5. *Tsunami in Tsubaki-tomari Bay.*

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The aspects of the tsunami acommpanied by the Nankai Earthquake of Dec. 21, 1946, were investigated in great detail by the members of the Institute. Among these investigations, the writers visited the eastern coast of Shikoku-Island with special interest to the mode of intrusion of sea-water in a bay. They measured the maximum heights attained by the tsunami in various parts of Tsubaki-tomari Bay, that is situated in the most eastern part of Tokushima prefecture, adjacent Cape Gamoda. As is shown in Fig. 1, the distance between the mouth and the head of bay amounts to about 4 km, while the depth increases almost linearly with distance from the head amounting to about 25 m at the mouth. The maximum heights from the sea level at the time of the tsunami were obtained with the aid of the tide-prediction at Muroto[1], the most southeastern part of Shikoku-Island. They are also indicated in Fig. 1.

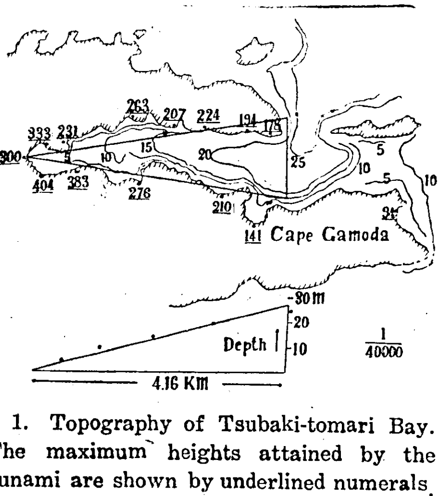

Fig. 1. Topography of Tsubaki-tomari Bay. (The maximum heights attained by the tsunami are shown by underlined numerals. Unit: cm.)

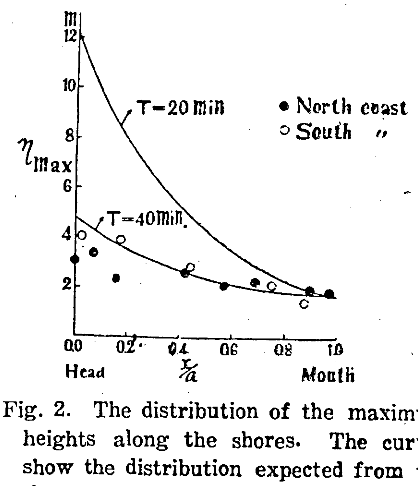

Fig. 2. The distribution of the maximum heights along the shores. The curves show the distribution expected from the theory mentioned later.

In Fig. 2, plotting these values against the distance measured along the shores, they could obviously see that the heights increase with

1) The prediction was made by Prof. H. Kawasumi.

distance from the mouth as was observed in some bays in the cases of the Sanriku Tsunami, 1933[2], and also in the present one[3].

When the tsunami-waves arrive at the mouth of a bay, the free oscillations with certain periods will be set up in the bay. The closer the period of the tsunami-waves to that of the free oscillation of the bay, the larger become the amplitude of it. Thus it is of interest to study to what extent the free oscillation may be excited in a bay.

G. Nishimura, K. Kanai and T. Takayama[4),5)], discussed the motion of sea-water in a rectangular bay with uniform depth by means of Stokes' method. In this paper, the writers estimate, by means of opertional method, the oscillation of water in a bay whose breadth and depth vary linearly from its mouth to head with application to the tsunami in Tsubaki-tomari Bay just mentioned.

As is shown in Fig. 1, assuming that this bay can be replaced by an idealized bay whose breadth b and depth h vary respectively

$$b = b_0 x/a \quad \text{and} \quad h = h_0 x/a\,, \tag{1}$$

where x, b_0, h_0 and a denoted respectively the distance measured along the central line of the bay, breadth and depth at the mouth and the length of the bay. As is well known in the theory of wave-motion in a canal of variable section, the elevation of surface η satisfies the next differential equation

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{g}{b(x)}\frac{\partial}{\partial x}\left\{b(x)h(x)\frac{\partial \eta}{\partial x}\right\}, \tag{2}$$

where g denotes acceleration of gravity.

