9. *On the Coda Waves of Earthquake Motions.* (*Part 4.*)

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Chapter 5. Periodgram Analysis of Coda Oscillations.

§ 18. In the preceding Chapters[1] studies have been made on the coda oscillations of 53 earthquakes, and their predominant periods have been determined from the curves of the successive one minute means of the periods of coda oscillations. In the present Chapter, with a view of establishing more definitely the periodicities in coda oscillations, a method of periodgram analysis has been resorted to.

The amplitudes of coda waves measured at equal intervals of time have been represented by the sequence u_0, u_1, ... To test the existence in sequence of a periodicity with the period extending over p consecutive terms of the sequence, we will write down the $u's$ in order in horizontal lines, each of which will contain $p\,u's$. Seen vertically the completed diagram will be found to contain p columns and n rows. We will add up the terms in the individual vertical columns, and denote the sums by U_0, U_1, ..., U_{p-1}. Then, dividing these sums by n, we obtain the means N_0, N_1, ..., N_{p-1} of the values of u in the individual columns. If the tentative period p should fall near the actual period of the oscillations, the difference between the greatest and the smallest numbers of the sequence N_0, ..., N_{p-1} would have a large value. This difference we call "the amplitude" of the sequence N_0, ..., N_{p-1} corresponding to the trial period p. Taking p as abscissa and the amplitude as ordinate, we get the periodgram for the trial period p.

§ 19. In the neighbourhood of the true period p, when the trial period is not a whole number of the unit of time but is, say, $p+0.5$, for the purpose of finding out the amplitude of that trial period, T. Whittaker[2] suggested modifying the arragement of $u's$ slightly so that terms in the same phase might be found in the same vertical column. But according to his method a u in the second row is not exactly in the same phase as that of the first row in the same vertical column, the difference of the

1) S. OMOTE, *Bull. Earthq. Res. Inst.*, **21** (1943) 458, **22** (1944), 140, **23** (1945), 47.

2) E. T. WHITTAKER and G. ROBINSON, "The Calculus of Observations," (1926), 343.

two being by no means small, especially when p is smaller than 5.[3)]

In such a case, two classes of u's must be distinguished in one vertical column, and the sums of each class of u's must be worked out separately; the sum of the u's in odd rows, and the sum of the u's in even rows. Thus we get two sequences of N's, namely, N odd's and N even's. If we denote by $\mathfrak{R}_{od}$ and $\mathfrak{R}_{ev}$ the amplitudes of the two respective sequences, the amplitude for the trial period $p+0.5$ can be given more accurately by $\mathfrak{R}=\mathfrak{R}_{od}+\mathfrak{R}_{ev}$. For the convenience of calculation the author suggests the following manner of arrangement of u's to give the amplitude for the trial period $p+0.5$:

$$
\begin{array}{llllllll}
 & u_0, & u_1, & \ldots\ldots\; u_{p-1}, & u_p, & u_{p+1}, & \ldots\ldots\; u_{2p-1}, & u_{2p} \\
 & u_{2p+1}, & u_{2p+2}, & \ldots\ldots\; u_{3p}, & u_{3p+1}, & u_{3p+2}, & \ldots\ldots\; u_{4p}, & u_{4p+1} \\
 & \ldots\ldots & & & & & & \\
 & \ldots\ldots & & & & & & \\
 & u_{2(m-1)p+m-1}, & \ldots\ldots & u_{(2m-1)p+m-2}, & u_{(2m-1)p+m-1}, & u_{(2m-1)p+m}, & \ldots\ldots & u_{(2mp+m-1)}. \\
\hline
\text{Sums.} & U_0, & U_1, & \ldots\ldots\; U_{p-1}, & U_p, & U_{p+1}, & \ldots\ldots\; U_{2p-1}, & U_{2p} \\
1/\text{m of } U's. & M_0, & M_1, & \ldots\ldots\; M_{p-1}, & M_p, & M_{p+1}, & \ldots\ldots\; M_{2p-1}, & M_{2p}
\end{array}
$$

The u's are to be written in horizontal rows of $2\times(p+0.5)$ terms each, and terms in vertical columns are to be added up, yielding the sums U_0, ……, U_{p-1}, U_p, ……, U_{2p-1}, U_{2p}. Divinding these sums by m (which is a number of horizontal rows) we obtain the means M_0, ……, M_{p-1}, M_p, ……, M_{2p-1}, M_{2p} of the values of u's in the individual columns.

