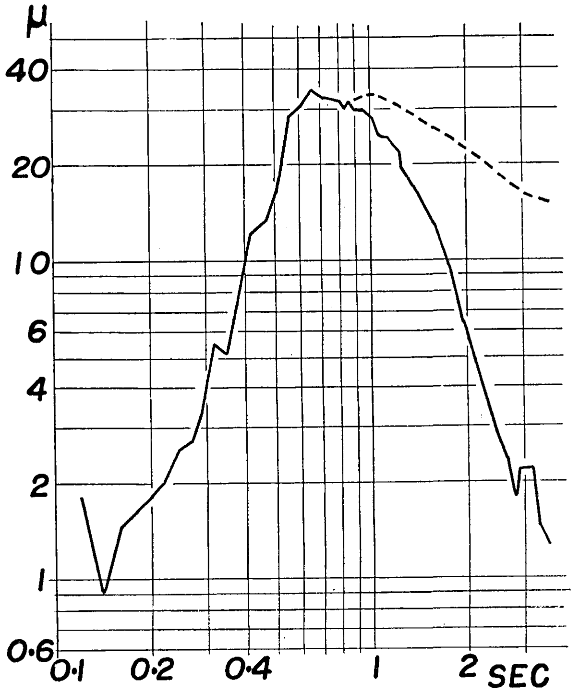

Fig. 7. Displacement spectrum for No. 175 earthquake.

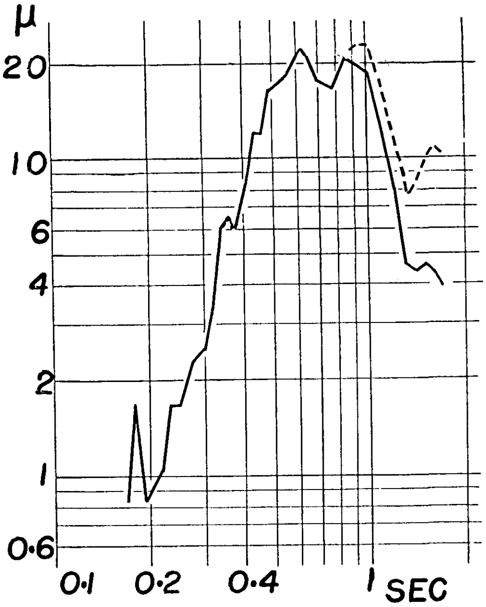

Fig. 8. Displacement sprectrum for No. 200 earthquake.

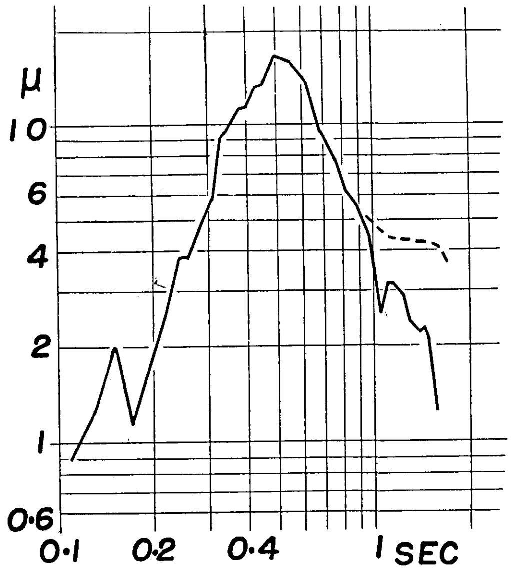

Fig. 9. Displacement spectrum for No. 279 earthquake.

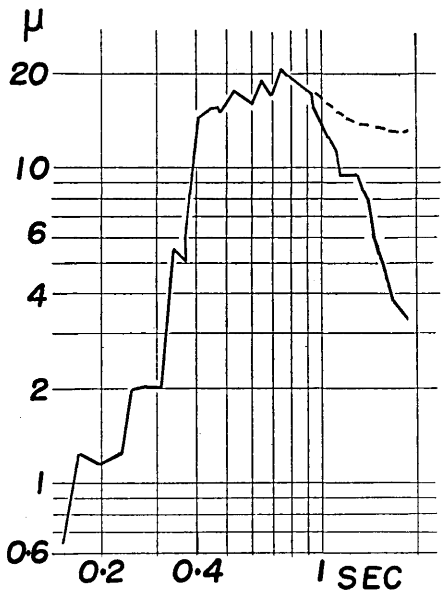

Fig. 10. Displacement spectrum for No. 280 earthquake.

