13. *Relation between the Amplitude of Earthquake Motions and the Nature of Surface Layer. IV.*

(The Case of Finite Train.)

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1. Introduction

The present paper is the continuation of our research work on the problem of earthquake movement of the superficial stratum and deals with the cases where the primary seismic waves are incident upwards normally to doubly stratified layers residing on the surface of a semi-infinite body.

Concerning the type of primary seismic waves, the following studies have been made:

I. The case where an infinite train of harmonic plane waves is transmitted to the surface layer consisting of two layers, taking the solid viscosity into consideration.

II. The case that a finite train of harmonic plane waves is transmitted toward the surface layer consisting of two layers.

2. The case of an infinite train of harmonic waves

Let ρ, μ, ξ, v and H be the density, elastic constant, coefficient of solid viscosity, velocity and thickness of layer, and u the displacement. Suffix 1, 2 and 3 represent the first layer, the second layer and the bottom medium respectively.

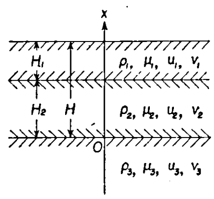

Fig. 1

In order to make the calculation easier, the damping of the third layer is neglected and dampings of both the first and the second layers are assumed to be very small. That is to put the conditions $\xi_3 p/\mu_3=0$, $\xi_2 p/\mu_2 \ll 1$ and $\xi_1 p/\mu_1 \ll 1$ into equations (1)~(9) given in the previous paper[1)].

1) K. KANAI, "Relation between the Nature of Surface Layer and the Amplitudes of Earthquake Motions", *Bull. Earthq. Res. Inst.*, **30** (1952), 32.

If we take the real parts only of all the complex equations, the displacement due to incident waves in the bottom medium, u_0, the displacement due to the vibrations in the first layer, u_1, and that in the second layer, u_2, become as follows:

$$u_0=\cos(pt+f_3x), \tag{1}$$

$$u_1=2\sqrt{\frac{R_1^2+S_1^2}{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}+\tan^{-1}\frac{S_1}{R_1}\right), \tag{2}$$

$$u_2=2\sqrt{\frac{R_2^2+S_2^2}{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}+\tan^{-1}\frac{S_2}{R_2}\right), \tag{3}$$

where

$$\left.\begin{aligned}P=&2(1+\beta Q_1+\gamma Q_2)\cos P_1\cos P_2-2(\alpha+\beta Q_2+\gamma Q_1)\sin P_1\sin P_2\\&-(\beta N_1+\gamma N_2)(\cos P_1\sin P_2+\sin P_1\cos P_2),\\Q=&(-\beta N_1+\gamma N_2)\cos P_1\cos P_2+\{-(2\alpha+\beta)N_1\\&+(2\alpha+\gamma)N_2\}\sin P_1\sin P_2+2(Q_2+\alpha Q_1+\gamma)\cos P_1\sin P_2\\&+2(Q_1+\alpha Q_2+\beta)\sin P_1\cos P_2,\end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned}R_1&=\cos\left(1+\frac{H_2}{H_1}-\frac{x}{H_1}\right)P_1,\\S_1&=P_1N_1\left(1+\frac{H_2}{H_1}-\frac{x}{H_1}\right)\sin\left(1+\frac{H_2}{H_1}-\frac{x}{H_1}\right)P_1,\end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned}R_2=&\cos P_1\cos\left(1-\frac{x}{H_2}\right)P_2-\alpha\sin P_1\sin\left(1-\frac{x}{H_2}\right)P_2,\\S_2=&\left\{P_1N_1+\alpha P_2N_2\left(1-\frac{x}{H_2}\right)\right\}\sin P_1\cos\left(1-\frac{x}{H_2}\right)P_2\\&+\left\{P_2N_2\left(1-\frac{x}{H_2}\right)+\alpha P_1N_1\right\}\cos P_1\sin\left(1-\frac{x}{H_2}\right)P_2,\end{aligned}\right\} \tag{6}$$

and

$$\left.\begin{aligned}&P_2=\omega,\quad P_1=\frac{\omega\eta\zeta}{\alpha},\quad Q_2=\left(\frac{\xi_2}{v_2^3}\right)\frac{\omega^2\phi}{2},\quad Q_1=\left(\frac{\xi_1}{v_1^3}\right)\frac{\omega^2\eta\phi}{2\zeta},\\&N_2=\left(\frac{\xi_2}{v_2^3}\right)\frac{\omega\phi}{2},\quad N_1=\left(\frac{\xi_1}{v_1^3}\right)\frac{\omega\phi\alpha}{2\zeta^2},\end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned}&\omega=\frac{pH_2}{v_2},\quad \eta=\frac{H_1}{H_2},\quad \zeta=\frac{\rho_1}{\rho_2},\quad \phi=\frac{v_2^2}{\rho_2H_2},\quad \alpha=\frac{\rho_1v_1}{\rho_2v_2},\\&\beta=\frac{\rho_1v_1}{\rho_3v_3}(\equiv\alpha\gamma),\quad \gamma=\frac{\rho_2v_2}{\rho_3v_3}.\end{aligned}\right\} \tag{8}$$

in which $2\pi/p=T$ is the period of waves and $2\pi/f_3=v_3T$ is the wave length in the bottom medium.

In the case of dilatational waves, it is necessary to replace μ by $\lambda+2\mu$ and ξ by $\xi'+2\xi$.

(i) The case in which $\xi_1=\xi_2=0$ and $v_1\rho_1<v_2\rho_2<v_3\rho_3$.

We selected nine cases, namely, $H_2/H_1=15, 6, 4, 3, 1.5, 1$; $H_2/H_1=4, 2$ and $H_2/H_1=4$. In addition the conditions $v_1:v_2:v_3=1:3:6$, $1:2:4$ and $1:4:8$ respectively, and the conditions $\rho_1=\rho_2=\rho_3$ are in common. We calculated the resulting amplitudes of vibrations at the free surface, $x=H_1+H_2(=H)$, at the intermediate boundary, $x=H_2$, and at the bottom boundary, $x=0$, for different values of wave length, the results being shown in Figs. 2–10. In these figures, v_1T is the wave length in the first layer, and the amplitude of the incident waves in the bottom medium is unity.

We also studied the distribution of displacements at different depths in the surface layer, the results of calculation of cases corresponding to points a, b, ····, (indicated by vertical strips in Figs. 2–10) being shown in Figs. 11–19.

From these figures it will be seen that the amplitudes of vibration at the free surface become maximum at such periods when the bottom boundary becomes nearly a nodal plane of stationary vibrations.

