26. *Vibration of a Building upon the Elastic Foundation.*

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1. Introduction

Today the oscillations of buildings are being very actively studied, and the problem of the effect of foundations is also being discussed. In these investigations, however, when the oscillations of buildings are discussed, the motion of the ground is assumed not to be affected by the buildings, while in the discussion of the foundation the existence of buildings are always neglected. It is evident, however, that such a method of investigation is not adequate. We must solve for the coupled motion of buildings and their foundations.

The calculation of such a coupled system is not easy, but fortunately I. Toriumi[1)] recently published an elaborate study, which makes it possible to obtain the result without special difficulty.

In this paper we utilize his result, consequently the assumptions adopted in his study are also assumed here: namely

1) Buildings are supposed to be circular cylinders.
2) In the problems of vertical and horizontal oscillations, the stress components are assumed to be constant within the circle of a base and vanish outside of it.
3) In the problem of rocking, the stress is proportional to x within the circle and zero outside.

We further assume, in the present paper, that the seismic waves come vertically upward and the ground motion is a pure vertical of horizontal one.

2. Notations

Notations in the paper of Toriumi are also adopted with only slight change, but many new ones are added, so all are given in this section.

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1) I. TORIUMI, "Vibrations in Foundations of Machines", *Technology Reports of the Osaka University*, **5** (1955), 103.

I. TORIUMI, "Vibrations in Foundation of Machine on the Ground," *Journ. Seism. Soc., Japan*, [ii], **7** (1955), 216, (in Japanese).

A	$=2\mu\pi r_0/(f_{1H}+if_{2H})$. cf. (4.6).
A'	$=A/m_0p^2$. cf. (4.17).
A_V, A_H	Amplitudes of incident waves vertical and horizontal respectively.
a_0	$=pr_0\sqrt{(\rho/\mu)}=pr_0/V_s=2\pi r_0/$(Wave length of S-waves).
a_{0MAX}	Value of a_0 that gives the maximum magnification.
B	$=(1/4)\mu\pi r_0^3/(f_{1R}+if_{2R})$. cf. (4.11).
B'	$=B/m_0p^2l_0^2$. cf. (4.17).
c	Magnification coefficient of a forced oscillation of a seismograph.
f_{1H}, f_{2H}	cf. (4.6). f_1 and f_2 in Toriumi's paper. (Horizontal motion)
f_{1R}, f_{2R}	cf. (4.11). id. (Rocking motion)
f_{1V}, f_{2V}	cf. (3.4). id. (Vertical motion)
h	Damping coefficient of a pendulum. $(=\varepsilon/n)$
h_{apparent}	Apparent damping coefficient of a building.
I_0	Moment of inertia of a building around its center of gravity.
k_0	Radius of gyration of a building.
l_0	Height of a building from its base to the center of gravity.
m_0	Mass of a building.
$\overline{M}_{z=0}$	Moment of a force.
n	Frequency of the free oscillation of a building.
p	Circular frequency of incident waves.
Q_H, Q_V	Horizontal and vertical force transmitted from a building to the ground.
r_0	Radius of a building.
S	$=m_0p^2/2\pi\mu r_0$.
S_0	$=S/a_0^2=(\rho_0/\rho)\cdot(l_0/r_0)=(\rho_0/\rho)\cdot\beta$.
T_0	Period of the free oscillation of a building.
T_{MAX}	Period of the incident wave giving the maximum amplitude.
U_1	Displacement at the free surface. (Horizontal motion)
U_G	Displacement of the center of base. (Rocking motion)
U_D	Additional displacement of the center of the base caused by the existence of a building. (Horizontal motion)
U_H	Displacement of the center of gravity. (Horizontal motion)
U_R	Horizontal displacement of the center of gravity. (Rocking motion)
u	Ratio of the frequency of the free oscillation of a building and incident waves. $(=n/p)$
V_{MAX}	Maximum amplitude.

W	Displacement of the center of gravity. (Vertical motion)
W_1	Displacement of the free surface. (Vertical motion)
X	$=\mathfrak{B}_{UR}\cdot\exp(-i\vartheta_{UR})$.
Y	$=\mathfrak{B}_{\Gamma R}\cdot\exp(-i\vartheta_{\Gamma R})$.
Z	$=\mathfrak{B}_{H}\cdot\exp(-i\vartheta_{H})$.
α	$=k_0/l_0$.
β	$=l_0/r_0$.
Γ_R	Angle of inclination of a building. (Rocking motion)
γ	id. (Toriumi's notation)
ν	$=a_0(n/p)$. cf. (5.12)
λ, μ	Lamé's constants of the foundation.
ρ, ρ_0	Density of the foundation and building respectively.
ϑ_H, ϑ_V	Phase difference between the incident waves and the oscillation of the building. (Horizontal and vertical motion)
ϑ_{UR}, $\vartheta_{\Gamma R}$	Phase difference between the incident waves and the horizontal or angular motion. (Rocking motion)
$\mathfrak{B}_H$, $\mathfrak{B}_V$	cf. (5.1) and (3.2).
$\mathfrak{B}_{UR}$, $\mathfrak{B}_{\Gamma R}$	cf. (4.2) and (4.3).

3. Vertical motion

First we consider the problem of vertical motion. Incident waves are assumed to be propagated vertically upward. The amplitude of the waves is A_V and the circular frequency is p. The method of solving this problem is briefly illustrated in Fig. 1.

The oscillation of the free surface would be, were it not for the building

$$W_1=2A_V\exp(ipt). \tag{3.1}$$

Displacement of the building caused by this wave is assumed to be

$$W=2A_V\exp(ipt)\cdot\mathfrak{B}_V\exp(-i\vartheta_V), \tag{3.2}$$

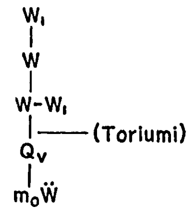


Fig. 1.

where $\mathfrak{B}_V$ and ϑ_V are both supposed to be real. Our present purpose is to obtain these two quantities as functions of material constants and of dimensions of the building.