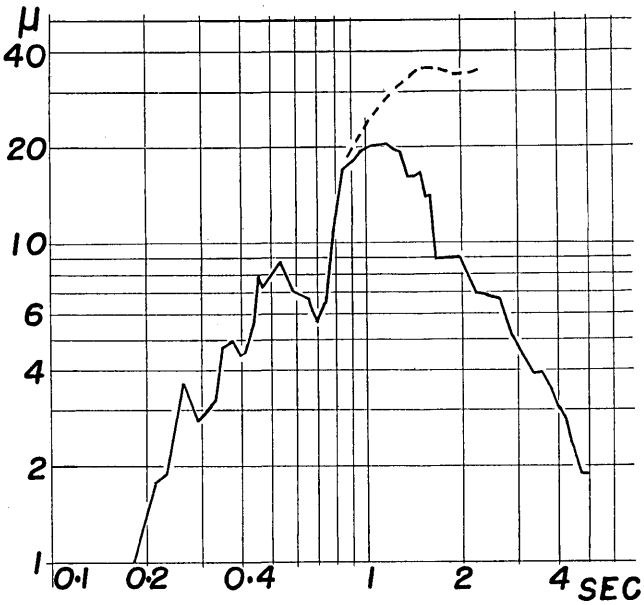

Fig. 11. Displacement spectrum for No. 314 earthquake.

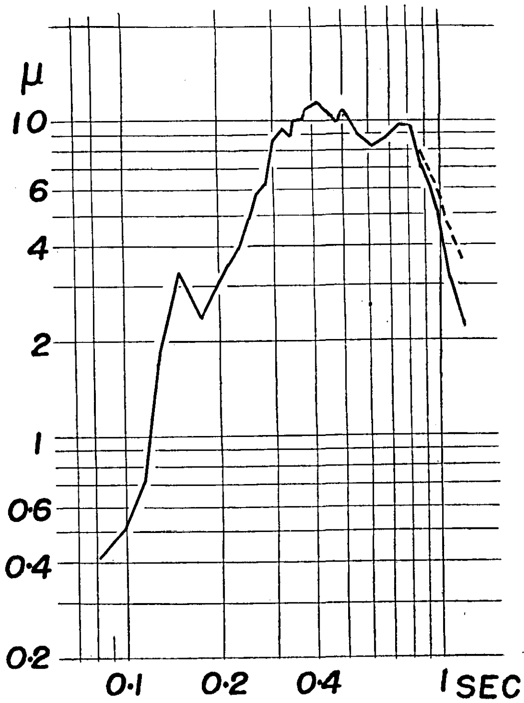

Fig. 12. Displacement spectrum for No. 378 earthquake.

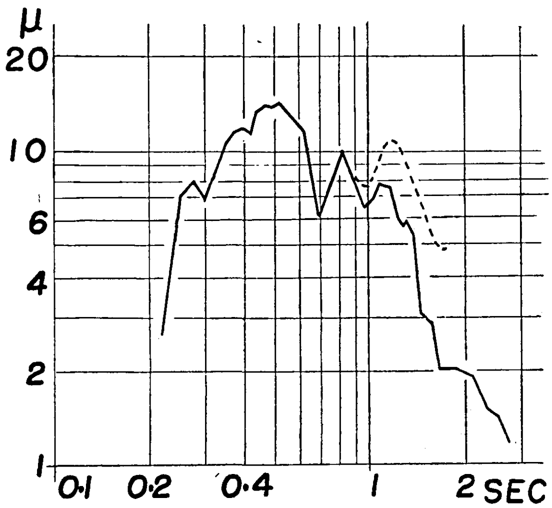

Fig. 13. Displacement spectrum for No. 409 earthquake.

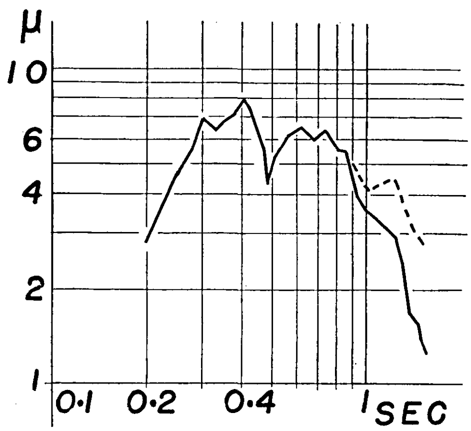

Fig. 14. Displacement spectrum for No. 420 earthquake.