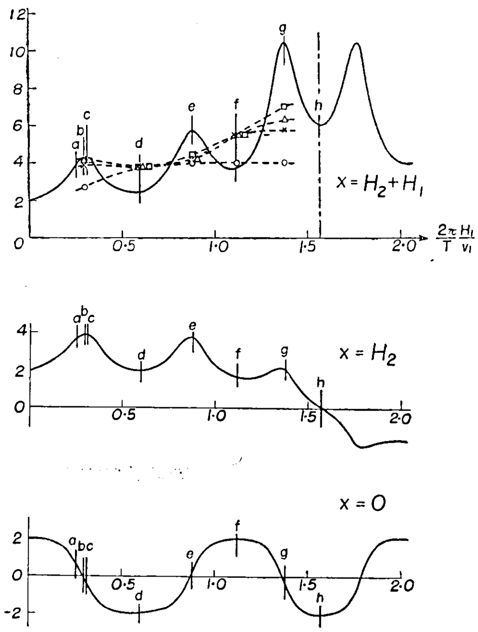

Fig. 2. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$.

Since the waves reflected at various boundaries interfere with one another, the spectral response of the amplitude of the surface layer is very irregular. In the cases of multiple layers, except in extremely special cases, the largest vibration amplitude on the free surface is not so large as seen in the case of the surface layer consisting of one layer. In a special case, that is, when both at the bottom layer boundary and the common boundary of the upper two layers become nodal planes of station-

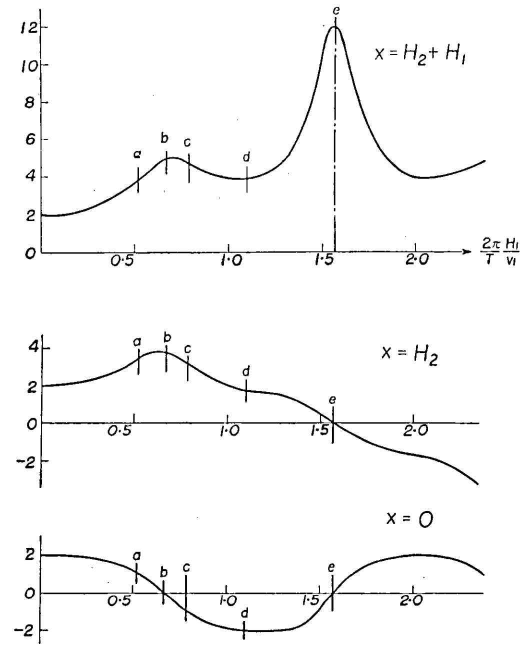

Fig. 3. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:6$.

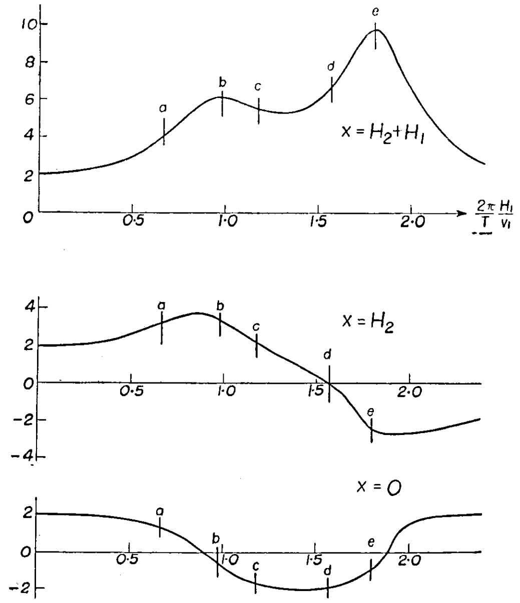

Fig. 4. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:4$.

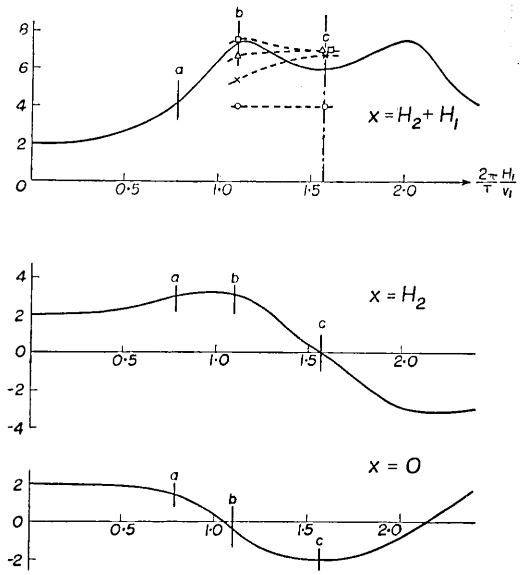

Fig. 5. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:3$.

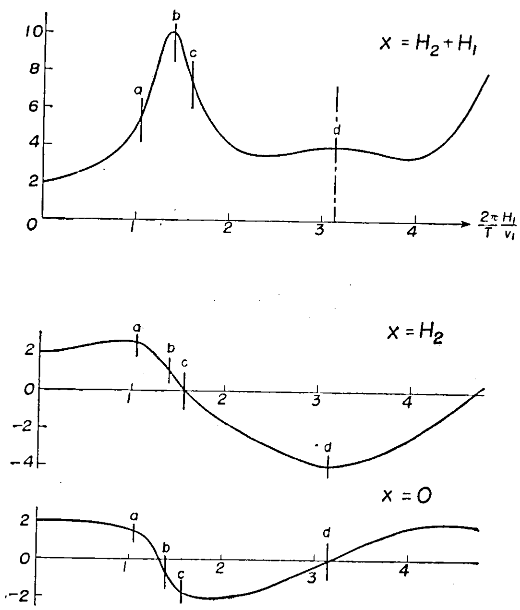

Fig. 6. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=2:3$.

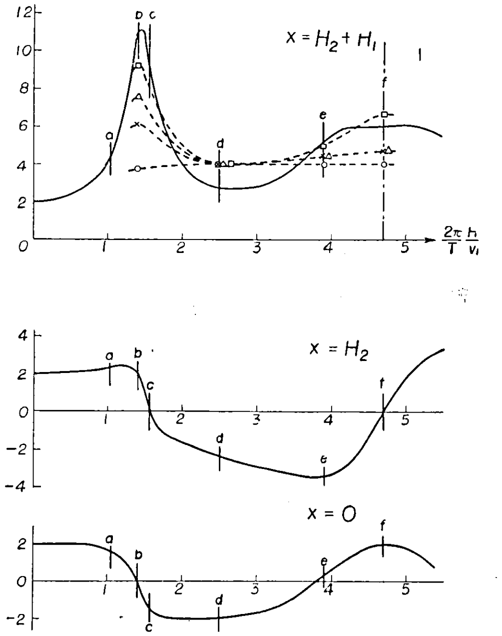

Fig. 7. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$.