An additional displacement caused by the existence of a building is

$$W-W_1=2A_V\exp(ipt)\cdot\{\mathfrak{B}_V\exp(-i\vartheta_V)-1\}. \tag{3.3}$$

According to Toriumi's theory, the relation between the vertical force transmitted from the structure to the foundation and the vertical displacement of the center of the base is given by the following expression.

$$\text{force}^{2)}: \qquad -Q_V \exp(ipt)\,,$$
$$\text{displacement}: \qquad Q_V \exp(ipt)\cdot\{f_{1V}+if_{2V}\}/\mu\pi r_0. \tag{3.4}$$

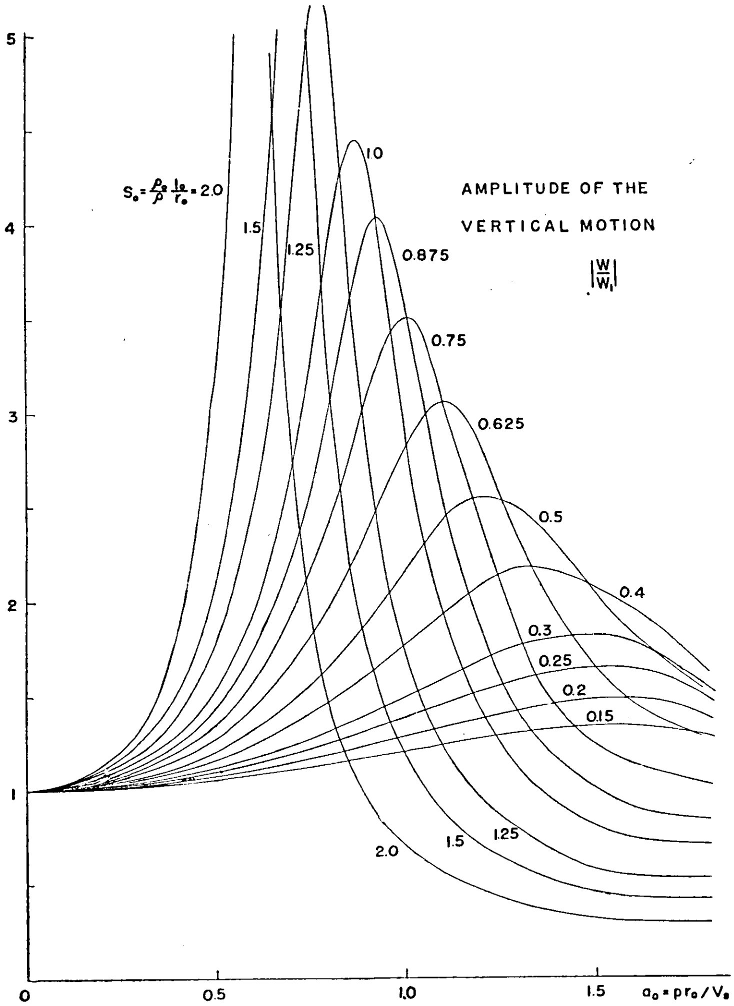


Fig. 2.

Therefore the force from the building to the foundation, corresponding to the displacement given in (3.3) is

2) His force is assumed to be plus when it acts toward the direction of $-z$.

$$-(W-W_1)\cdot\mu\pi r_0/\{f_{1V}+if_{2V}\}\ . \tag{3.5}$$

The force from the foundation to the building is

$$+(W-W_1)\cdot\mu\pi r_0/\{f_{1V}+if_{2V}\}$$
$$=2A_V\exp(ipt)\cdot\{\mathfrak{B}_V\exp(-i\vartheta_V)-1\}\cdot\mu\pi r_0/\{f_{1V}+if_{2V}\}\ . \tag{3.6}$$

This force makes the building with a mass m_0 have an acceleration $\ddot{W}$, therefore

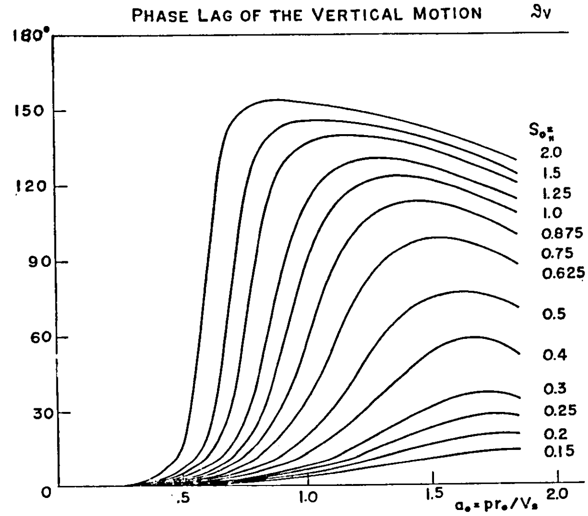


Fig. 3.

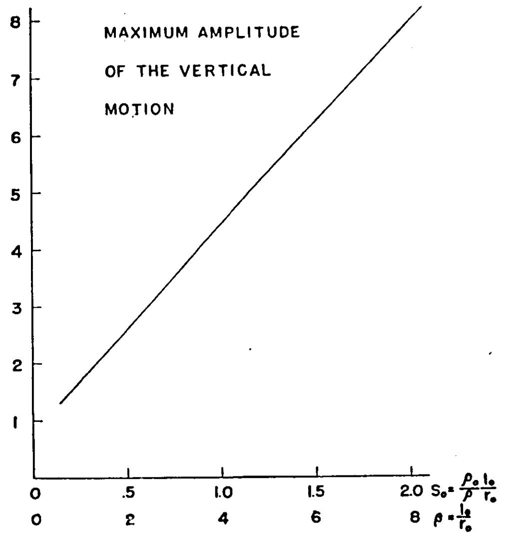


Fig. 4.

$$m\ddot{W}=m_0(-p^2)\cdot 2A_V \exp(ipt)\cdot \mathfrak{V}_V \exp(-i\vartheta_V) \qquad (3.7)$$

must be equal to the expression given in (3.6).

$$\mathfrak{V}_V \exp(-i\vartheta_V)=1/[1+2S\{f_{1V}+if_{2V}\}], \qquad (3.8)$$

where

$$S=m_0p^2/2\mu\pi r_0=a_0^2(\rho_0/\rho)\cdot(l_0/r_0)=a_0^2S_0\,.$$

ρ_0 and ρ do not change so widely, consequently S is chiefly determined by $\beta=l_0/r_0$, namely by the ratio of the height and the radius of a building. Given the value of the parameter S_0 we can express $\mathfrak{V}_V$ and ϑ_V as functions of a_0 $(=pr_0/V_s=2\pi r_0/\text{wave-length of } S\text{-waves})$. In this expression, f_{1V} and f_{2V} are f_1 and f_2 with regard to the vertical motion in Toriumi's paper and are obtained from Fig. 2 in his paper. The result of our calculation is given in Figs. 2–3. The form of the curve of $\mathfrak{V}_V$ is similar to that of the magnification function of a seismograph, but that of the phase lag is somewhat different. In Fig. 4 the maximum amplitude is given as a function of S_0. For comparison with the result given in later chapters, we gave the value of β in the abscissa assuming ρ/ρ_0 to be equal to 4. Compared with the cases of rocking and horizontal motion maximum amplitude is fairly small.

4. Rocking motion

Next we consider the case of the incident waves being purely horizontal. Amplitude and the circular frequency of the incident wave are A_H and p respectively. and the building is assumed to be rigid

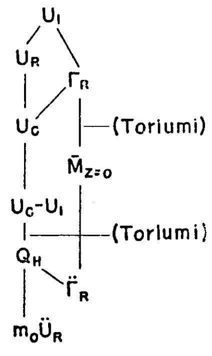


Fig. 5.

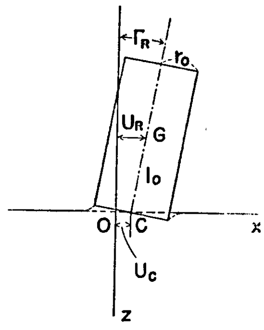


Fig. 6.

enough to ignore its free oscillations. Horizontal as well as rocking motion are considered and the scheme of the solution is given in Fig. 5.

