4. *The Relation between the Amplitude and the Period of Earthquake Motion.*

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1. Introduction.

Needless to say, the amplitude of earthquake motion becomes larger in a great earthquake, and it is also a well-known fact that the period becomes longer as the earthquake becomes greater. Several articles concerning the relation between the magnitude of earthquake and the amplitude of earthquake motion[1] have been published till now, but concerning the relation between these two and the period of earthquake motion we can give only a few studies[2], and naturally there are many problems yet unsolved.

In order to study analytically the mechanism of earthquake origin from the property of earthquake motion, it is desired to use the waves of unchanged wave-form on their way from the earthquake origin to the observing station.

The waves starting from the earthquake origin change their forms on their way of propagation and the degree of change is the smallest in the initial motion of P-waves.

Naturally the regional distribution of "push" and "pull" in the initial motion of P-waves has served greatly for the study on the mechanism of earthquake origin.

In ordinary districts, the wave form of seismic waves is changed to a great degree because of the complicated geological conditions of the

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[1] C. F. RICHTER, "An Instrumental Earthquake Magnitude Scale", *Bull. Seism. Soc. Amer.*, **25** (1935), 1.

B. GUTENBERG and C. F. RICHTER, "Earthquake Magnitude, Intensity, Energy and Acceleration", *Bull. Seism. Soc. Amer.*, **32** (1942), 163.

H. KAWASUMI, "Measures of Earthquake Danger and Expectancy of Maximum Intensity throughout Japan as Inferred from the Seismic Activity in Historical Times", *Bull. Earthq. Res. Inst.*, **29** (1951), 469.

C. TSUBOI, "Determination of the Richter-Gutenberg's Instrumental Magnitudes of Earthquakes Occurring in and near Japan", *Geophys. Notes, Geophys. Inst., Tokyo Univ.*, **4** (1951), No. 5.

[2] H. HONDA and H. ITO, "On the Period of the P-waves and the Magnitude of the Earthquake", *Geophys. Mag.*, **13** (1939), 155.

R. YOSHIYAMA, Monthly Coll. Earthq. Res. Inst., Tokyo Univ., Nov. 18, 1947.

ground near the surface. But Hitachi Mine (Ibaraki prefecture), where we have been making observations for these several years, has a very good condition[3] in this sense. There, the difference between the wave forms near the ground surface and those at a depth of several hundred meters can be easily explained analytically.

In this paper, we are going to examine the relation between the amplitude and the period of earthquake motion using the initial motion of P-waves shown in the seismograms obtained at Hitachi Mine, and from the results we will enter into the discussion of the mechanism of seismic wave generation.

Table I.

Earthq. No.	Date	Hypocenter (Central Meteor. Observ.)			Duration of prelim. tremor at Hitachi Mine (sec)
		Epicenter		Depth (km)	
		N	E		
170	Jan. 26, 1951	37.2	140.1	shallow	12.0
171	Jan. 28, 1951	36.6	141.2	40	7.0
181	Feb. 20, 1952	36.1	139.9	55	10.3
200	Mar. 10, 1952	36.7	141.1	40	7.0
252	June 12, 1952	36.2	140.1	55	8.7

Table II. Instrumental constants of the seismographs.

Earthq. No.	Direction	Natural period (sec)	Magnification	Damping ratio
170	EW	1.0	250	13:1
	UD	0.25	210	〃
171	EW	1.0	250	〃
	UD	0.25	210	〃
181	EW	1.0	215	〃
	NS	〃	288	〃
	UD	0.10	220	〃
200	EW	1.0	215	〃
	NS	〃	288	〃
	UD	0.10	260	〃
252	NS	1.0	250	〃
	UD	0.26	120	〃

2. Correction of Seismograms.

Table I shows the positions of the origin of the earthquakes treated herein which were determined by the Central Meteorological Obser-

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[3] K. Kanai and T. Tanaka, "Observations of the Earthquake-motion at the Different Depths of Earth. I", *Bull. Earthq. Res. Inst.*, **29** (1951), 107.

K. Kanai, "The Results of Observation of Wave Velocity in the Ground", *Bull. Earthq. Res. Inst.*, **29** (1951), 503.

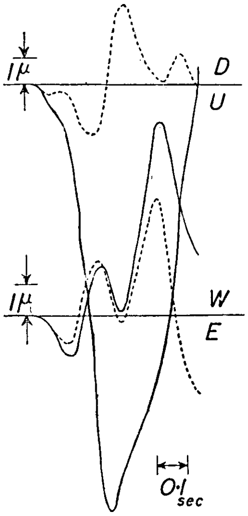

Fig. 1. Earthq. No. 170: Jan. 26, 1951: Δ=82 km, D=76 km. Full and broken lines represent the actual ground motions and the records of seismograms respectively.