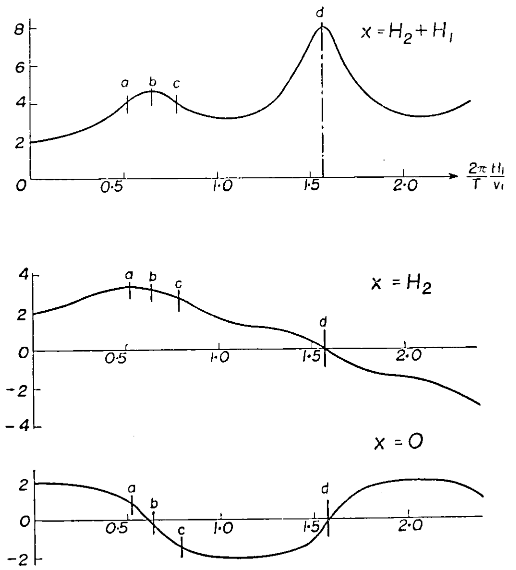

Fig. 8. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:2:4$, $H_1:H_2=1:4$.

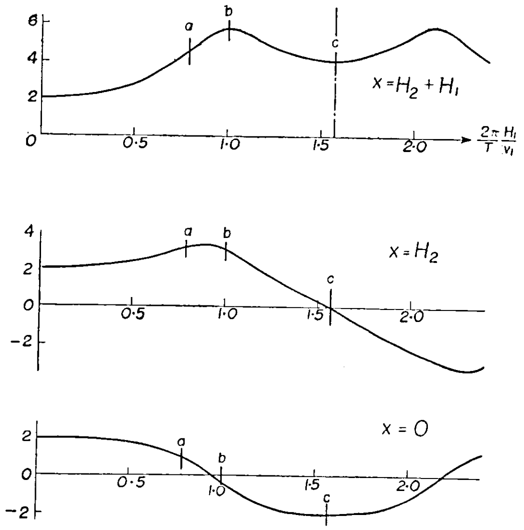

Fig. 9. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:2:4$, $H_1:H_2=1:2$.

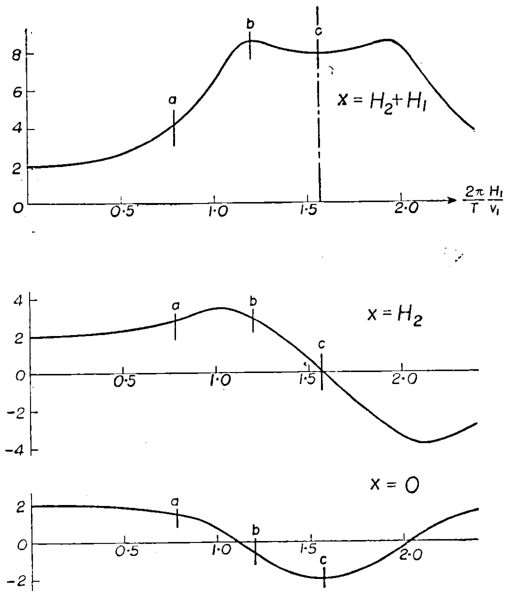

Fig. 10. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:4:8$, $H_1:H_2=1:4$.

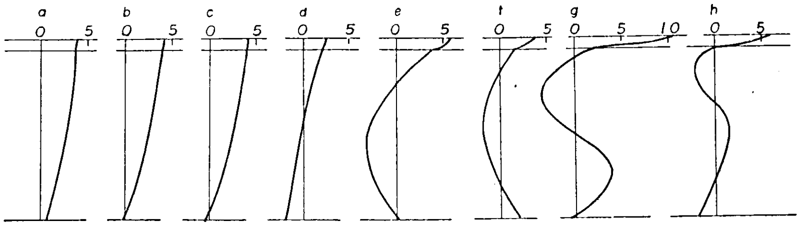

Fig. 11. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$.
a, b····indicated in Fig. 2.

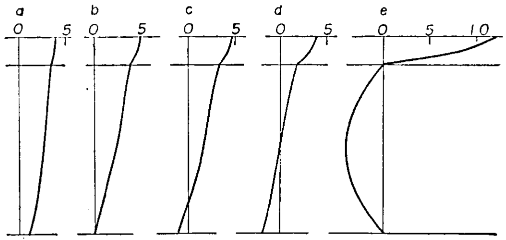

Fig. 12. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:6$.
a, b····indicated in Fig. 3.

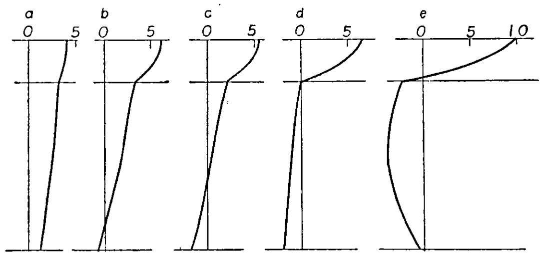

Fig. 13. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:4$.
a, b····indicated in Fig. 4.

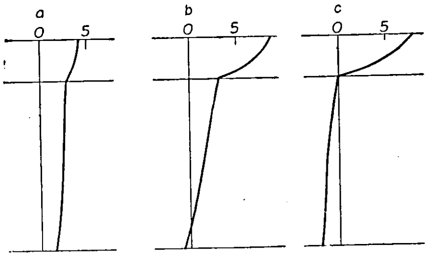

Fig. 14. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:3$. a,b····indicated in Fig. 5.

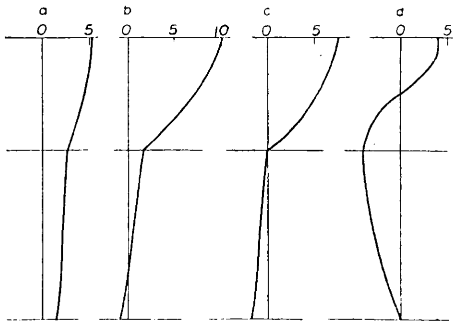

Fig. 15. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=2:3$. a, b····indicated in Fig. 6.