If there were no building the oscillation at the free surface $z=0$ would be

$$U_1=2A_H \exp(ipt) . \tag{4.1}$$

Displacement of the building caused by the incidence of waves is (cf. Fig. 6)

$$U_R=2A_H \exp(ipt)\cdot \mathfrak{V}_{UR} \exp(-i\vartheta_{UR})\equiv 2A_H \exp(ipt)\cdot X . \tag{4.2}$$

Deflection angle Γ_R (clockwise is taken to be plus) is

$$\Gamma_R=2A_H \exp(ipt)\cdot \mathfrak{V}_{\Gamma R} \exp(-i\vartheta_{\Gamma R})\equiv 2A_H \exp(ipt)\cdot Y . \tag{4.3}$$

As is clear from Fig. 6, the displacement of the center of the base is

$$U_G=U_R-l_0\Gamma_R\equiv 2A_H \exp(ipt)\cdot\{X-l_0Y\} . \tag{4.4}$$

The additional displacement of the foundation caused by the existence of a building is

$$U_G-U_1=2A_H \exp(ipt)\cdot\{X-l_0Y-1\} . \tag{4.5}$$

According to the study of I. Toriumi the displacement U' caused by a force $Q_H \exp(ipt)$ acting from a body to the ground in the direction $+x$ is

$$U'=Q_H \exp(ipt)\cdot\frac{1}{2\pi\mu r_0}\{f_{1H}+if_{2H}\}\equiv Q_H \exp(ipt)/A . \tag{4.6}$$

If the displacement is given, the corresponding force is

$$Q_H \exp(ipt)=U'A . \tag{4.7}$$

Therefore the force acting from the ground to a building corresponding to the displacement in (4.5) is, changing the sign

$$\begin{aligned}-Q_H \exp(ipt)&=-(U_G-U_1)A\\ &=-2A_H \exp(ipt)\cdot\{X-l_0Y-1\}\cdot A .\end{aligned} \tag{4.8}$$

Since this force causes the displacement U_R of the center of gravity

$$m_0\ddot{U}_R=-Q_H \exp(ipt) . \tag{4.9}$$

Substituting from (4.2) and (4.8) we have

$$m_0p^2X=\{X-l_0Y-1\}A . \tag{4.10}$$

Next we will consider the rotational motion. We will quote again from the paper of I. Toriumi.

When the moment of the force acting from a building to the ground is given by $\bar{M}_{z=0}$ (Counter-clockwise is taken to be plus.) the angular deflection γ is

$$\gamma=\frac{4\bar{M}_{z=0}}{\pi r_0^3\mu}\exp(ipt)\cdot\{f_{1R}+if_{2R}\}\equiv\bar{M}_{z=0}\exp(ipt)/B\,. \qquad (4.11)$$

This expression also gives the deflection when the moment of a force acting from the ground to a building is $\bar{M}_{z=0}$ (taking the clockwise direction plus).

If the deflection angle is given the moment of a force is

$$\bar{M}_{z=0}\exp(ipt)=\gamma B\,. \qquad (4.12)$$

When the deflectional angle is expressed by Γ_R in the equation (4.3)

$$\bar{M}_{z=0}\exp(ipt)=\Gamma_R B=2A_H\exp(ipt)\cdot YB\,. \qquad (4.13)$$

Then the moment of a horizontal force $-Q_H\exp(ipt)$ around the center of gravity G acting on the base of a building is

$$l_0Q_0\exp(ipt)=l_0 2A_H\exp(ipt)\cdot\{X-l_0Y-1\}A\,. \qquad (4.14)$$

The angular motion Γ_R is caused by the moment expressed in (4.13) and (4.14), therefore we have

$$I_0\ddot{\Gamma}_R=\bar{M}_{z=0}\exp(ipt)+l_0Q_H\exp(ipt)\,. \qquad (4.15)$$

Substituting from (4.3), (4.13) and (4.14)

$$-m_0k_0^2p^2Y=YB+l_0\{X-l_0Y-1\}A\,. \qquad (4.16)$$

(4.10) and (4.16) are the simple linear equations of X and Y, so we can easily obtain the next solutions.

$$\left.\begin{aligned} l_0Y\equiv Y'&=-1\Big/\left\{\left(1-\frac{1}{A'}\right)(B'+\alpha^2)+1\right\}\\ X&=(B'+\alpha^2)\Big/\left\{\left(1-\frac{1}{A'}\right)(B'+\alpha^2)+1\right\}, \end{aligned}\right\} \qquad (4.17)$$

where

$$A' \equiv A/m_0p^2 = 1/S(f_{1H} + if_{2H}) ,$$
$$B' \equiv B/m_0p^2l_0^2 = 1/8\beta^2 S(f_{1R} + if_{2R}) ,$$

and

$$\alpha^2 \equiv k_0^2/l_0^2 ,$$

$$S = m_0p^2/2\pi\mu r_0 = a_0^2 S_0 = a_0^2 \frac{\rho_0}{\rho}\frac{l_0}{r_0} . \qquad (4.18)$$

Introducing the relations (cf. (4.6) and (4.11).)

$$A = 2\pi\mu r_0 / \{f_{1H} + if_{2H}\} ,$$

and

$$B = \pi\mu r_0^3/4\{f_{1R} + if_{2R}\} ,$$

into the above expressions and somewhat modifying we have

$$Y' = 1 \Big/ \left[a_0^2 S_0 (f_{1H} + if_{2H})\alpha^2 + \left\{ -(1+\alpha^2) + \frac{1}{8\beta^2} \frac{f_{1H} + if_{2H}}{f_{1R} + if_{2R}} \right\} - \frac{1}{a_0^2} \frac{1}{8\beta^2 S_0 (f_{1R} + if_{2R})} \right] ,$$

$$X = -\left\{ \alpha^2 + \frac{1}{8\beta^2 a_0^2 S_0 (f_{1R} + if_{2R})} \right\} \Big/ \Big[\qquad '' \qquad \Big] . \qquad (4.19)$$

Y' is, in a few words, a magnification coefficient of a building with regard to the rotational motion, and if the center of gravity remains unmoved

$$Y' = -1 . \qquad (4.20)$$

X is the magnification with respect to the translation, and if the building moves with just the same motion as the foundation, the magnification proves to be

$$X = 1 . \qquad (4.21)$$

The result of numerical calculations is given in Figs. 7–15.