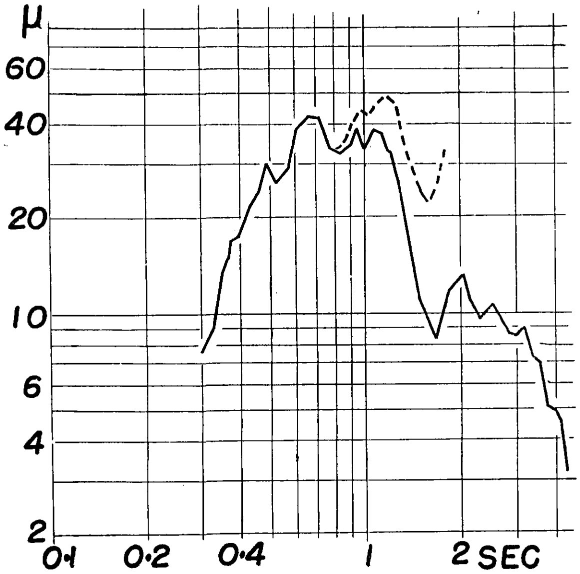

Fig. 15. Displacement spectrum for No. 428 earthquake.

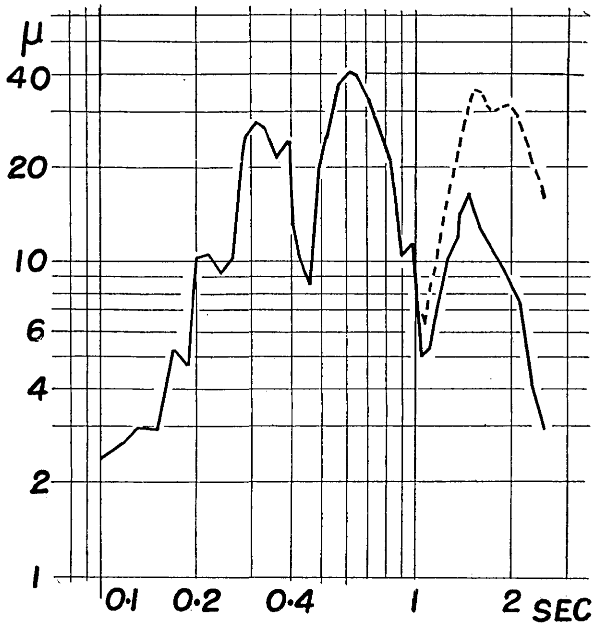

Fig. 16. Displacement spectrum for No. 440 earthquake.

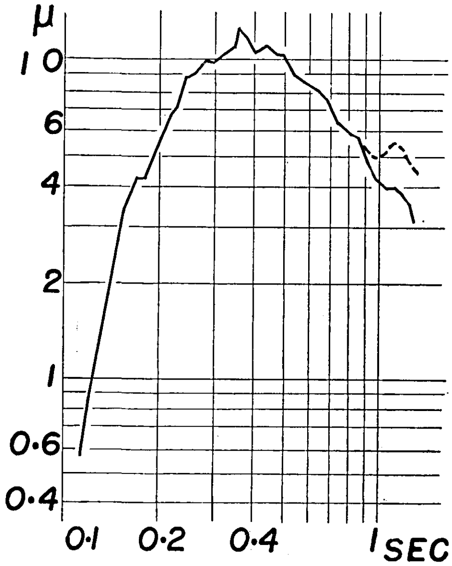

Fig. 17. Displacement spectrum for No. 441 earthquake.

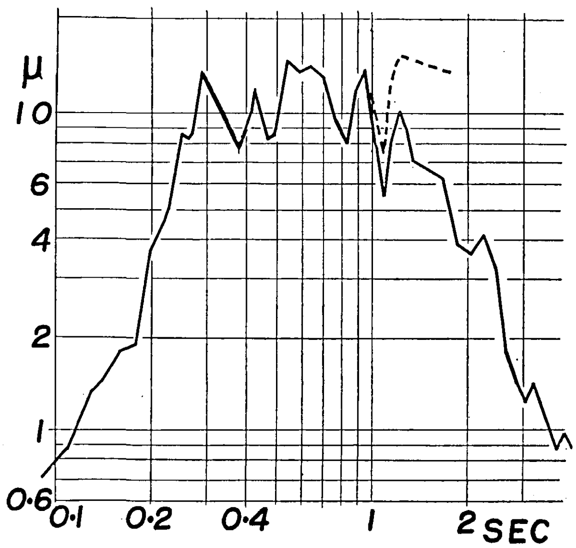

Fig. 18. Displacement spectrum for No. 446 earthquake.