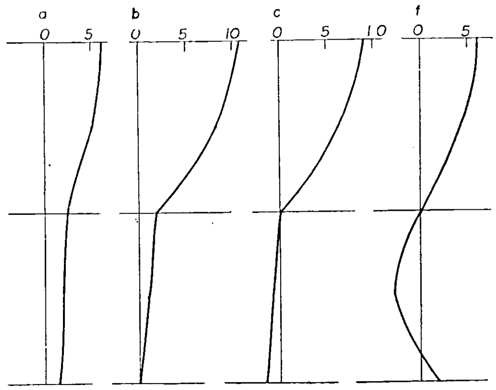

Fig 16. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$.
a, b····indicated in Fig. 7.

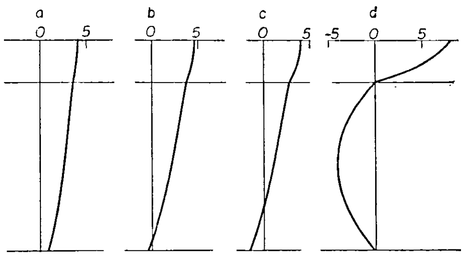

Fig. 17. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:2:4$,
$H_1:H_2=1:4$. a, b····indicated in Fig. 8.

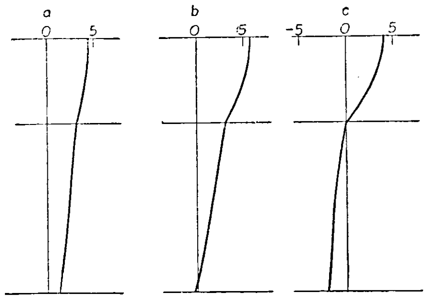

Fig. 18. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:2:4$,
$H_1:H_2=1:2$. a, b····indicated in Fig 9.

ary vibrations, the vibration amplitude at the free surface gets extremely large. The value of amplitude mentioned above become nearly equal to $2v_3\rho_3/v_1\rho_1$, and is as large as in the case of the single stratified layer.

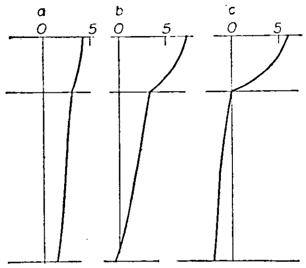

Fig. 19.
$\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:4:8$,
$H_1:H_2=1:4$.
a, b····indicated in Fig. 10.

(ii) The case in which the second layer is the lowest velocity.

We calculated two cases, namely, $H_1:H_2=1:1$, $v_1:v_2:v_3=2.5:1:5$ and $H_1:H_2=3:1$, $v_1:v_2:v_3=2:1:5$. In addition the condition $\rho_1=\rho_2=\rho_3$ are kept constant in common.

The resulting amplitudes of vibrations at $x=H_1+H_2$, $x=H_2$ and $x=0$ for different periods being shown in Figs. 20 and 21. And, the

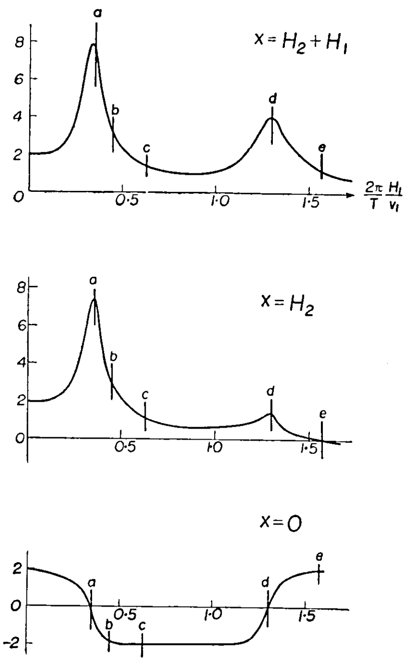

Fig. 20. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=2.5:1:5$, $H_1:H_2=1:1$.

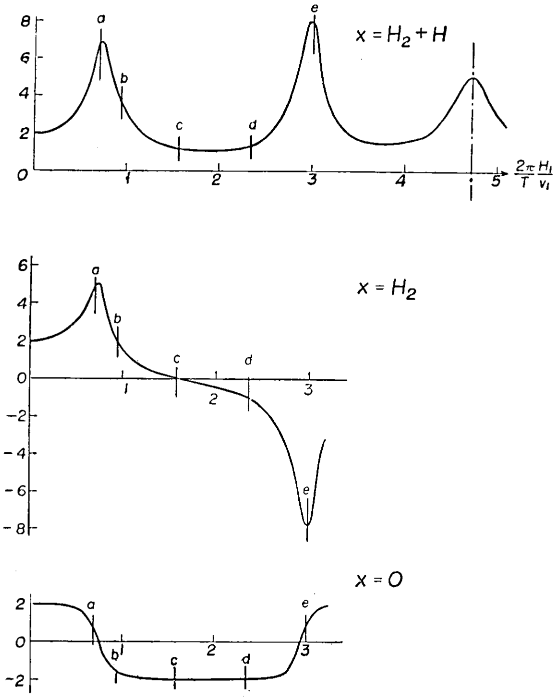

Fig. 21. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=2:1:5$, $H_1:H_2=3:1$.

distribution of displacements at different depths in the surface layer are shown in Figs. 22 and 23.

From these figures, it will be seen that, the amplitude of vibration at the intermediate layer of the lowest velocity becomes maximum at such periods when the bottom boundary becomes nearly a nodal plane of stationary vibrations.

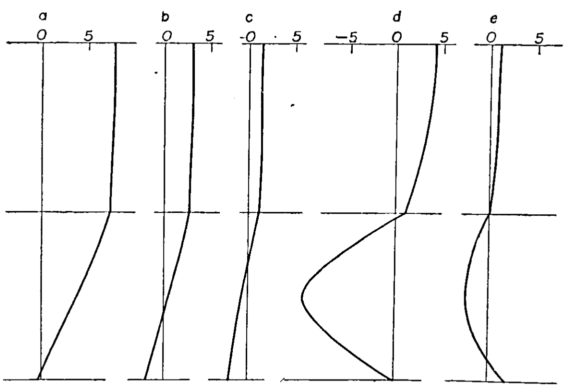

Fig. 22. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=2.5:1:5$, $H_1:H_2=1:1$. a, b.... indicated in Fig. 20.

(iii) The effect of solid viscosity.