Fig. 7 shows $\mathfrak{V}_{UR} = |U_R/U_1| = |X|$, namely the magnification of the displacement of the center of gravity. The parameter in this figure is $\beta = l_0/r_0$. In this calculation a simple relation $\rho_0/\rho = 1/4$ is assumed, since

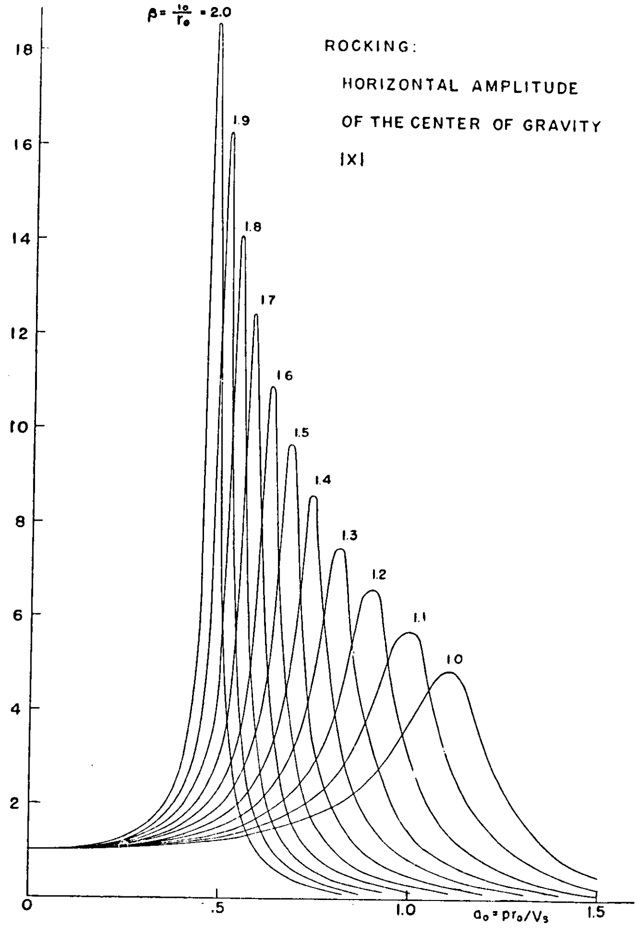


Fig. 7.

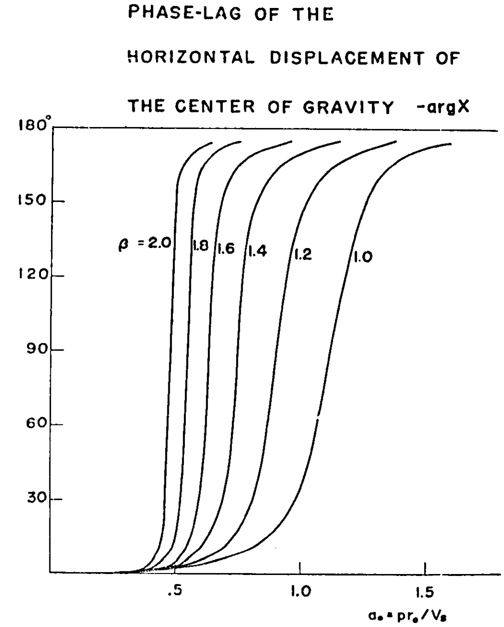


Fig. 8.

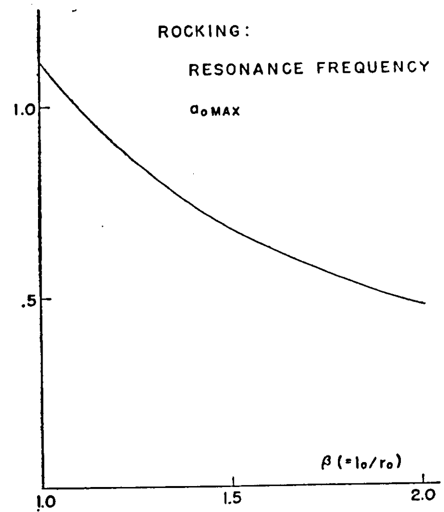


Fig. 10.

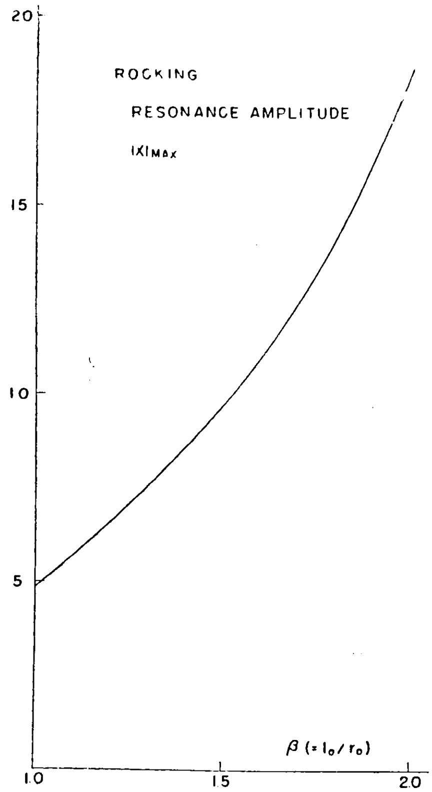
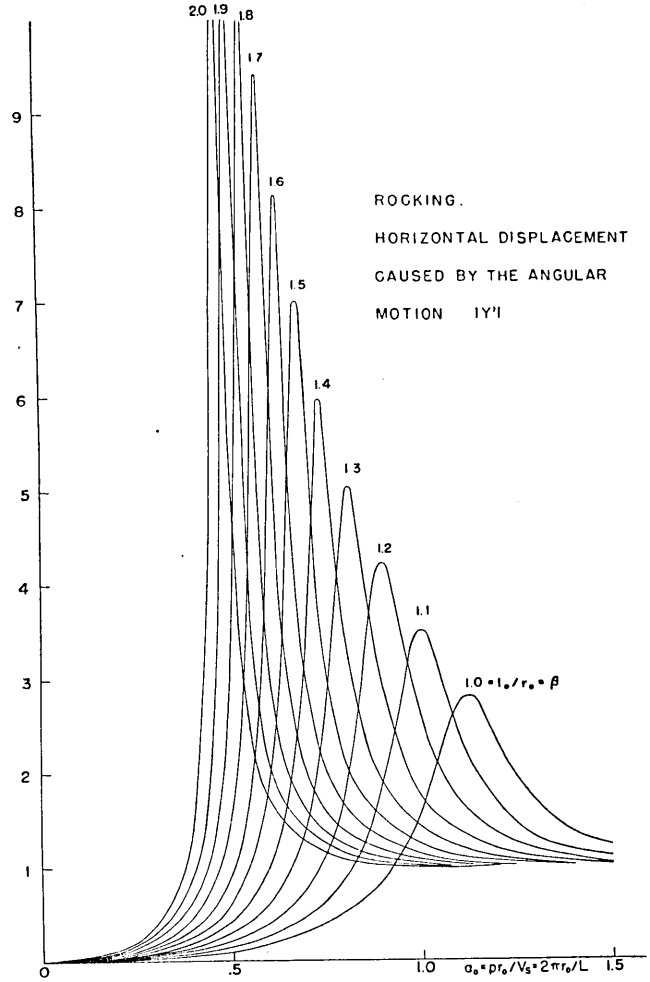


Fig. 9.