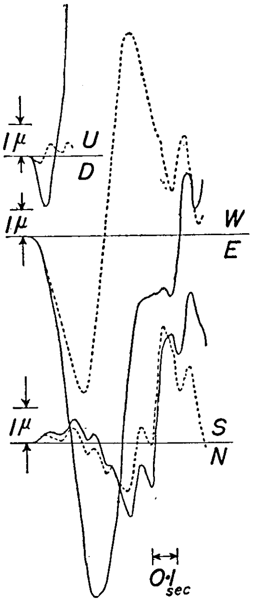

Fig. 4. Earthq. No. 200: Mar. 10, 1952: Δ=42 km, D=46 km. Full and broken lines represent the actual ground motions and the records of seismograms respectively.

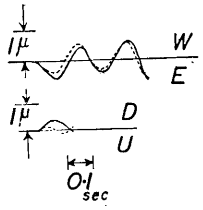

Fig. 2. Earthq. No. 171: Jan. 28, 1951: Δ=55 km, D=29 km. Full and broken lines represent the actual ground motions and the records of seismograms respectively.

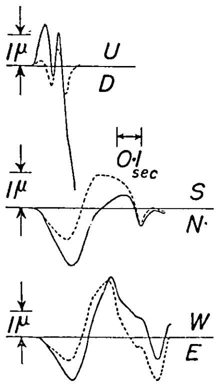

Fig. 3. Earthq. No. 181: Feb. 20, 1952: Δ=82 km, D=44 km. Full and broken lines represent the actual ground motions and the records of seismograms respectively.

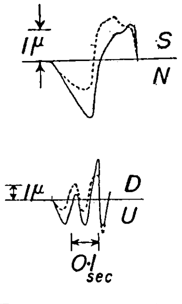

Fig. 5. Earthq. No. 252: June 12, 1952: Δ=67 km, D=40 km. Full and broken lines represent the actual ground motions and the records of seismograms respectively.

vatory, Tokyo, and the duration of preliminary tremor observed at Hitachi Mine. Table II shows the intrumental constants of seismographs.

Let the wave form of seismograms be $f(t)$. Then we get the actual ground motion $F(t)$ from the following equation:

$$F(t)=\frac{-1}{V}\left[f(t)+2\varepsilon\int_0^t f(t)dt+n^2\int_0^t dt\int_0^t f(t)dt\right], \qquad (1)$$

where V, ε and n are the geometrical magnification, damping coefficient and natural frequency of the seismograph respectively.

From equation (1), the graphical calculation is carried out by applying the seismograms, then the actual ground motion will be given as shown in Figs. 1—5 by full lines. The broken lines in these figures represent the records of seismograms. The values of the period and the amplitude of the initial motion of P-waves can be read from Figs. 1—5. In this case, the period of the initial motion of P-waves is twice the time from the beginning of motion to the moment to cross the zero-line.

3. Relation between the period and the amplitude of seismic waves.

Suppose the relation between the viscosity of the material of earth crust and the amplitude at a certain spot is as follows:

$$A\propto e^{-\frac{kx}{T^2}}, \qquad (2)$$

where

x=distance from the earthquake origin to a certain spot,
$k=(2\pi)^2\xi/2\rho v^3$,
ξ=coefficient of solid viscosity,
ρ=density,
v=velocity.

If the penetrating power of seismic waves at discontinuity surfaces of the material within the earth crust is assumed to be τ_1, τ_2, ..., τ_n, and the ratio of the actual amplitude at the ground surface to that of the incident wave to be κ_0, A_0, the amplitude at a certain spot on ground surface may be

$$A_0=\frac{\mathfrak{A}}{\sum_{i=1}^{n}x_i}\times e^{-\frac{1}{T^2}\sum_{i=1}^{n}k_i x_i}\times(\tau_1\tau_2\ldots\tau_n)\times\kappa_0 \qquad (3)$$

where $\mathfrak{A}$ indicates the constant determined from the mechanism of earthquake origin.