When an infinite train of harmonic plane waves in transmitted to the surface layer which consists of two layers, taking the solid viscosity into

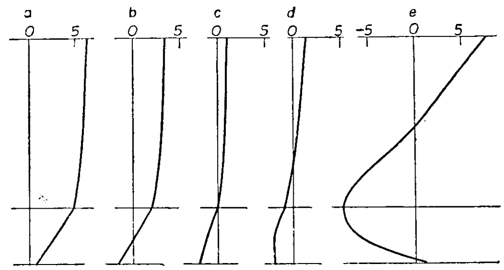

Fig. 23. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=2:1:5$, $H_1H:_2=3:1$. a, b····indicated in Fig. 21.

consideration, some special cases are plotted in the following drawings, using equations (1)-(8). The cases of $H_2/H_1=15, 6, 3$ and 1 besides the conditions $\xi_1/v_1^3=\xi_2/v_2^3=10^{-7}$ C.G.S., $\phi=2\times10^5$ C.G.S. and $v_1:v_2:v_3=1:3:6$ are given in Figs. 24–27. In the results of the experimental investigations[2], the assumed values of coefficient of solid viscosity proved to be not improbable values in the superficial layer of the earth. The value of $\phi=2\times10^5$ C.G.S. corresponds to the conditions where $\rho_2=2$, $v_2=400$ m/s and $H_2=40$ m.

Figs. 24–27 tell us that, when the superficial layer is a visco-elastic medium, the synchronous conditions of higher order do not grow to a large value, and then the curve of the spectral response of the amplitude at the free surface becomes flat.

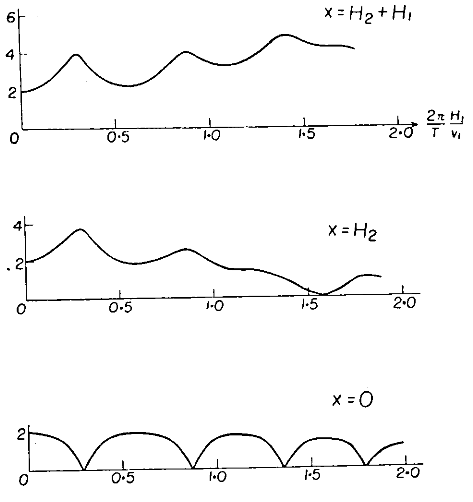

Fig. 24. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$.

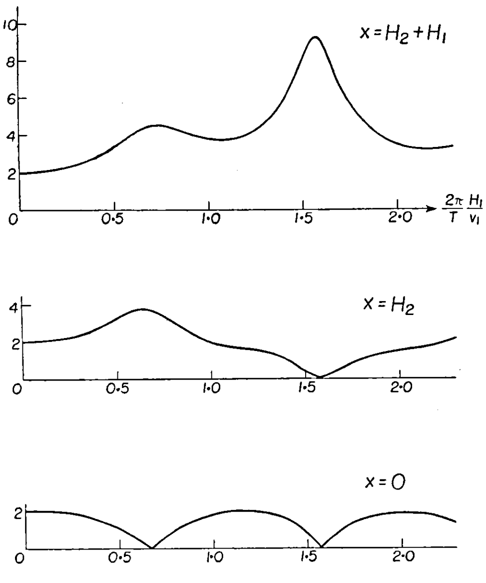

Fig. 25. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:6$.

2) K. KANAI and K. OSADA, "The Result of Observation concerning the Waves Caused in the Ground by Building Vibration", *Bull. Earthq. Res. Inst.*, **29** (1951), 514.

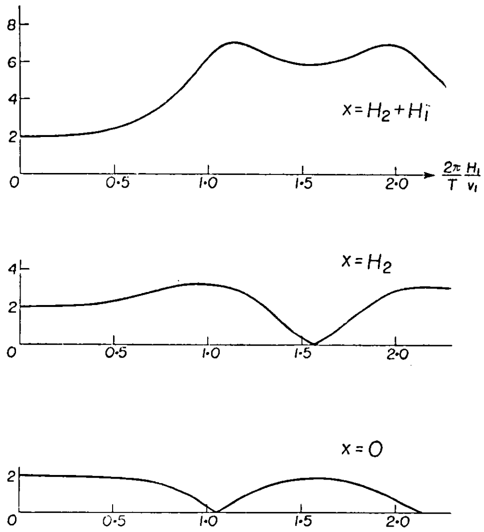

Fig. 26. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:3$.

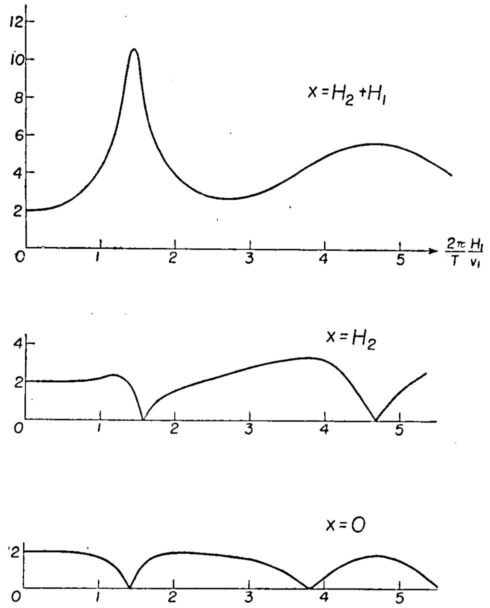

Fig. 27. $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$.

3. The case of a finite train of harmonic waves

For simplicity, the effect of solid viscosity of the surface layer is neglected.

Let the form of the primary waves for any time reaching to the bottom medium be

$$\left.\begin{aligned} u_0&=\cos c(v_3t-x)\,, & &[(v_3t-a)<x<v_3t] \\ &=0\,, & &[v_3t<x] \\ &=0\,, & &[(v_3t-a)>x] \end{aligned}\right\} \tag{9}$$

then, the displacement of the vibratory movement at the free surface is expressed by[3)]

$$u_{2(x=H)}=\frac{8M}{(1+\alpha)(1+\gamma)}\sin cH\left\{\frac{v_3t}{H}+\tau\right\}\,, \tag{10}$$

where

3) K. SEZAWA and K. KANAI, "Possibility of Free Oscillations of Strata exited by Seismic Waves. Part IV.", *Bull. Earthq. Res. Inst.*, **10** (1932), 277.