Table I.

Earthquake that the periodgram analysis was executed and its portions in which the analysis was made.

Earthq. No.	Date	Epicenter	Epicentral distance	Portions studied
			km	m m m m
16	1925 III 31	Siwoyasaki	259	6∼ 7, 8∼10
20	1936 II 21	Kôti-Yamato	391	11∼13
54	1941 XII 17	Kagi	2230	22∼29, 30∼37
36	1938 V 20	Celebes	4500	42∼44, 52∼53
37	1938 V 13	New Guinea	4600	55∼56, 80∼83, 102∼103
49	1937 XII 23	Mexico	11600	80∼83
51	1937 VI 21	Peru	14900	69∼75, 123∼126
52	1936 VII 13	Chili	16800	64∼70, 78∼81, 138∼141

3) S. OMOTE, *Special Bulletin of the Earthq. Res. Inst.*, No. 7 (1949), 7.

If we denote by $\mathfrak{M}_1$, and $\mathfrak{M}_2$ the amplitudes of the two sequences M_0, ..., M_{p-1} and M_p, ..., M_{2p} respectively, the most reasonable amplitude for the trial period $p+0.5$ will be given by $\mathfrak{M}=\mathfrak{M}_1+\mathfrak{M}_2$.

§ 20. In this manner periodgrams have been prepared for the eight selected earthquakes (Table I), two of which had epicentral distances less than 1,000 km, one about 2,000 km, two others about 4,000 km, and the remaining three exceeding 10,000 km. The amplitude for each trial period

Table II.

Amplitudes $\mathfrak{N}$ for respective trial period p of each portions.

Earthq. No.	No. 16		No. 20	No. 54		No. 36		No. 37			No. 49	No. 51		No. 52		
Portion / p	6~7	8~10	11~13	22~29	30~37	42~44	52~53	55~56	80~83	102~103	80~83	69~75	123~130	64~70	78~81	138~141
5	64.5	103.8	117.6	8.4									0.34			
6	115.2	84.9	69.6	6.6	5.3								1.01			
6.5		168.2														
7	89.3	118.9	298.6	11.6	7.1								0.29			
7.5		77.0	222.7		19.3											
8	221.7	363.8	148.9	13.5	17.4								1.19			
8.5		130.1		65.3	14.2											
9	125.4	131 8	83.8	40.3	23.5								1.23			
9.5				115.8	46.2											
10	113.7	101.8	153.1	118.4	47.5			27.4					0.94	15.8	9.1	
10.5				88.6	39.7											
11	80.7	89.4	139.4	70.1	15.9								0.95			
11.5					10.7											
12	130.3	109.9		57.3	23.9	106.4	77.0	24.0	16.7	9.6	34.9		1.23	11.2	10.1	19.3
12.5					23.4											
13				9.1	16.6	140.8	58.3	27.5	22.1	10.5	31.5		0.62			
13.5					18.9											
14			178.6	15.6	8.7	193.8	149.7	44.4	29.2	25.6	27.7	16.7	3.55	63.4	21.3	29.3
15			144.6	45.3	15.6	141.0	100.8	73.8	18.3	32.1	37.8	17.7	4.20	57.2	36.8	45.9
15.5												42.0		128.9	22.5	
16			92.5	91.4		117.1	64.3	51.3	72.7	32.9	158.2	46.3	10.03	133.0	28.7	115.6
16.5												59.2	5.20	60.5	70.8	
17			157.1			111.0	100.7	17.6	38.2	34.7	130.7	66.9	3.37	89.0	123.6	58.8
17.5												52.0			54.2	
18				22.6		215.3	122.2	19.2	32.1	21.2	112.5	94.6	3.49	397.1	40.8	
18.5												44.9			92.7	
19			136.9			264.4	175.3	31.4	45.8	10.5	33.3	17.9	2.57	141.0	108.7	36.6
19.5															90.9	
20			186.9			256.1	106.9	63.5	38.5	7.8	18.0	18.5		90.9	58.0	31.3
21						273.7	71.0	50.2	81.3	10.5	58.6	65.5		212.2		
22						125.0	33.2	49.3	22.0		61.1	23.7		117.6		
23												26.4				

will be seen in Table II. The curves for the periodgrams will be reproduced in Figs. 1～16. Generally speaking, the periods obtained from periodgram analysis have been found to coincide fairly well with the

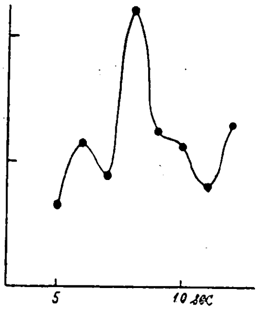

Fig. 1.
No. 16 (Siwoya-Saki) 6 min～7 min.

