24. *Notes on the Energy and Frequency of Earthquakes.**

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1) B. Gutenberg and C. F. Richter[1] made statistical studies of the energy and frequency of earthquakes. They arranged a great many shocks according to the "Instrumental Magnitude Scale" (hereafter expressed as M_I) introduced by the junior author[2]. They obtained the following relation

$$log\ N = a + b(8 - M_I) \tag{1}$$

where N is the annual number of earthquakes, of which the magnitude is between $M_I + 0.05M_I$ and $M_I - 0.05M_I$, and a and b the numerical constants. The values of the constants determined by them are

$$a = -0.48 \pm 0.02,\ b = 0.90 \pm 0.02$$

for the shallow schocks, where M_I is greater than 6, and

$$a = -2.04 \pm 0.09,\ b = 0.88 \pm 0.03$$

for the shocks occurring in South California, where M_I is between 4 and 6. Hereafter we shall discuss the value of "b" only.

Combining (1) with the relation according to Gutenberg and Richter[1]

$$log\ E = 12 + 1.8M_I \tag{2}$$

where E is the energy released in an earthquake having the magnitude of M_I, we get

$$log\ NE = k + 0.9M_I \tag{3}$$

for the shocks with the magnitude greater than 6, where "k" is a constant. For the shocks in South California we have the same relation as (3) excepting the value of "k". Since NE is the total energy annually released in shallow shocks within the range from M_I (1+0.05) to M_I (1−0.05), one can say from the relation (3) that the annual energy increases with M_I, if the earthquakes are classified at equal intervals of the M_I scale. Thus, Gutenberg and Richter wrote; "the results support in detail the previous conclusion that the smaller shocks almost never are sufficiently frequent to approximate the energy released in larger shocks. This

* communicated by T. Matsuzawa..

1) B. GUTENBERG *and* C. F. RICHTER. *Seismicity of the Earth* (1949).

2) C. F. RICHTER. *Bull. Seis. Soc. Am.*, **25** (1935), 1.

means that great shocks are essentially independent events, uninfluenced by the occurrence of smaller earthquakes, which are at most symptons of the regional strains released in the major shocks." But according to the opinion of the present writers, the expression quoted above is liable to be misunderstood.

The relation between the earthquake-magnitude and the frequency of earthquakes with such magnitude depends entirely on the nature of the magnitude-scale by which the shocks are classified. In order to avoid misunderstanding. the following sentence is to be added to the above-quoted description, "if the earthquakes are classified according to Richter's magnitude scale". If the classification is done by other scales, the relation will be apparently quite different,

For instance, the energy E itself which is released in a shock may be taken as a magnitude-scale.

As the relation between M_I and E is known, we can transform the relation (3) into the expression with the new variable E in the same way as the usual distribution functions in statistics are transformed. The process is as follows. The formula (1) is rewritten as

$$N(M_I)\ dM_I = const \times 10^{-0.9M_I} dM_I \qquad (4)\ \text{a}$$

where $N(M_I)$ is the annual number of earthquakes of which the magnitude ranges between M_I and M_I+dM_I. When $N(M_I)$ and E are expressed as

$$N(M_I)dM_I = const \times 10^{-bM_I} dM_I \qquad (4)\ \text{b}$$

$$E = const \times 10^{cM_I} \qquad (2)\ \text{b}$$

put (2) b into (4) and get

$$N(M_I)dM_I = const \times E^{-\frac{b}{c}} dM_I$$

or

$$N(E)dE = const \times E^{-\frac{b}{c}} \frac{dM_I}{dE} dE$$

thus we have

$$N(E)dE = const \times E^{-\frac{b}{c}-1} dE$$

From the numerical values of b and c in (4) a and (2), we obtain the new formula

$$N(E)dE = const \times E^{-1.5} dE \qquad (5)$$

and

$$N(E)E\ dE = const \times E^{-0.5} dE \qquad (6)$$

This result shows that the total energy annually released in shocks having the energy between E and $E+dE$ becomes smaller with the increase in E. The apparent difference between the above-mentioned result and the one obtained by Gutenberg and Richter is due solely to the difference in the two magnitude scales by which the earthquakes are classified. It must be remarked that the formula (1) is wholly dependent on the nature of Richter's scale.

2) On the other hand, H. Kawasumi defined another magnitude-scale (hereafter called M_K). The intensity "I" of an earthquake in Japanese C.M.O. scale at the epicentral distance of 100 km is taken by him to be the M_K of that shock. According to Kawasumi's statistical investigations[3], the frequency of earthquake occurrences is expressed by

$$N(M_K)dM_K = const \times 10^{-0.5\,M_K}\,dM_K \qquad (7)\text{ a}$$

where $N(M_K)$ is the frequency of earthquakes occurring during a certain period, of which M_K ranges between M_K and M_K+dM_K. Kawasumi's result shows further that the relation between E and M_K is

$$E = const \times 10^{\,M_K} \qquad (8)$$

Combining (8) and (2), we get the relation between M_K and M_I

$$M_K = const + 1.8M_I$$

Using this relation we can transform (7) into the expression with the parameter M_I. As

$$10^{\,M_K} = const \times 10^{1.8M_I}$$

we obtain

$$N(M_K)dM_K = const \times 10^{-0.9M_I}\,dM_K$$

or

$$N(M_I)dM_I = const \times 10^{-0.9M_I}\,\frac{dM_K}{dM_I}dM_I$$

$$N(M_I)dM_I = const \times 10^{-0.9M_I}\,dM_I \qquad (7)\text{ b}$$

This relation is quite the same as that obtained by Gutenberg and Richter.