Writing p in place of $\partial/\partial t$ and substituting (1) in (2), we get

$$\frac{d^2\eta}{dx^2}+\frac{2}{x}\frac{d\eta}{dx}-\frac{ap^2}{gh_0}\frac{\eta}{x^2}=0\,. \tag{3}$$

We can easily find that the solution of (2) remaining finite at the head of the bay or at $x = 0$ becomes

$$\eta = \frac{A}{p\sqrt{\dfrac{ax}{gh_0}}} I_1\left(2p\sqrt{\frac{ax}{gh_0}}\right), \tag{4}$$

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2) *Bull. Earthq. Res. Inst. Suppl. Vol.*, **1**, Part 2 (1934), 9.

3) *Special Bull. Earthq. Res. Inst.*, No. **5** (1947), 98.

4) G. Nishimura and K. Kanai, *Bull. Earthq. Res. Inst. Suppl. Vol.*, **1**, Part 1 (1934), 182.

5) G. Nishimura, K. Kanai and T. Takayama, *Bull. Earthq. Res. Inst.*, **13** (1935), 64.

where I_1 denotes modified Bessel function of the first order. A is determined by the boundary condition at the mouth of the bay or at $x = a$ as follows.

If we assume that $\eta = 0$ for $t < 0$

$$\eta = C \sin \omega t \quad \text{for} \quad t < 0 \tag{5}$$

at $x = a$, it becomes

$$A = \frac{p\sqrt{\dfrac{a^2}{gh_0}}}{I_1\left(2p\sqrt{\dfrac{a^2}{gh_0}}\right)} \frac{C\omega p}{\omega^2+p^2}, \tag{6}$$

considering that the operational form of (5) is expressed by $\dfrac{C\omega p}{\omega^2+p^2}$.
From (4) and (6), we get

$$\eta = \sqrt{\frac{a}{x}} \frac{I_1\left(2p\sqrt{\dfrac{ax}{gh_0}}\right)}{I_1\left(2p\sqrt{\dfrac{a^2}{gh_0}}\right)} \frac{C\omega p}{\omega^2+p^2}. \tag{7}$$

Solving this operational equation, η becomes

$$\eta = \frac{\omega C}{2\pi i} \int_L e^{\lambda t} \frac{I_1\left(2\lambda\sqrt{\dfrac{ax}{gh_0}}\right)}{I_1\left(2\lambda\sqrt{\dfrac{a^2}{gh_0}}\right)} \frac{d\lambda}{\lambda^2+\omega^2}, \tag{8}$$

where L is Bromwich's path of integration having all singularities to its left-hand side.

According to the theory of contour integration, (8) becomes

$$\eta = C\sqrt{\frac{a}{x}} \left\{ \frac{J_1\left(2\omega\sqrt{\dfrac{ax}{gh_0}}\right)}{J_1\left(2\omega\sqrt{\dfrac{a^2}{gh_0}}\right)} \sin \omega t + \sum_{s=1}^{\infty} \frac{J_1\left(\alpha_s\sqrt{\dfrac{x}{a}}\right)}{J_0(\alpha_s)} \frac{\omega\sqrt{\dfrac{gh_0}{a^2}}}{\dfrac{gh_0}{4a^2}\alpha_s - \omega^2} \sin \frac{\sqrt{gh_0}}{2a}\alpha_s t \right\} \tag{9}$$

where α_s denotes the s-th root of $J_1(x) = 0$. The we can estimate the elevation at any time from (9).