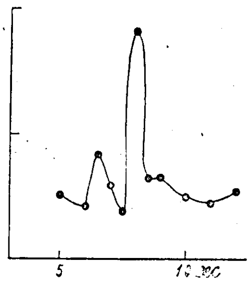

Fig. 2.
No. 16 (Siwoya-Saki) 8 min～10 min.

predominant periods determined in the preceding chapters from the curves of successive one minute means of periods of the respective earthquakes.

With regard to some figures two peaks are seen in a curve. For instance Celebes earthquake (Figs. 6, 7) has two peaks at 14 sec. and 19 or 20 sec. New Guinea earthquake (Figs. 8, 9 and

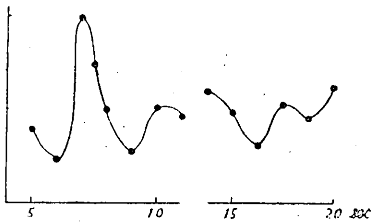

Fig. 3.
No. 20 (Kôti-Yamato)
11 min～13 min.

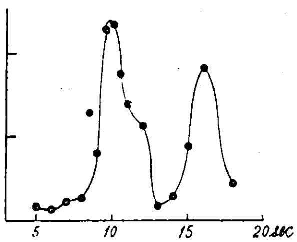

Fig. 4.
No. 54 (Kagi) 22 min～29 min.

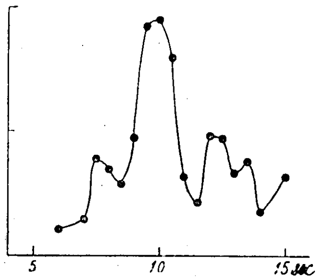

Fig. 5.
No. 54 (Kagi) 30 min～37 min.

10) at 15～16 sec. and 20～21 sec. The periodgrams of the first coda waves of extremely distant earthquakes such as Mexico (Fig. 11), Peru (Fig. 12) and Chili (Figs. 13, 14) earthquakes also show two high peaks. Of these two peaks, one is always seen at a period that agrees well

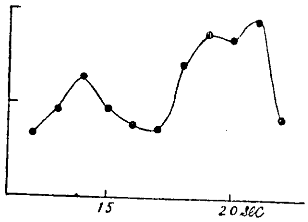

Fig. 6.
No. 36 (Celebes) 42 min~44 min.

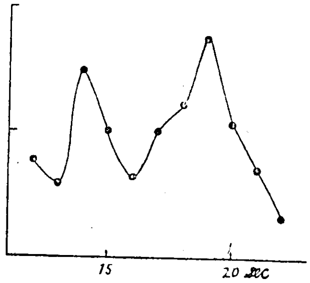

Fig. 7.
No. 36 (Celebes) 52 min~53 min.

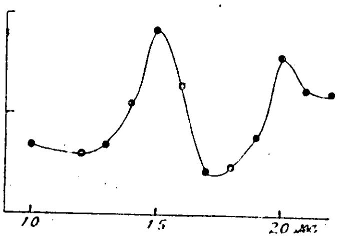

Fig. 8.
No. 37 (New Guinea) 55 min~56 min.