Then, suppose the seismic wave reaches a spot 100 km from the earthquake origin, through a non-viscous homogeneous material. The amplitude, A, at this spot may be written, based on equation (3), as follows:

$$A = \frac{A_0 e^{\frac{1}{T^2}\sum_{i=1}^{n} k_i x_i}}{(\tau_1\tau_2\cdots\cdots\tau_n)\times\kappa_0} \times \frac{\sum_{i=1}^{n} x_i}{10^7}\,. \tag{4}$$

Then let us compare the amplitudes of several earthquakes by using the value of amplitude, A, at this supposed spot.

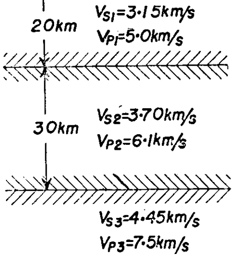

Fig. 6. The structure model of the earth crust.

Fig. 6[4] illustrates the structure model of the earth crust adopted by us, Fig. 7 shows the origins of the earthquakes determined from the incident angle and the duration of preliminary tremor at Hitachi Mine, both of which were given by the calculated results of reflection phenomena at ground surface[5] and the analytical results of seismograms. (The origins thus determined approximately coincide with those determined by the Central Meteorological Observatory, Tokyo, using many data obtained by the various meteorological stations,)

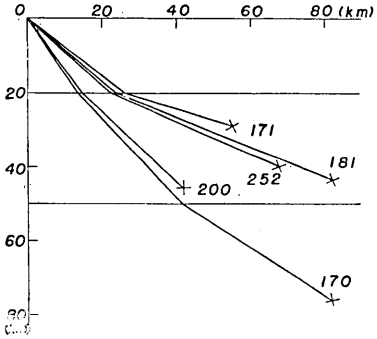

Fig. 7. The origins of the earthquake.

τ_1 and τ_2 are given by the path shown in Fig. 7[6] and κ_0 by the calculation of reflection at ground surface[7], and they are shown in Table III. (As seen in Table III, all the values of τ_1 and τ_2 are approximate to 1 and those of κ_0 approximate to 2. Therefore, some changes in the structure model of earth crust may have no influence upon the results stated in this paper.)

Table III.

Earthq. No.	τ_1	τ_2	κ_0	Incident angle
170	1.07	0.97	2.03	34°02′
171	0.75	—	2.04	52°18′
181	0.83	—	2.03	49°39′
200	1.01	—	2.03	36°16′
252	0.87	—	2.04	48°25′

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[4] T. MATUZAWA, K. YAMADA and T. SUZUKI, "On the Forerunners of Earthquake-motions (The Second Paper)", *Bull. Earthq. Res. Inst.*, **7** (1929), 241.

[5] T. MATUZAWA, *Zisin* (*Jour. Seism. Soc., Japan*), **4** (1932), 125 (in Japanese).

[6] H. KAWASUMI and T. SUZUKI, *Zisin* (*Jour. Seism. Soc., Japan*), **4** (1932), 277 (in Japanese).

[7] *loc. cit.*, 5).

The values of A are determined by introducing the values shown in Table III into equation (4), and in Fig. 8 the relation between the amplitude, A, and the period, T, of the initial motion of P-waves is shown. In this case the viscosity coefficient of materials from the earthquake origin to the ground surface, is assumed to be constant. In Fig. 8, the marks ● show the values, reduced to the spot 100 km from earthquake origin, which were obtained by H. Honda and H. Ito[8] from the results of conspicuous deep earthquakes.

Fig. 8 shows that general earthquakes adopted by us and conspicuous deep earthquakes adopted by H. Honda and H. Ito form nearly a straight line. This fact seems to prove that the relationship