$$\left.\begin{aligned} M&=\sum_{m=0}^{\infty}(-1)^m \sum_{n=0}^{m} \frac{n\,!}{n\,!\,(m-n)\,!} \sum_{p=0}^{n} \frac{m\,!}{p\,!\,(n-p)\,!}\left(\frac{1-\gamma}{1+\gamma}\right)^{m-p}\left(\frac{1-\alpha}{1+\alpha}\right)^{m+p-n}, \\ \tau&=(2p-2m-1)\frac{v_3 H_2}{v_2 H}+(2n+1)\left(\frac{H_2}{H}-1\right)\frac{v_3}{v_1}, \end{aligned}\right\} \tag{11}$$

$$\alpha=\frac{\rho_1 v_1}{\rho_2 v_2}, \quad \beta=\frac{\rho_1 v_1}{\rho_3 v_3}, \quad \gamma=\frac{\rho_2 v_2}{\rho_3 v_3}. \tag{12}$$

Some numerical examples were calculated using the equations (10)~(12), and are shown in Figs. 28–38. In Figs. 28–32, the cases of $N=1/2$, 1, 3/2, 2 wave length at different periods of waves corresponding to points *b*, *d*, *e*, *f* and *g* indicated in Fig. 2, under the condition that $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$ and $H_1:H_2=1:15$ in common, are plotted respectively. While in Figs. 33 and 34, the cases of $N=1/2$, 1, 3/2, 2 wave length at different periods of waves corresponding to points *b* and *d* indicated in

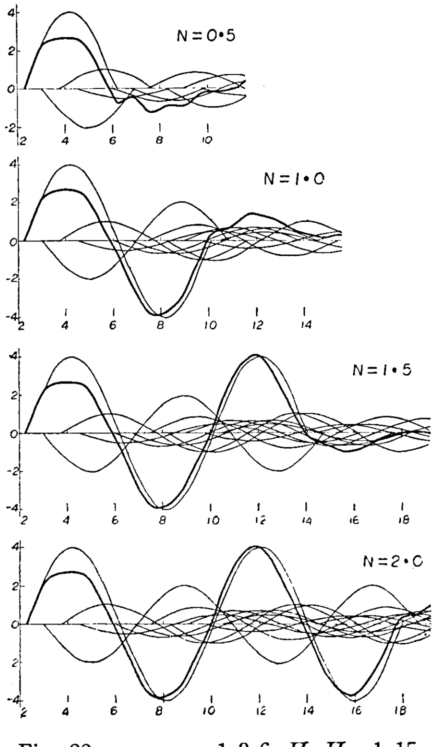

Fig. 28. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$. Corresponding to *b* indicated in Fig. 2.

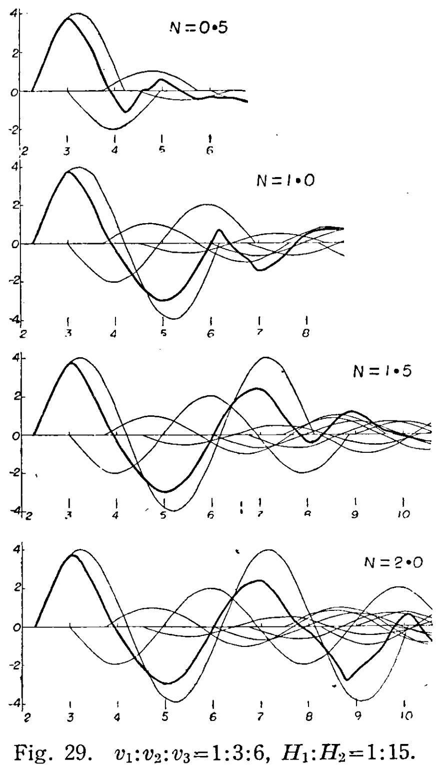

Fig. 29. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$. Corresponding to *d* indicated in Fig. 2.

Fig. 5, under the condition that $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$ and $H_1:H_2=1:3$ in common, are illustrated. And, in Figs. 35–38, the cases of $N=1/2, 1, 3/2, 2$ wave length at b, d, e and f indicated in Fig. 7, under the conditions that $\rho_1=\rho_2=\rho_3$, $v_1:v_2:v_3=1:3:6$ and $H_1:H_2=1:1$ in common are shown.

Figs. 28, 33, 34 and 35 show that when the period of incident waves of a finite train of the harmonic type is too large to have any node in the surface layer, the maximum amplitude at the free surface will be approximate to the value obtained in the case of an infinite train, if only about two waves succeed.

Figs. 30, 31 and 32 tell us that if the period of waves is so short that there may be nodes in the surface layer, the amplitude at the free surface will not be approximate to an asymptote value, unless considerably many trains of waves succeed. Therefore, the same conditions seldom occur actually in which the nodes exist near to both the bottom boundary and the first boundary at the same time in the case of an infinite train and the amplitude gets extremely large.

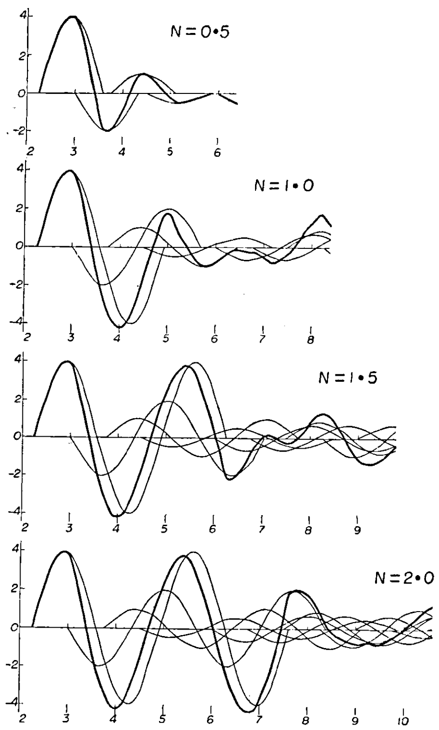

Fig. 30. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:15$. Corresponding to e indicated in Fig. 2.

From Figs. 29, 36, 37 and 38, it will be seen that it is not always true that the longer the train is, the larger the amplitude at the free surface becomes. There is even a case where the amplitude on the free surface becomes maximum in the initial half waves and takes the value of $2\times(2/(1+\alpha))\times(2/(1+\gamma))$ times of the amplitude of primary waves in the bottom medium. This is due to the fact that the waves reflected at various boundaries interfere with one another.