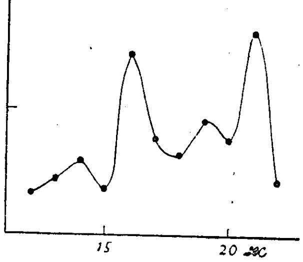

Fig. 9.
No. 37 (New Guinea) 80 min~83 min.

with the predominant period of the same earthquake determined in the preceding chapters, while the other peak is seen at the period T equals to about 20 seconds in all earthquakes regardless of the difference in their epicentral distances. Contrary to this the periodgrams on the second coda waves of extremely distant earthquakes such as Peru and Chili earthquakes (Figs. 15, 16) show a very high single peak at

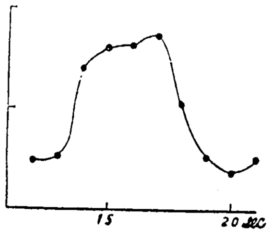

Fig. 10.
No. 37 (New Guinea) 102 min~103 min.

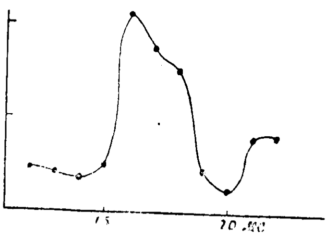

Fig. 11.
No. 49 (Mexico) 80 min~83 min.

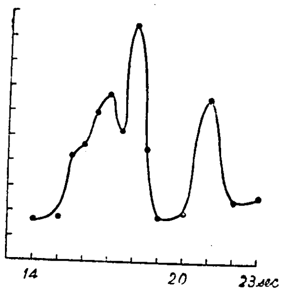

Fig. 12.
No. 51 (Peru) 69 min~75 min.

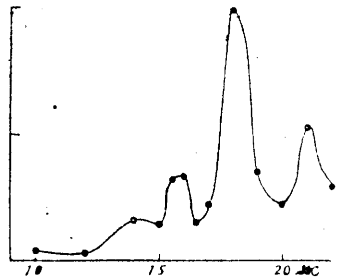

Fig. 13.
No. 52 (Chili) 64 min~70 min.

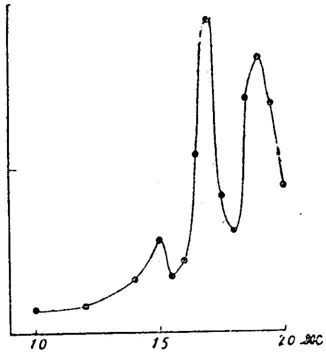

Fig. 14
No. 52 (Chili) 78 min~81 min.

T equals to 16 seconds. This period agrees well with the period of the second peak of the Kagi earthquake. (Fig. 4).

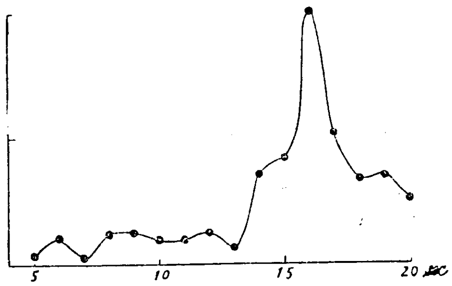

Fig. 15.
No. 51 (Peru) 123 min~130 min.

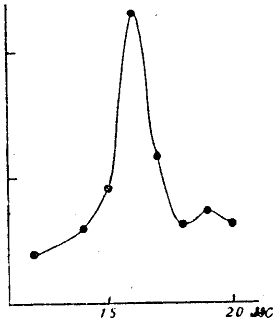

Fig. 16.
No. 52 (Chili) 138 min~141 min.

Predominant periods determined by means of these periodgram analyses will be tabulated in Table III and for the purpose of comparison predominant periods obtained in the previous papers[4] are given also. Anyhow it seems that two sorts of waves are likely to be present in the first coda waves and single sort of waves in the second coda waves. These are very notable facts, and, in order to determine their periods more exactly, we proceeded to make analyses by the method introduced by K. Takahashi, the result of which will be seen in the following chapter.

4) S. OMOTE, *loc. sit.*, 1).

Table III.

Comparison of the predominant periods determined by different methods.

T_m: Predominant period determined from the curve of the successive one minute mean of the periods of coda oscillations.

T_p: Predominant period determined by means of the periodgram analysis.

Earthq. No.	T_m		T_p		
	1st coda	2nd coda	1st coda		2nd coda
	sec.	sec.	sec.	sec.	sec.
16	6.5	—	8	—	—
20	7.2	—	7	—	—
36	15.9	—	14	20	—
37	15.3	—	15	20	—
49	17.5	16.9	16	—	—
51	18.1	16.5	18	21	16
52	17.7	16.6	18	20	16