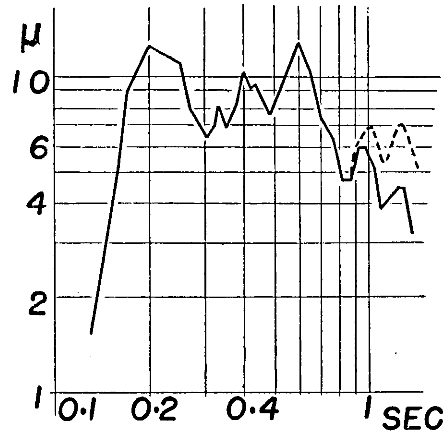

Fig. 19. Displacement spectrum for No. 499 earthquake.

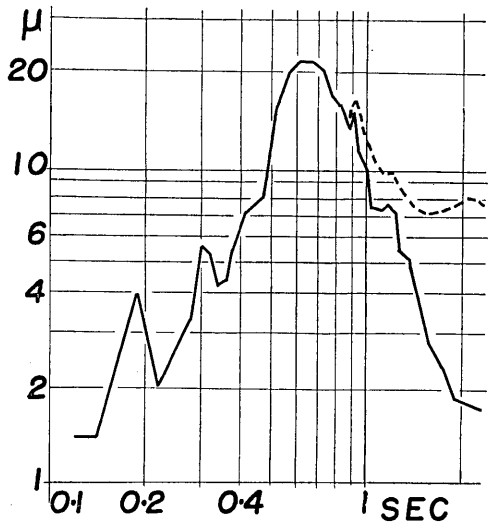

Fig. 20. Displacement spectrum for No. 520 earthquake.

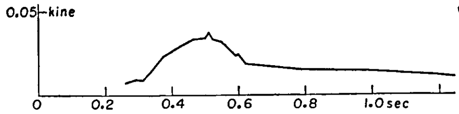

Fig. 21. Velocity spectrum for No. 170 earthquake.

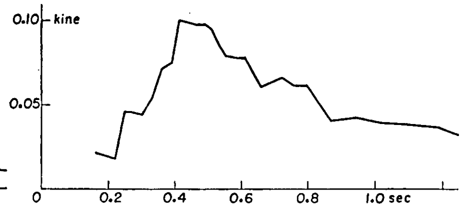

Fig. 22. Velocity spectrum for No. 173 earthquake.

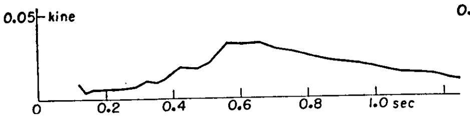

Fig. 23. Velocity spectrum for No. 175 earthquake.

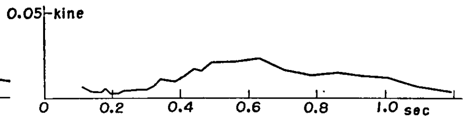

Fig. 24. Velocity spectrum for No. 200 earthquake.

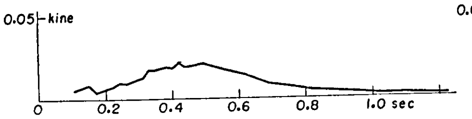

Fig. 25. Velocity spectrum for No. 279 earthquake.

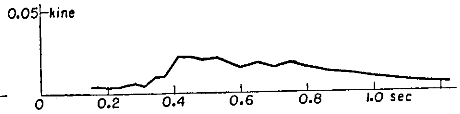

Fig. 26. Velocity spectrum for No. 280 earthquake.

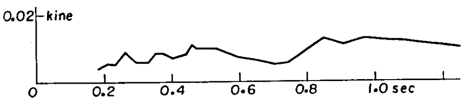

Fig. 27. Velocity spectrum for No. 314 earthquake.

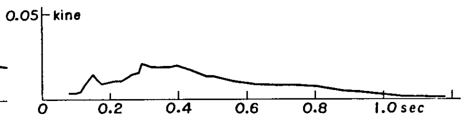

Fig. 28. Velocity spectrum for No. 378 earthquake.