It is noteworthy that Kawasumi's result agrees with the result obtained by Gutenberg and Richter, although they dealt with different data, i. e. shocks occurring in different places and in different periods.

From the relation (7) and (8), we have

$$N(M_K)EdM_K = const \times 10^{0.5\,M_K}\,dM_K \qquad (10)$$

In this relation, NE becomes larger as was the case by Richter's scale. This result is to be expected from the definitions of both magnitude-scales.

3) An experimental formula

$$N(a)da = ka^{-m}da \qquad (11)$$

for the trace amplitude and frequency of earthquakes occuring in Kanto District

3) H. KAWASUMI, Read at the meeting of Earthquake Research Institute (1943), No. 197.

was obtained by M. Ishimoto and K. Iida[4]. In this formula "a" is the trace amplitude, and $N(a)$ is the number of the earthquakes with trace amplitude between a and $a+da$, and "m" and "k" are numerical constants.

The present authors found that the relation (11) holds also in some cases of aftershocks of great earthquakes.[5][6] Moreover, it was found that the same relation holds in "micro-earthquakes",[7][8] which could be recorded only by high sensitive seismometers with the magnification of scores of thousand or more. In many cases the value of the exponent "m" was 1.8 or 1.9.

On the other hand, it was mathematically deduced[6] that if Ishimoto-Iida's formula holds between the trace amplitude and the frequency of shocks, a similar formula

$$N(A_0)dA_0 = const \times A_0^{-m} dA_0$$

holds in the area where the shocks occur (A_0 is the amplitude of a shock at its focus). Then,

$$N(A)dA = const \times A^{-m} dA$$

must hold between the amplitude A at the place having an epicentral distance of 100 km and the frequency of the shocks with the amplitude between A and $A+dA$, when the formula (11) holds.

As "M_I" is defined as

$$M_I = const + \log A \tag{12}$$

we can compare Ishimoto-Iida's formula with the result (1) obtained by Gutenberg and Richter.

Transforming A in (11) into M_I, we have

$$N(A)dA = const \times A^{-m} dA$$
$$= const \times 10^{-mM_I} dA$$

and,

$$N(M_I)dM_I = const \times 10^{-mM_I} \frac{dA}{dM_I} dM_I$$

or,

$$N(M_I)dM_I = const \times 10^{-(m-1)M_I} dM_I \tag{13}$$

Since the value of m is 1.8 or 1.9, this formula may be written as follows;

$$N(M_I)dM_I = const \times 10^{-0.8M_I}\, dM_I \qquad (14)\ a$$
$$N(M_I)dM_I = const \times 10^{-0.9M_I}\, dM_I \qquad (14\ b$$

4) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939) 443.
5) Z. SUZUKI and T. ASADA, Geophysical Notes Vol. 1. No. 32 (1947).
6) Z. SUZUKI, Read at the meeting of Seismological Society of Japan, Oct. 1948.
7) T. ASADA and Z. SUZUKI, Geophysical Notes, Vol. 2, No. 16 (1949).
8) T. ASADA and Z. SUZUKI, *Bull. Earthq. Res. Inst.*, **28** (1951) 415.

This is quite the same as the formula (1). It is remarkable that the relation (14) coincides well with the relation (1) although the earthquakes treated in both cases are quite different in their magnitude, place and time of occurrence.

4 The summarized conclusion of this investigation will be as follows:

a) The relation between the energy and frequency of earthquakes obtained by B. Gutenberg and C. F. Richter

$$\log NE = k + 0.9\ M_I$$

is wholly dependent on the nature of the "Instrumental Magnitude Scale". If the earthquakes are classified by another scale, the relation will be apparently quite different from the above formula.

b) The result of Gutenberg and Richter coincides well with that of Kawasumi, although the latter treated only earthquakes which occurred near Japan. The relation coincides also with Ishimoto-Iida's formula for the smaller earthquakes such as aftershocks or micro-earthquakes.

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24. 地震の頻度とエネルギーについて

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B. Gutenberg 及 C. F. Richter は地震の頻度とエネルギーとの關係について

$$\log NE = \mathrm{const} + 0.9\ M_I$$

とゆう式を求めて居り，これから「小地震の出すエネルギーは大地震のものに比して數の多さを考慮しても尚遥かに小さく從つて大地震は小地震に影響されない」と述べている．然し上の式は地震を組分けする尺度によつて當然變るもので，例えばエネルギー E の等間隔にとれば

$$\log NE = \mathrm{const} - 0.5 \log E$$

となり見かけ上逆の様な印象を與える．從つて Gutenberg 及 Richter の式については Instrumental Magnitude Scale によつたものである事を注意せねばならない．

河角博士の統計結果は Magnitude Scale を變換すれば Gutenberg, Richter の結果と同じものである事が分る，又餘震や微小地震について成立する石本飯田の式もやはり同じ結果を與える．この様に地震の頻度とエネルギーの關係は場所や時の如何を問わず又世界的な大地震から微小地震に至る迄の廣い範圍に亙つて同じものである事は面白い．