4. Conclusion

From the present investigations on the problem of earthquake

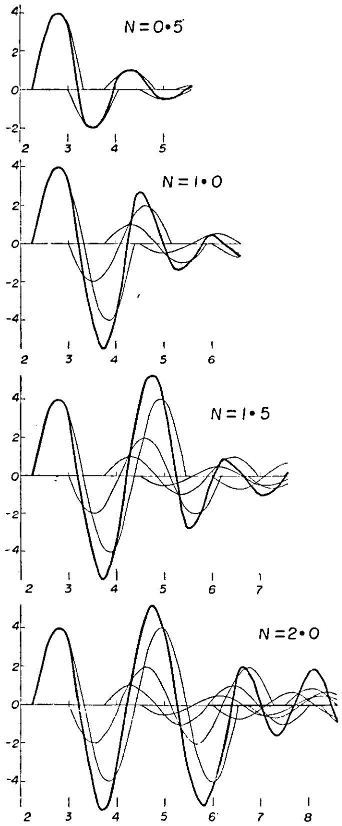

Fig. 31. $v_1:v_2:v_3=1:3:6$,
$H_1:H_2=1:15$.
Corresponding to f indicated
in Fig. 2.

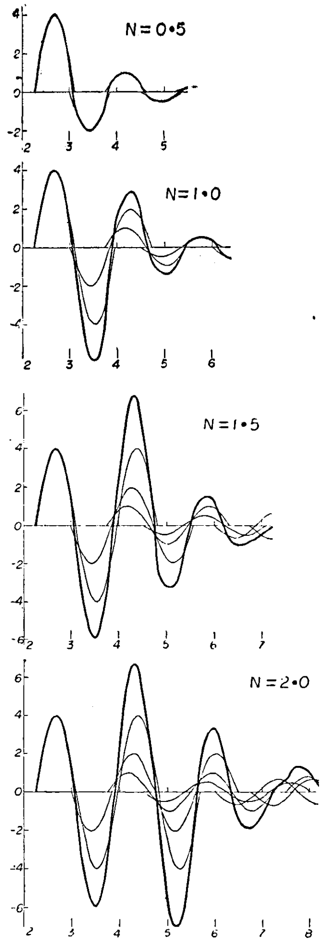

Fig. 32. $v_1:v_2:v_3=1:3:6$,
$H_1:H_2=1:15$.
Corresponding to g indicated
to Fig. 2.

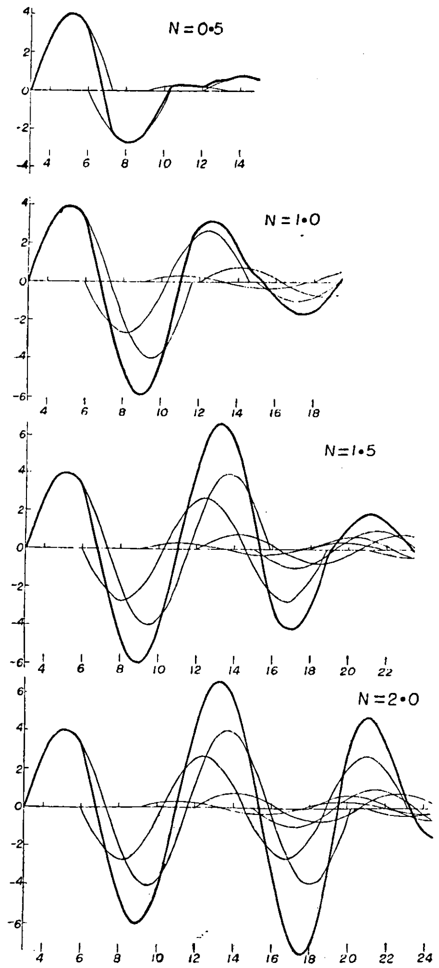

Fig. 33. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:3$. Corresponding to b indicated in Fig. 5.

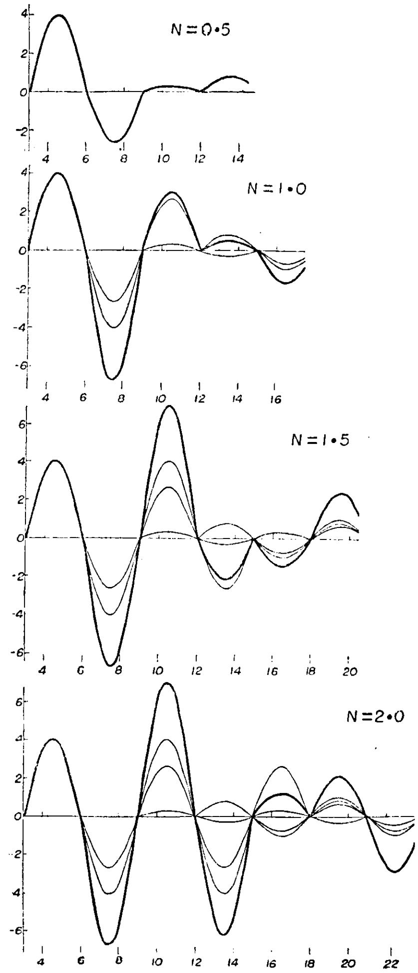

Fig. 34. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:3$. Corresponding to c indicated in Fig. 5.

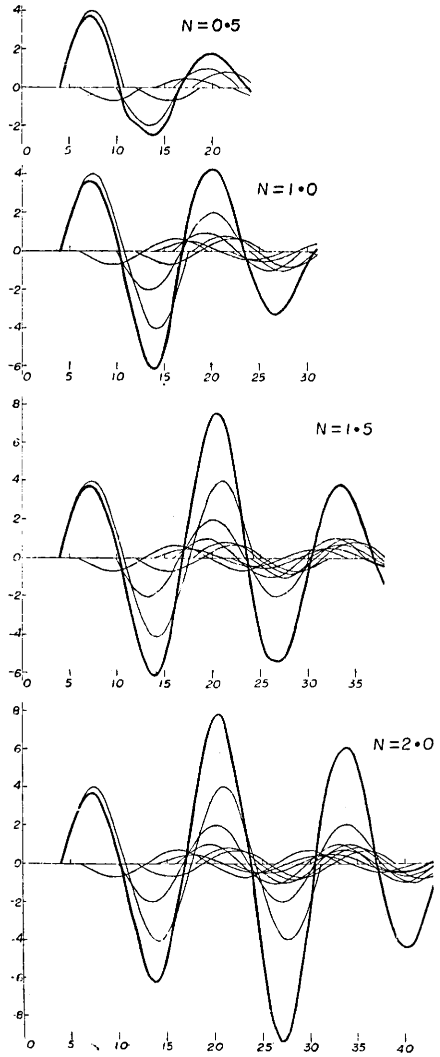

Fig. 35. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$. Corresponding to b indicated in Fig. 7.