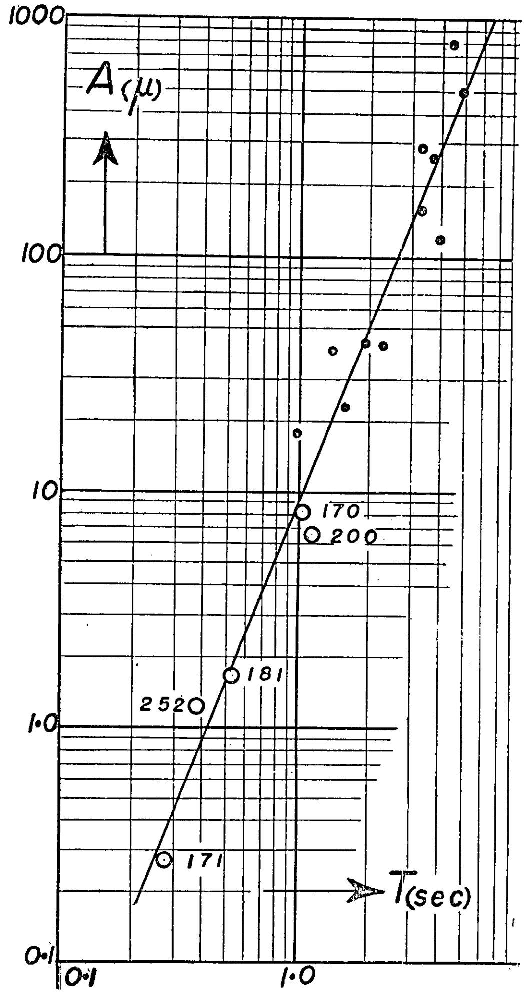

Fig. 8a. The relation between the amplitude, A, and the period, T, of the initial motion of P-waves. The case in which damping factor k is 0. (Marks ● show the values of conspicuous deep earthquakes).

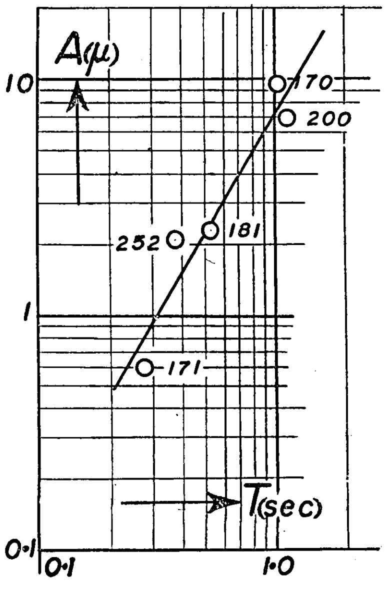

Fig. 8 b. The relation between the amplitude, A, and the period, T, of the initial motion of P-waves. The case in which damping factor k is 10^{-8} C.G.S.

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[8] H. HONDA and H. ITO, *loc. cit.*, 2), 159.

between the amplitude and the period of seismic waves has a considerable generality.

Then the relation between the period and amplitude of the initial motion of S-waves is presented for reference in Fig. 9. The case of S-waves is very complicated because the influence of the preliminary tremor upon the initial motion cannot be wholly removed. Therefore, the correction of amplitude is carried out with regards to only the relation that the amplitude decreases in inverse proportion to the distance from the earthquake origin.

From Figs. 8 and 9, it is found that with the initial motion of seismic wave the amplitude is approximately in proportion to the square of the period. As stated before, of all the seismic waves observed on the ground surface, the initial motion (particularly of P-wave) is the best in communicating the characteristics of seismic waves generated at the earthquake origin.

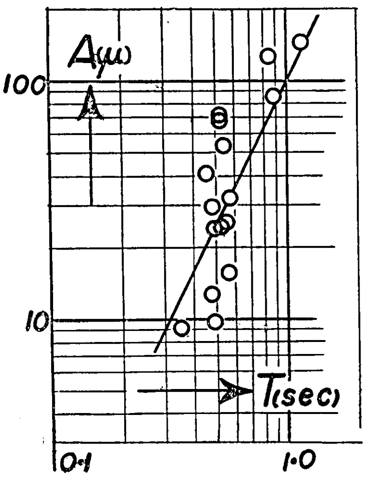

Fig. 9. The relation between the amplitude and the period of the initial motion of S-waves. The case in which damping factor k is 4.5×10^{-8} C.G.S.

Therefore, the characteristics of the initial motion of earthquake motion is supposed to be the first approximation of the characteristics of the seismic waves generated at the earthquake origin. Then the relation between the amplitude and the period of seismic wave may be expressed as

$$\text{Amplitude}\propto(\text{Period})^2. \tag{5}$$

4. Presumption of the mechanism of seismic wave generation.

When pressure change or shearing force suddenly acts at a spherical origin, the amplitude of the generated seismic waves at a fairly far-off spot from the origin may be written for both P-waves and S-waves as follows:

$$A\propto a^2p_0\sin\frac{\alpha t}{a}\ ^{9)}, \tag{6}$$

where p_0: pressure or shearing force which acts on spherical surface,

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9) K. SEZAWA and K. KANAI, "Transmission of Arbitrary Elastic Waves from a Spherical Source, Solved with Operational Calculus, I, II, III," *Bull. Earthq. Res. Inst.*, **19** (1941), 151, 417, **20** (1942), 1.

a : radius of spherical origin, t : time, α : constant determined from the property of the material existing near the spherical origin.