When t is very small, we can easily get more convenient expression by use of asymptotic expansion. As is well known, it becomes that

$$I_1(z) = \sqrt{\frac{1}{2\pi z}}e^z\left\{1-\frac{3}{8}\frac{1}{z}-\frac{15}{128}\frac{1}{z^2}\cdots\cdots\right\} \tag{10}$$

for large values of $|z|$. Then taking the first term, (8) becomes

$$\eta = \frac{\omega C}{2\pi i}\left(\frac{a}{x}\right)^{3/4}\int_L e^{\lambda\left\{t-2\sqrt{\frac{a}{gh_0}}(\sqrt{a}-\sqrt{x})\right\}}\frac{d\lambda}{\omega^2+\lambda^2}$$

or

$$\eta = C\left(\frac{a}{x}\right)^{3/4}\sin\omega\left\{t-2\sqrt{\frac{a}{gh_0}}(\sqrt{a}-\sqrt{x})\right\}. \tag{11}$$

This expresses the wave proceeding into the bay with velocity $\frac{\sqrt{gh_0}}{2}$ while the amplitude increases proportionally to $x^{-3/4}$.

Taking $a = 4.2$ km and $h_0 = 28$ m, the writers estimate the elevation of the surface in the bay both in the cases of the tsunami having the periods 20 and 40 minutes. The results are shown in Fig. 3a and Fig. 3b where it is clearly seen that the increasing rate in amplitude with distance from the mouth is remarkably larger in the case of 20 min. than that of 40 min. On the other hand, the period of the seiche

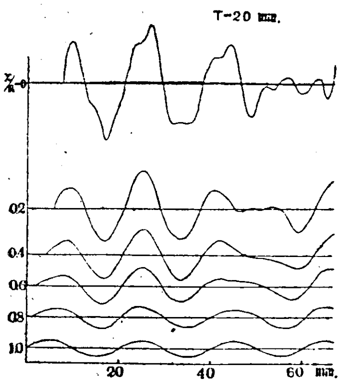

Fig. 3a. The elevation of sea-water at various parts of the bay. The period of the tsunami-wave is assumed to be 20 minutes.

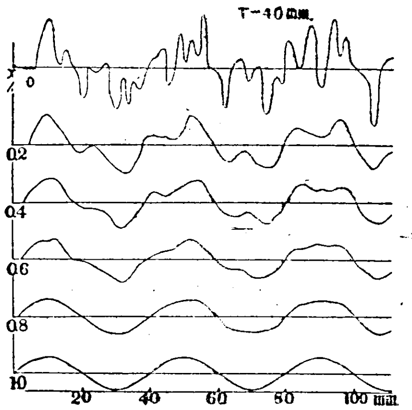

Fig. 3b. The elevation of sea-water at various parts of the bay. The period of the tsunami-wave is assumed to be 40 minutes.

of this bay T_0 is given by putting $\eta = 0$ at the mouth, that is, they get from (4)

$$J_1\left(\frac{4\pi}{T_0}\sqrt{\frac{a^2}{gh_0}}\right) = 0. \tag{12}$$

Then the longest period becomes 14.2 min. . The actual observations executed by the writers at the head of the bay gave about 11 min. as shown in Fig. 4. Then, with resonance phenomena in mind, it seems reasonable that the amplitude becomes larger in the case of 20 min. than that of 40 min. .

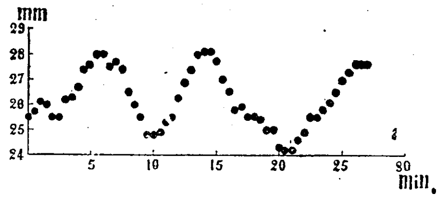

Fig. 4. The seiches measured at the head of the bay.

Assuming that the amplitude of the tsunami-waves amounts to 1.8 m at the mouth, in the next place, it is found that the maximum heights of sea water during a few oscillations vary along the shore as is also shown in Fig. 2. As the agreement between the theory and the observation seems good in the case of 40 min. , we may consider that the tsunami-waves have a period which does not differ materially from 40 min. . At the head of the bay, however, the observation gives smaller values than the calculated ones. This may be explained if we take into account the difference between the actual condition and the boundary condition that there does not occur horizontal flow at the head of the bay.

In conclusion, the writers wish to express their sincere thanks to Professors R. Takahashi and T. Hagiwara for their kind advices and helpful suggestions.