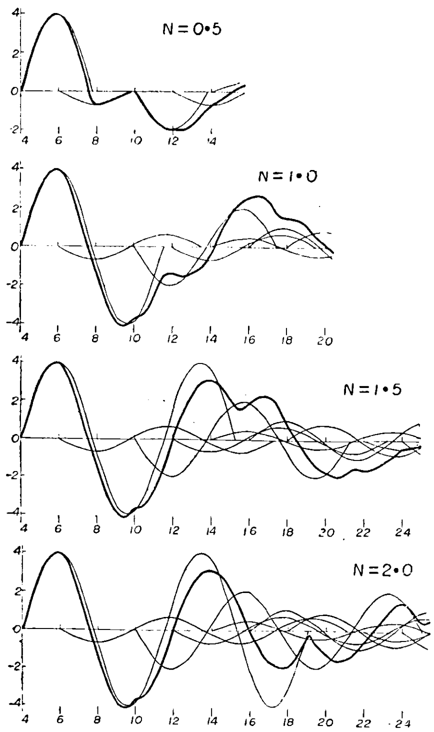

Fig. 36. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$. Corresponding to d indicated in Fig. 7.

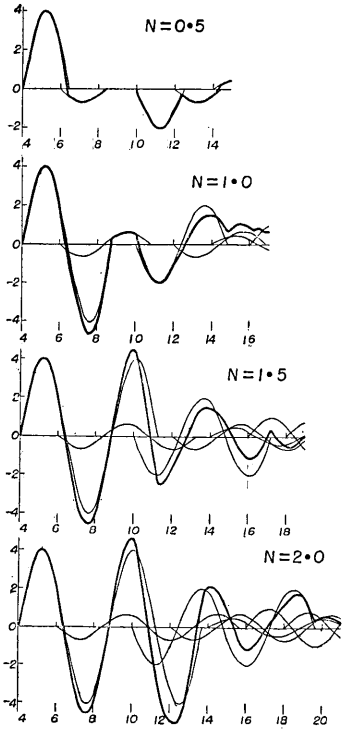

Fig. 37. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$. Corresponding to *e* indicated in Fig. 7.

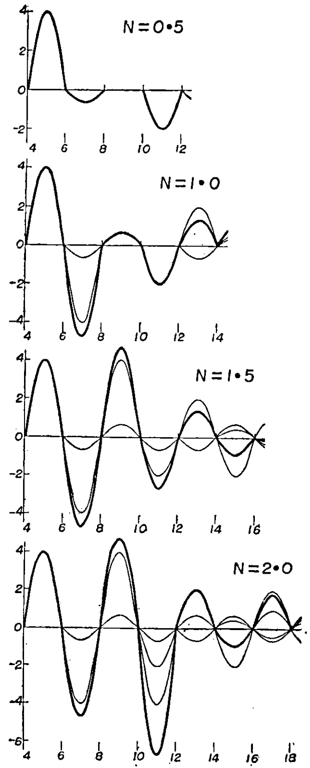

Fig. 38. $v_1:v_2:v_3=1:3:6$, $H_1:H_2=1:1$. Corresponding to *f* indicated in Fig. 7.

movement of the surface layer which consists of two layers, the following has been made clear :

(i) The vibration amplitude on the free surface becomes maximum, when waves of such a period as to have a node at the bottom boundary are transmitted. In a special case, there will be nodes both at the first boundary and at the bottom boundary, in which case the maximum amplitude at the free surface gets extremely large.

(ii) Since the waves reflected at various boundaries interfere with one another, the spectral response of the amplitude of the surface layer is very irregular. In these cases, exept for extremely special cases, the maximum value of the peak is not so large as seen in the case of the surface layer consisting of one layer.

(iii) If the period of incident waves of a finite train of the harmonic type is very short there may be nodes in the surface layer, and the amplitude at the free surface will not be approximate to an asymptote value, unless considerably many trains of waves succeed. Therefore, it seldom occurs actually that the amplitude gets extremely large.

(iv) When the period of incident waves of a finite train of the harmonic type is too large to have any node in the surface layer, the maximum value of amplitude at the free surface will be approximate to the value obtained in the case of an infinite train, if only about two finite trains may succeed.

(v) It is not always true that the longer the train is, the larger the amplitude becomes. There is even a case where the amplitude on the free surface becomes maximum in the initial half waves.

(vi) Since the form of seismic waves are complicated virtually, the unusual increment of amplitude of the ground hardly occurs regarding the vibrations of a period short enough to produce nodes in a surface layer (so-called higher harmonics). Accordingly, in analysing actual earthquake motion, it is regarded as reasonable to consider that the surface layer consists of more than two layers, if there are more than two peaks of the spectral response of amplitude.

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13. 地震動振幅と地表層の性質との関係 (第 4 報)
(有限長調和波の場合)

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表面層内が2層に分かれている場合について，無限長調和波型の入射波による表面層内の振幅分布及び有限長調和波型の入射波による地表面の振動性状を数理的に詳しくしらべた結果を列挙すると次のようになる．

(1) 最下層とその下の半無限体との境界面が振動の節になる周期の波が入射した時に，地表面の振幅は極大になる．

(2) その特別の場合として，最下層とその下の半無限体との境界面及び第 1 層と第 2 層との境界面が同時に振動の節になるような周期の波が入射した時に，地表面の振幅は特に大きくなる．その値は $2\rho_n v_n/\rho_1 v_1$ に近い．1 は第 1 層，n は下の半無限体である．この場合の周期は $4H_1/v_1$ に近い．実際問題として，厚い堆積層がある場合に，短い周期の波が卓越することが稀にあるのは，この性質のあらわれであろう．

(3) 一般に，表面層が2層以上に分れている場合には，2以上の境界面からの反射波が互に干渉

し合うので，波の周期特性には 1 層の場合のような，特別に大きな振幅になりにくい．この事は，厚い堆積層がある実際の地盤の振動特性に類似している．

次に，有限長調和波の計算結果としてわかつたことを示す．

（1） 比較的に長い周期の波は，2 波長位で無限長のときの振幅に近ずく．

（2） 地表面の振幅が極大になるような，比較的に短い周期の波は，相当に連続しないと無限長のときの振幅に近ずかない．

（3） 有限長調和波の特別な場合には，最初の半波の振幅が最大になることがある．この時の地表面の振幅は無論 $2\times(2/(1+\alpha))\times(2/(1+\gamma))$ をとる．

（4） 実際問題としては，2 層以上から成つている場合に，地表面の振幅が，前に書いた $2\rho_n v_n/\rho_1 v_1$ に近い値をとる位に増幅されることは，非常に起りにくいことになる．