If the pressure or shearing force p_0 acting upon the spherical surface is constant in every earthquake, from equation (6) it becomes

$$A \propto a^2, \quad T \propto a, \qquad (7)$$

and

$$A \propto T^2. \qquad (8)$$

Then it is found that the relation expressed in equation (8) coincides with that of the period and the amplitude of the initial motion of actual earthquake which was obtained from the seismograms.

As to the earthquake, No. 170, one of the earthquakes we used in this paper, such things as the relation between the P-waves and the S-waves concerning both amplitude and period, and the regional distribution of "push" and "pull" of initial motion were explained from the mechanism of earthquake generation in the author's previous paper[10], That is

$$\left.\begin{aligned} \widehat{rr}_{r=a} &= p_0 P_2(\cos\theta), && t>0, \\ &= 0 && t<0, \end{aligned}\right\} \qquad (9)$$

Considering the mechanism of earthquake generation expressed in equation (9), the radius of spherical origin a is given by

$$a=\frac{T}{2.6\,C_2}=\frac{T}{2.6}\sqrt{\frac{\mu}{\rho}}\,. \qquad (10)$$ [11]

When the radius a of spherical origin is determined from equation (10), strain, p_0/μ, will be given by

$$\frac{p_0}{\mu}=\frac{u_{f_1} r}{0.06 a^2}\,. \qquad (11)$$ [12]

Now, we calculate p_0/μ by applying equation (10) and (11) to the results obtained from the observation of the earthquake No. 170 as an example. In this case, as $t=0.53$ sec, if it is assumed to be $\sqrt{\mu/\rho}=4.45$ km/sec, the radius of spherical origin becomes from equation (10) $a \doteqdot 1$ km. In the earthquake concerned, since $u_{f_1}=9$ micron, with such numerical data as $r=100$ km and $a=1$ km as already known, equation (11) becomes p_0/μ

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[10] K. KANAI, "Examinations of the Assumption concerning Mechanism of Earthquake Origin through Seismograms", *Bull. Earthq. Res. Inst.*, **30** (1952), 39.

[11] K. KANAI, "Possibility of the Period of P-waves becoming smaller than those of S-waves", *Bull. Earthq. Res. Inst.*, **29** (1951), p. 533, Fig. 3. (−0.5 on the ordinate scale of Fig. 3 should be read −0.05.)

[12] *ibid.*, 11)

(strain of spherical surface)$\doteqdot 10^{-4}$.

From the results of calculation about various earthquakes, it is found that, when the mechanism of earthquake generation is assumed thus, the strain at the spherical surface where seismic waves are generated is expressed in the following equation. $p_0/\mu = 10^{-4} \sim 10^{-3}$. It may be considered that this value is nearly equal to the elastic limit of the material which constitutes the earth crust. It means that seismic waves are generated near the zone where the strain reaches the rupture limit.

In case of such a mechanism of earthquake generation, the energy of earthquake, E, and the amplitude, A, are in the relation of

$$E \propto A^{1.5} (\equiv a^3), \tag{12}$$

and the relation between E and the period, T, are as follows

$$E \propto T^3. \tag{13}$$

The amplitude of earthquake motion is considerably influenced by direction, damping, construction of earth crust and property of the ground near the observation point. Then it is easily seen that the relationship between the amplitudes at earthquake origin and at the observation point is not so simple. Therefore, in general, there will be a considerable inaccuracy in determining the energy or magnitude of earthquake by using the amplitude of earthquake motion even if we adopt the initial motion of P-waves which is considered to communicate the characteristics of seismic waves generated at the earthquake origin comparatively well.

On the contrary, the period of seismic waves, particularly that of the initial motion of P-waves, can be considered to keep the characteristics of the waves unchanged from the earthquake origin to the observation point. Consequently, if the relation between the period and the energy can be clarified, the energy or the magnitude of earthquake will be given more accurately by using the period of seismograms than by using the amplitude. In this sense, we can say that equation (13) is more important than equation (12).

5. Concluding remarks.

It was found from the initial motion of seismograms obtained at Hitachi Mine that there is such a relation between the amplitude, A and the period T of seismic waves as $A \propto T^2$.