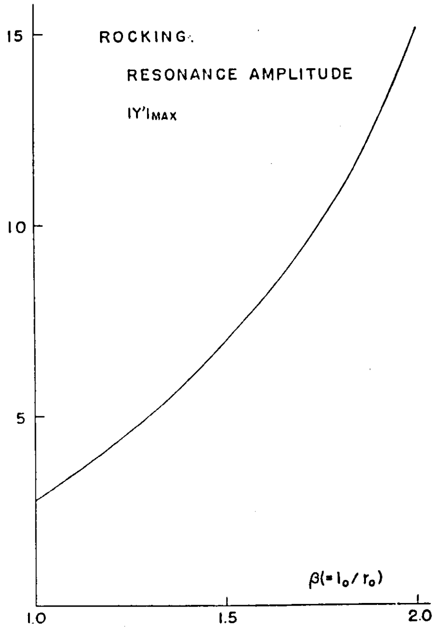


Fig. 13.

Fig. 11.

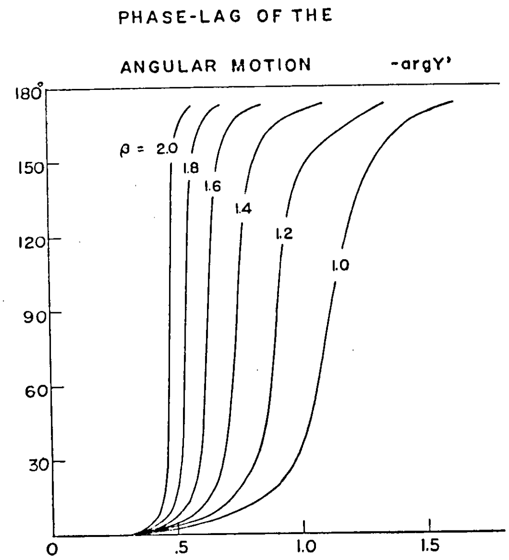


Fig. 12.

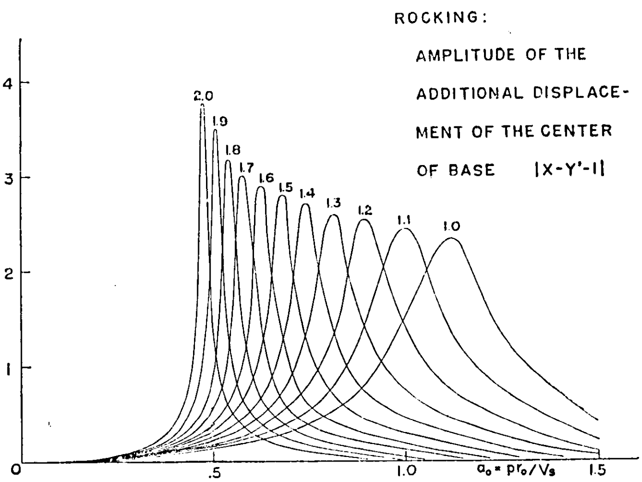


Fig. 14.

it is too complicated to have two independently changeable parameters ρ_0/ρ and l_0/r_0. From (4.18) we have simply $S_0=l_0/4r_0=\beta/4$, and the curves in the figure give the cases $S_0=0.25, 0.275, 0.3, \cdots 0.475$ and 0.5.

Fig. 8 gives the phase lag of the same problem, namely $\vartheta_{UR}=-\arg X=-\arg U_R/U_1$.

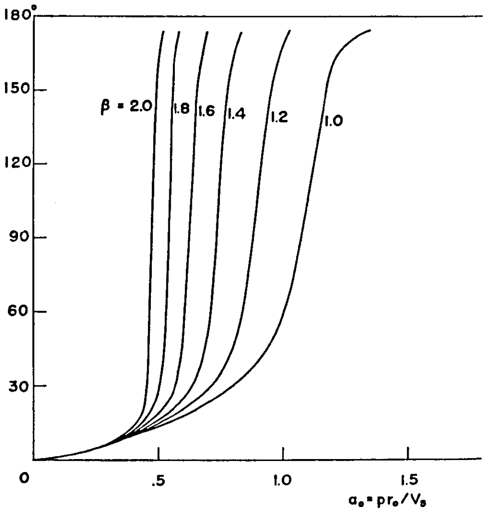


Fig. 15.

Fig. 9 gives the resonance amplitude as a function of $\beta=l_0/r_0$ and Fig. 10 shows the value of a_0 corresponding to this maximum amplitude. $\beta=4S_0$ is assumed in this calculation, too.

Figs. 11–13 give the results of similar calculations with regard to the rotation, or rocking, of a building. Fig. 11 is the graph of the angular amplitude $l_0\mathfrak{B}_{FR}=|Y'|=|l_0\Gamma_R/U_1|$ and Fig. 12 is the phase lag $\vartheta_{FR}=-\arg Y=-\arg(\Gamma_R/U_1)$

Fig. 13 gives the maximum magnification, but the corresponding value of a_0 is not given, since it is almost equal to the value given in Fig. 10.

Fig. 14 gives $|X-Y'-1|$, the ratio of the motion of the base center to that of the ground without building. If the base of a building moves exactly the same as the free surface, this quantity will become identically equal to 1, but the calculated curves have maxima and show that the movement is not the same. This suggests that even a seismograph installed at the basement of a building cannot give the unaltered motion of incident waves.

Fig. 15 gives the phase lag.

5. Horizontal motion

Suppose we have a system composed of the single mass shown in Fig. 16. We will calculate the horizontal motion corresponding to the shear vibration caused by a purely horizontal vibration incident from vertically below. The boundary condition at the base of a building is assumed, as is done by I. Toriumi, to be X_z=const.

The course of our calculation is given in Fig. 17.

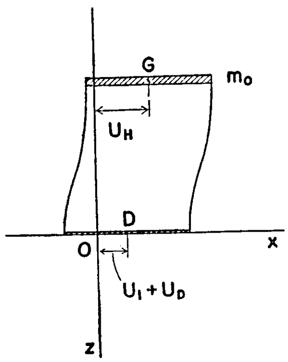


Fig. 16.

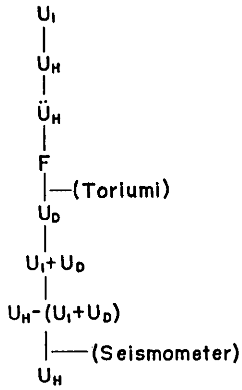


Fig. 17.