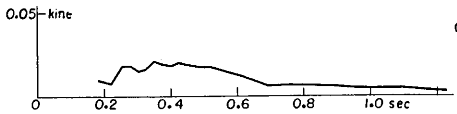

Fig. 29. Velocity spectrum for No. 409 earthquake.

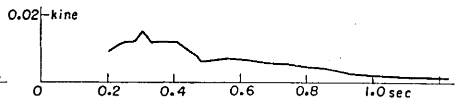

Fig. 30. Velocity spectrum for No. 420 earthquake.

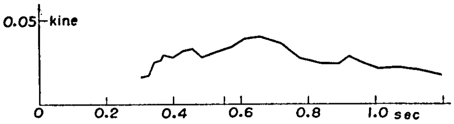

Fig. 31. Velocity spectrum for No. 428 earthquake.

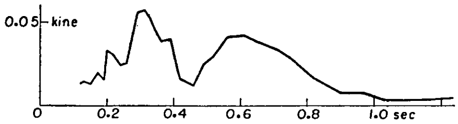

Fig. 32. Velocity spectrum for No. 440 earthquake

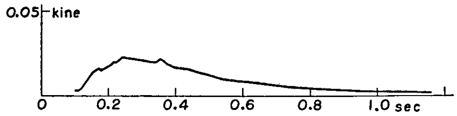

Fig. 33. Velocity spectrum for No. 441 earthquake.

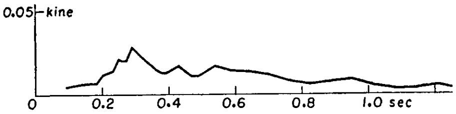

Fig. 34. Velocity spectrum for No. 446 earthquake.

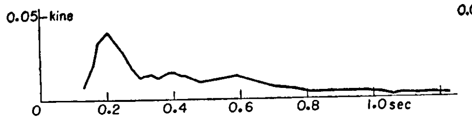

Fig. 35. Velocity spectrum for No. 499 earthquake.

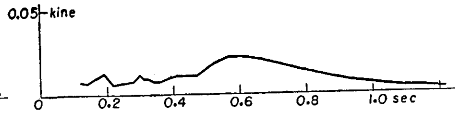

Fig. 36. Velocity spectrum for No. 520 earthquake.

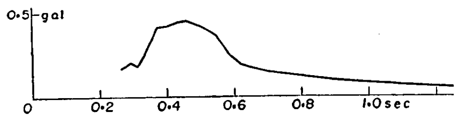

Fig. 37. Acceleration spectrum for No. 170 earthquake.

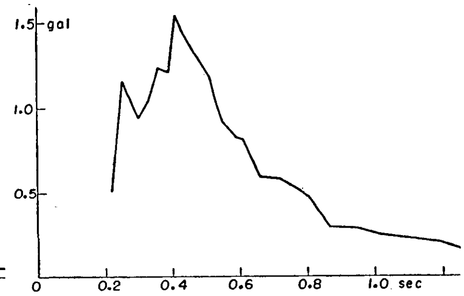

Fig. 38. Acceleration spectrum for No. 173 earthquake.

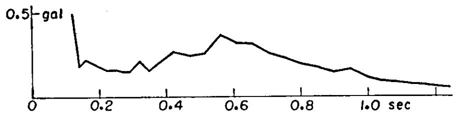

Fig. 39. Acceleration spectrum for No. 175 earthquake.

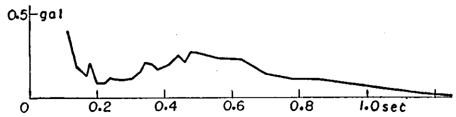

Fig. 40. Acceleration spectrum for No. 200 earthquake.

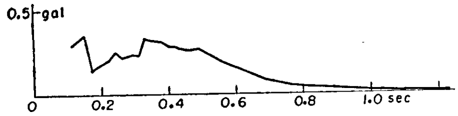

Fig. 41. Acceleration spectrum for No. 279 earthquake.

Fig. 42. Acceleration spectrum for No. 280 earthquake.

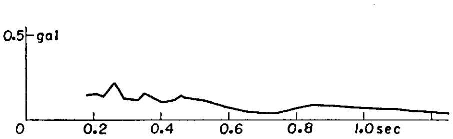

Fig. 43. Acceleration spectrum for No. 314 earthquake.