This coincides with the mathematical results in case the pressure or shearing force at a spherical origin is assumed to be constant in every earthquake and the factor to determine the amplitude to be the radius of a spherical origin.

Then by calculating the strain at the spherical surface where seismic waves are generated with numerical data obtained from observations, it was found to be $10^{-4} \sim 10^{-3}$.

As the results of these studies, it is considered that seismic waves are generated at the spherical surface where a pressure causes strain near the elastic limit of the material constituting the earth crust. In such a mechanism of earthquake generation, the energy of earthquake E and the amplitude A are in the relation of $E \propto A^{1.5}$ and the relation between E and the period T are $E \propto T^3$.

The author would like to take this opportunity to thank the members of Motoyama Office, Hitachi Mine for their contributions and also Messrs. T. Tanaka and T. Suzuki for carrying out the observations.

Added Note:

The comparison is made between the seismogram and the actual ground motion concerning the initial motion. By the definition of an initial motion, it is found that $f(t)$ in equation (1) is always larger than zero within the range of the initial motion. Consequently, the second

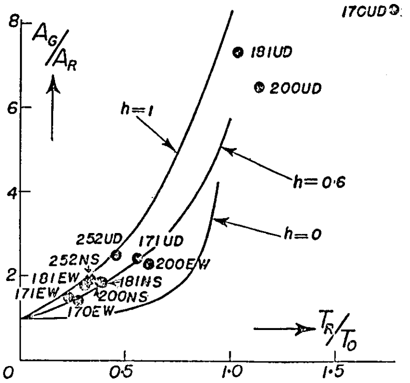

Fig. 10. T_R and T_0 represent the period of the initial motion of record and the period of seismograph. A_G and A_R represent the amplitude of the initial motion of actual ground motion and of record. The curves illustrate the calculated results of the first motion in a sudden occurrence of sinusoidal wave form.

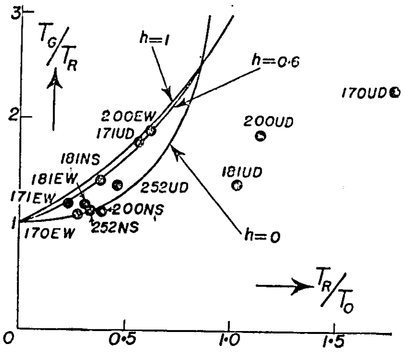

Fig. 11. T_G, T_R and T_0 represent the period of initial motion of actual ground motion, of record and the period of seismograph. The curves illustrate the calculated results of the first motion in a sudden occurrence of sinusoidal wave form.

and the third term of equation (1) are also larger than zero within the range of the initial motion. In other words, $f(t)$ is always smaller than $F(t)$ from the moment when $f(t)$ begins to move to the moment when it crosses the zero line (that is within the range of the initial motion of $f(t)$).

It means that, whatever kind of seismographs we may use or whatever form the seismic waves may assume, so far as it concerns the initial motion, the amplitude of seismogram is always smaller than that of the actual ground motion.

Similarily, as far as concerns the initial motion, the period given by the record is in every case shorter than that of the actual ground motion.

Figs. 10, 11 show the above mentioned relations of the actual earthquakes which can be found from Figs. 1～5. If the period of earthquake motion is approximately equal or smaller than that of the seismographs, by using Figs. 10, 11, the amplitude and period of actual ground motion can be easily found out from the seismograms within only a few per cent error.

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4. 地震動の振幅と周期との関係

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日立鉱山で観測した地震記象の初動を解析して，$A\propto T^2$ の関係が得られた．

地震波の振幅は，たとえＰ波の初動でも，伝播中に反射，屈折，吸収，拡散などの現象があり，又，方向性の問題もあるから，観測値から震源附近での大きさを推定することは容易ではない．特に，普通の観測場所では，地表面附近の複雑な地層の影響が地震波の振幅には大きくひゞくから，問題は益々やっかいになる．

その意味では，Ｐ波の初動の周期は，震源附近の性質を観測場所まで保持する度合が最も大きい筈である．

従って，地震のエネルギーと初動の周期との関係を，なにらかの方法で求めることができれば，任意の観測所の地震記象から，地震のエネルギーをたやすく，しかも比較的に精度をよく，きめることができるであろう．

１つの試みとして，簡単な発震機構の仮定のもとに，この関係を求めてみたら，$E\propto T^3$ ということになった．