The incident horizontal motion at the surface $z=0$ is, if there is not the effect of a building,

$$U_1=2A_H \exp(ipt) .$$

We denote the displacement of the center of gravity G, referred to the inertial system,

$$U_H=2A_H \exp(ipt)\cdot \mathfrak{B}_H \exp(-i\vartheta_H)$$
$$\equiv 2A_H \exp(ipt)\cdot Z. \tag{5.1}$$

A force acting from the ground to m_0 is

$$m_0\ddot{U}_H=2A_H \exp(ipt)\cdot m_0(-p^2)Z , \tag{5.2}$$

and the force from m_0 to the ground is

$$-m_0\ddot{U}_H . \tag{5.3}$$

According to the theory of Toriumi, as quoted in (4.6) before, the displacement U' of the ground caused by $Q_H \exp(ipt)$, a force acting in the direction of x from a mass to the ground, is

$$U'=Q_H \exp(ipt)/A .$$

If, instead of $Q_H \exp(ipt)$, we put (5.3) into the above expression, we obtain that additional displacement U_D at the base center of a building, caused by the existence of the building,

$$U_D = 2A_H \exp(ipt) \cdot m_0 p^2 Z/A \ . \tag{5.4}$$

The displacement of the base is, adding the displacement of the free surface,

$$U_1 + U_D = 2A_H \exp(ipt) \cdot \{1 + m_0 p^2 Z/A\} \ . \tag{5.5}$$

The displacement of m_0 referred to the moving coordinates given by (5.5) is

$$U_H - (U_1 + U_D) = 2A_H \exp(ipt) \{Z - 1 - m_0 p^2 Z/A\} \ , \tag{5.6}$$

which is equal to the motion of a pendulum, having the same free period and damping coefficient, and being excited by the motion of a supporting point given by the equation (5.5). Therefore we can apply the well-known theory of a seismograph.

If we denote the motion of the earth by x, and the displacement of a pendulum referred to moving coordinates by y, then the next equation holds, which connects x and y:

$$\frac{d^2y}{dt^2} + 2\varepsilon \frac{dy}{dt} + n^2 y = -\frac{d^2x}{dt^2} \ . \tag{5.7}$$

Assuming the form

$$x = x_m \exp(ipt) \ ,$$
$$y = y_m \exp(ipt) \ ,$$

we have

$$y = cx \ , \tag{5.8}$$

where

$$c = 1/\{(u^2 - 1) + i2hu\} \ ,$$
$$u = n/p \ , \quad h = \varepsilon/n \ .$$

Putting

$$x \rightarrow U_1 + U_D \ ,$$
$$y \rightarrow U_H - (U_1 + U_D) \ , \tag{5.9}$$

we have from (5.5) and (5.6)

$$\begin{aligned} 2A_H \exp(ipt) \cdot \{Z - 1 - m_0 p^2 Z/A\} \\ = c \cdot 2A_H \exp(ipt) \cdot \{1 + m_0 p^2 Z/A\} \ . \end{aligned} \tag{5.10}$$

Therefore

$$Z=1\Big/\left[\frac{1}{1+c}-\frac{m_0p^2}{A}\right]. \tag{5.11}$$

Now, if we put

$$u=\frac{n}{p}=\frac{n}{a_0V_s/r_0}=\frac{1}{a_0}\nu\,, \tag{5.12}$$

we have the following expression using (4.6) and (5.8)

$$Z=1\Big/\left[1-\left(\frac{\nu^2}{a_0^2}-i2h\frac{\nu}{a}\right)^{-1}-a_0^2S_0(f_{1H}+if_{2H})\right]. \tag{5.13}$$

If the building itself has no damping, we may put $h=0$, so that

$$Z\,(\text{no damp})=1/[1-a_0^2/\nu^2-a_0^2S_0(f_{1H}+if_{2H})]\,. \tag{5.14}$$

The additional displacement caused by the existence of the building is, using (5.4) and (5.13)

$$U_D=2A_H\exp{(ipt)}\cdot a_0^2S_0(f_{1H}+if_{2H})$$

$$\div\left[1-\left(\frac{\nu^2}{a_0^2}-i2h\frac{\nu}{a}\right)^{-1}-a_0^2S_0(f_{1H}+if_{2H})\right]. \tag{5.15}$$

If the building itself has no damping

$$U_D=2A_H\exp{(ipt)}\cdot a_0^2S_0(f_{1H}+if_{2H})/[1-a_0^2/\nu^2-a_0^2S_0(f_{1H}+if_{2H})]\,. \tag{5.16}$$

In the case of resonance

$$n=p \quad \text{or} \quad a_0=\nu\,. \tag{5.17}$$

Introducing this relation, we have

$$U_D\,(\text{resonance})=-2A_H\exp{(ipt)}\,. \tag{5.18}$$

Therefore

$$U_1+U_D\,(\text{resonance})=0\,, \tag{5.19}$$

which implies that the base center of a building does not move at all. An interesting property.

A general expression for the motion of the base is

$$U_1+U_D=2A_H\exp{(ipt)}\cdot\left\{1-\left(\frac{\nu^2}{a_0^2}-i2h\frac{\nu}{a_0}\right)^{-1}\right\}Z\,. \tag{5.20}$$

When $h=0$

$$U_1+U_D=2A_H \exp(ipt) \cdot \{1-a_0^2/\nu^2\} Z(\text{no damp}). \qquad (5.21)$$

The results of numerical calculations are given in Figs. 18–24.

We have now three parameters a_0, S_0 and ν. Besides there is another one h which gives the damping property inherent in the building itself. To avoid a useless complication we will assume that the building itself is without damping ($h=0$), and show, as an example, the calculation of the case $\nu^2=n^2r_0^2/(\mu/\rho)=1$.

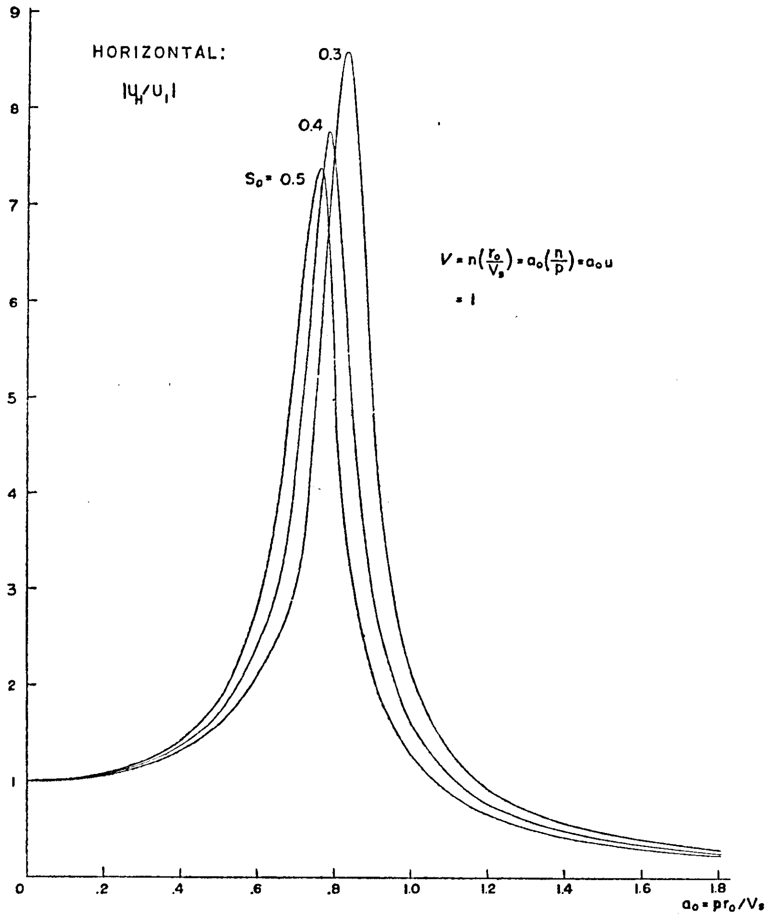


Fig. 18.