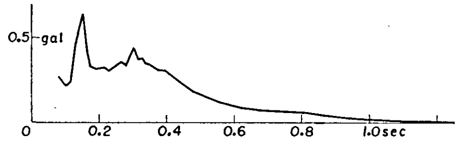

Fig. 44. Acceleration spectrum for No. 378 earthquake.

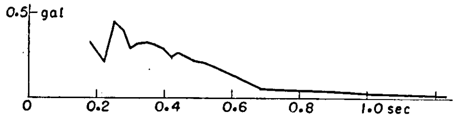

Fig. 45. Acceleration spectrum for No. 409 earthquake.

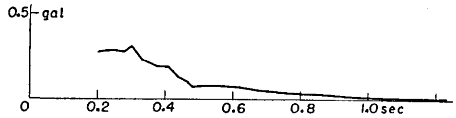

Fig. 46. Acceleration spectrum for No. 420 earthquake.

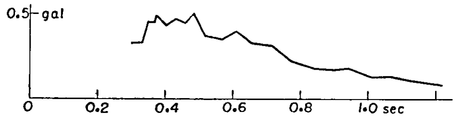

Fig. 47. Acceleration spectrum for No. 428 earthquake.

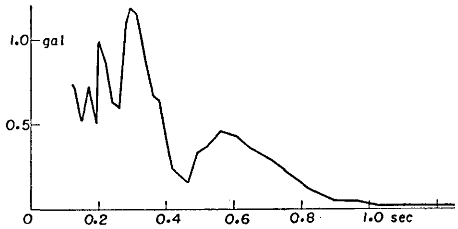

Fig. 48. Acceleration spectrum for No. 440 earthquake.

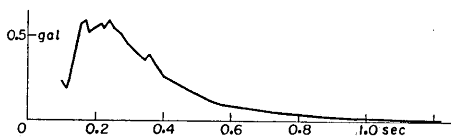

Fig. 49. Acceleration spectrum for No. 441 earthquake.

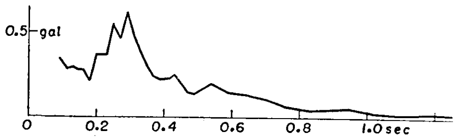

Fig. 50. Acceleration spectrum for No. 446 earthquake.

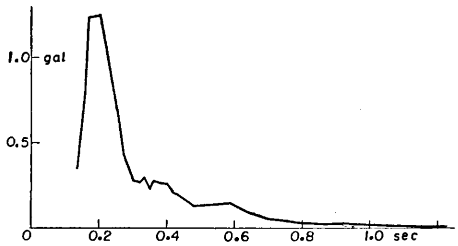

Fig. 51. Acceleration spectrum for No. 499 earthquake.

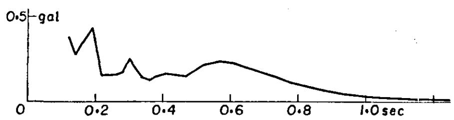

Fig. 52. Acceleration spectrum for No. 520 earthquake.

15. 地震動の振幅と周期の関係

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地震動は，普通，いわゆる地盤の影響で非常に複雑な波形になり，その本性を調べるのが困難である．そこで，地盤による地震動の変調をできるだけ避けようとして，日立鉱山の坑内で地震観測を始めてから，10 年以上になる．今回の報告は，その期間に，地下 300 m で得られた地震記象の中から比較的満足な記録をしたものをえらんで周期解析を行い，振幅と周期の関係をしらべた結果である．

先づ，本研究で，地震動の変位スペクトルには，或周期で山ができ，地震が大きくなる程，その山になる周期が長くなることが一層明かになつた．震央距離 100 km における地震動スペクトルの山になる振幅 $A_{m\cdot s}$ と，その周期 T_m との関係式として，次の実験式が得られた．即ち，

$$A_{m\cdot s}=53T_m^{2.56} \text{ ミクロン} \qquad (1)$$

次に，地震動に含まれる各周期の波を単弦振動として，変位スペクトルから速度スペクトルを作つてみると，かなり広い範囲の周期にわたつて，速度スペクトルがほぼ一定であることがわかつた．即ち，地震動は，工学的に問題になりそうな周期の範囲内では，大体，勢力の等分配の法則を満足させていると考えてよいことがわかつたわけである．