Fig. 18 gives the magnification $\mathfrak{V}_H$ with regard to the displacement of the center of gravity G (cf. (5.1)), and Fig. 19 the phase difference

ϑ_H (cf. (5.1)). Fig. 20 is the magnification with regard to the displacement of the center of the base of the building (cf. U_1+U_D in (5.5)). In this figure all the curves become zero at the point $a_0=1$, which fact was mentioned before in (5.19). The values $S_0=0.3$, 0.4 and 0.5 are adopted because, it seems, they can represent a short, medium and tall building.

Fig. 21 gives the maximum magnification. Abscissa is

$$1/\nu^2=(\mu/\rho)/n^2r_0^2 \ .$$

In the case of a simple pendulum there is a relation between the maximum magnification V_{MAX} and the damping coefficient

$$V_{MAX}=1/2h \ . \quad (5.22)$$

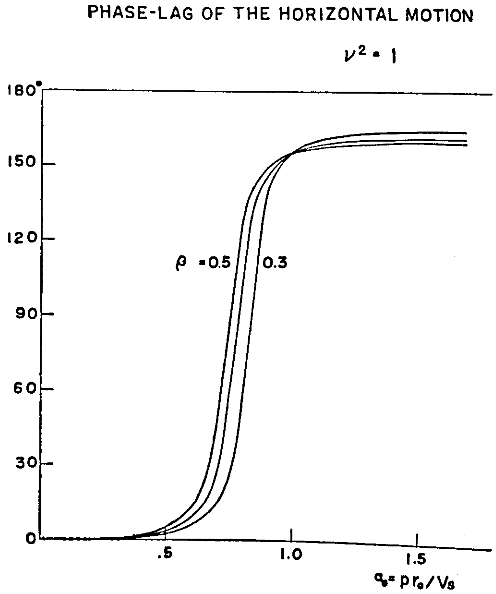


Fig. 19.

If we apply this relation to the above figure and calculate the apparent damping coefficient h_{apparent}, then we have the result shown in

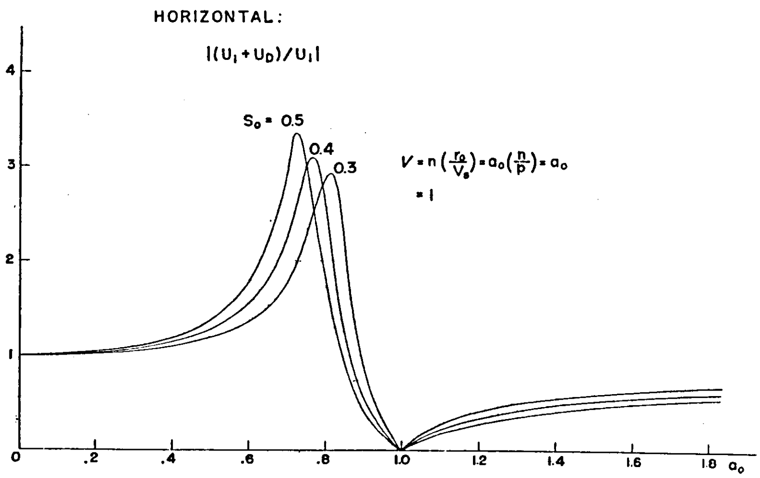


Fig. 20.

Fig. 22. This is the case when the building itself has no damping, and

h_{apparent} is caused only by the radiation of energy from the building into the elastic medium.

The value of a_0, giving V_{MAX} is shown in Fig. 23 as a function of

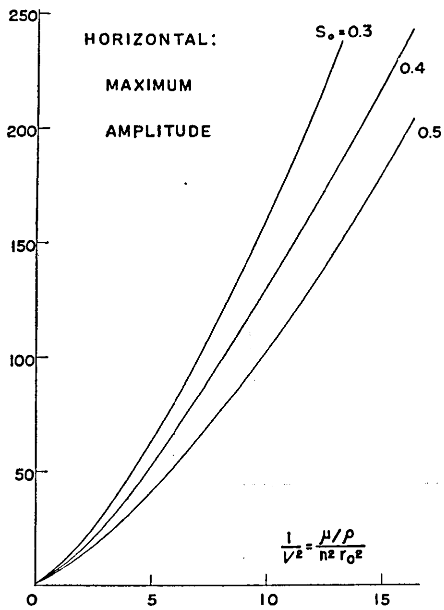


Fig. 21.

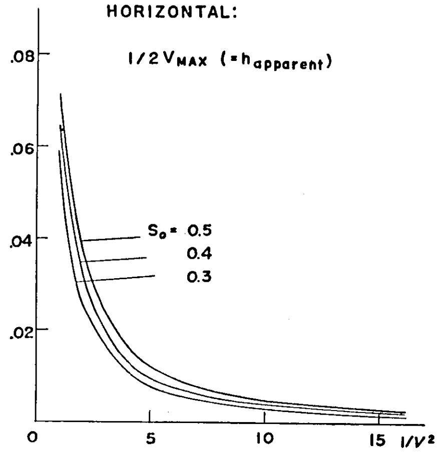


Fig. 22. The apparent damping coefficient when the building itself is without damping.

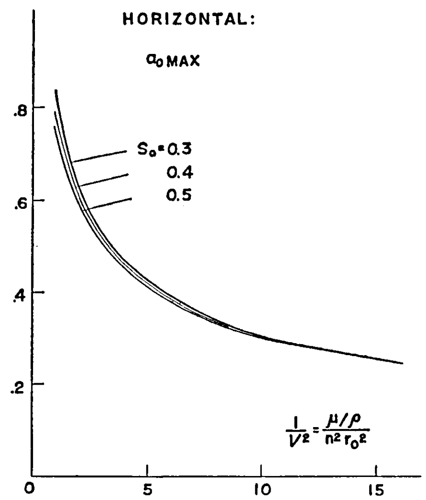


Fig. 23. The resonance frequency as a function of $1/\nu^2=(\mu/\rho)/(n^2r_0^2)$.

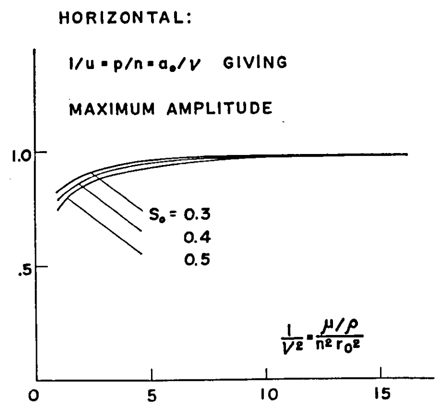


Fig. 24.

$1/\nu^2$. In a_0 and $1/\nu^2$, ρ and μ are both involved and the above figure is a little difficult to interpret, so a graph of $a_0/\nu=p/n=1/u$ is given in Fig. 24 as a function of $1/\nu^2$. When $1/\nu^2$ is large $1/u=p/n$ is nearly equal to 1. That means that in the case of resonance the periods of incident waves and of the building are nearly equal. (The former is always a little larger and the difference becomes important when $1/\nu^2$ becomes small.)

6. Elements which affect the maximum amplitude

In this section the effect of the density, rigidity and the dimension of the building on the maximum amplitude is discussed by means of the method of dimensional analysis.

If the period of the external force or the frequency of the incident seismic waves varies, the amplitude of the oscillation of a building varies accordingly and takes a maximum value corresponding to some value of the period of incident waves. We will denote this value as V_{MAX}.

As is in the case of a rigid building, if there is no such value as a natural frequency of a building, so long as we neglect the effect of the gravity, we can assume that V_{MAX} is a function of m_0, r_0, l_0; ρ, μ and p.

$$V_{MAX}=F(m_0, r_0, l_0;\ \rho, \mu;\ p)\ . \tag{6.1}$$

In this expression, μ and p are the only quantities which have the dimension of time. Therefore μ and p can be involved in the function F only by the combination $p/\sqrt{\mu}$ which does not have the dimension of time. Or we may say that p and μ are involved in the form $a_0=pr_0/\sqrt{(\mu/\rho)}$. Hence we can easily conclude that μ does not affect V_{MAX} directly, but only through a_0. The rigidity μ only affects the unit of the time axis, but does not affect V_{MAX}. This circumstance can be seen practically in the numerical calculation in § 3 and § 4.

The above discussion cannot hold if the building has its own natural frequency. Horizontal motion treated in § 5 is an example of such a case. V_{MAX} may represent any kind of the magnification: vertical, horizontal or rotational.

7. Approximate formulas giving the resonance period and the amplitudes[3)]

7.1 Rocking motion

In the case of rocking motion we can find that the resonance peaks of the translation as well as rotation appear corresponding to the period given by the following approximate formula

$$T_{MAX} \doteqdot 6(l_0/V_s)\,, \tag{7.1}$$

which is obtained from Fig. 10.

Also, using the Figs. 9, 10 and 13, we can deduce the next formula

$$|X|_{MAX} \doteqdot 5.7\, a_0^{-1.5}\,, \tag{7.2}$$

and

$$|Y'|_{MAX} \doteqdot 3.5\, a_0^{-2}\,, \tag{7.3}$$

which give maximum magnification factors of the displacement of the center of gravity and rotation of the building, respectively.

7.2 Horizontal motion

In a similar manner we can obtain the formulas in the case of the horizontal motion. A one-mass undamped system shows maximum response

$$|Z|_{MAX} \doteqdot 1.5\, S_0^{-1} a_0^{-3}\,, \tag{7.4}$$

for the incident wave with a period

$$T_{MAX} \doteqdot 1.3\, a_0^{0.2} T_0 \quad (0.3 < a_0 < 0.7)\,. \tag{7.5}$$

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3) The authors owe entirely to Dr. H. Kawasumi for this section. cf. H. KAWASUMI and K. KANAI, "Vibrations of buildings in Japan, Part I. Small amplitude vibrations of actual buildings".

26. 弾性地盤上に建つ建築物の振動

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1. 建築物の振動とこれに影響を及ぼす地盤問題も昨今盛んに多くの議論を生んでゐる．然しここでは建築物と地盤とを別々に扱ふ縦来の方法を離れて，それらを一体の振動系と見做し上下動，水平動及びロッキングの夫々の場合についての振動性能を求めてみた，計算のもとになる部分は鳥海氏によつて為されたものを使用することにし，従つて建物は円柱状と考へ，更に底面での力の作用の仕方も同氏がとられた仮定——上下及び水平動の場合には円内で応力一定，外部では 0，ロッキングの時には円内では x に比例する応力，外では 0——がそのまま成立つものとする．又地震波は鉛直下方から純粋の上下又は水平動が入射するものとして計算した．

2. 記号を §2 に示す．

3. 上下動に関する考への運びは Fig. 1 に従ひその結果を Fig. 2～Fig. 3 に示す．倍率曲線としては地震計のそれに良く似たものが得られるが，位相差には多少の違ひがある．Fig. 4 には極大倍率を S_0 を変数にとつて示してあるが，後のロッキング，水平動にくらべかなり小さい．

4. ロッキングの場合の考への筋道は Fig. 5 に示す，Fig. 7 及び Fig. 8 に重心の水平変位についての倍率及び位相差を，そして Fig. 9 及び Fig. 10 に極大倍率とそれを与へる a_0 の値を示す．ここでは建物と地盤の密度の比を 1/4 にとり，従つてパラメーターは建物の縦横の比になつてゐる．

回転に関するものについては上と同様に Fig. 11 から Fig. 13 までに，但し極大を与へる a_0 は殆んど変らないから図示はしてない．

又底面中心の変位に関するものは Fig. 14 と Fig. 15 に夫々倍率と位相差を示した．これから見ても建物の地階においた地震計も猶，入射地動そのままを与へるものではないことが明瞭である．

5. 水平動に於ては Fig. 16 に示されるやうな単一質量の模型を考へて，建物の剪断振動に相当する水平動を Fig. 17 に示される筋道で計算をした．

前二節と異り複雑化をさけて建物は非減衰と仮定し，更に $\nu=1$ といふ特別な場合を採つて計算した．Fig. 18 は重心の変位に関する倍率を，Fig. 19 にその位相差を示す．Fig. 20 には建物底面の中心に関する倍率が示されてゐる．この場合 $a_0=1.0$ で曲線が凡て倍率 0 の点に集まるのは，丁度共振に相当した場合に (5.19) の式が成立することを図示してゐるのである．

Fig. 21 に極大倍率を及びそれと関聯して建物の見かけの減衰係数を Fig. 22 に示す．又極大を与へる a_0 を $1/\nu^2$ の函数として表したのが Fig. 23 である．Fig. 24 には a_0/ν, 即ち建物の週期と入射波の周期との比をグラフ化してある．$1/\nu^2$ が大きくなれば，その比は殆んど 1 に等しく，つまり共振の時，建物と入射波の周期は殆んど等しいが，前者は常に後者よりも常に小さいといふことが分る．

6. 極大倍率に影響を与へる要素としては，建物が剛体と考へてよい場合のやうに，それ自身の振動数なるものが存在せず，又重力を考慮に入れない限り，m_0, r_0, l_0; ρ, μ; p がその要素と考へられる．そして次元解析の助けを借りて考へるなら，時の次元をもつものは μ 及び p のみだから，μ 及び p はただ $p/\sqrt{\mu}$ 或は $a_0=pr_0/\sqrt{\mu/\rho}$ として含まれなければならない，このことから μ の変動は a_0 を通じてのみ影響することが結論される．

7. 実用的な見地を考へて簡潔な近似式をつけ加へた．ロッキングの水平と回転，及び水平動の極大倍率は夫々 a_0 の 1.5, 2, 3 乗に逆比例してゐるといつた関係があることである．

御指導を給はつた河角広，金井清両先生に厚く御礼申し上げる．