41. The Deflections of the Vertical, the Undulation of the Geoid, and Gravity Anomalies.

By Chûji Tsuboi,

Earthquake Research Institute.

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Let \( AA \) in Fig. 1 be the plane of the earth's surface, the gravity anomalies along which are known from point to point, and \( BB \) another plane which is at a depth \( d \) from the surface, and on which are distributed the surface densities responsible for these gravity anomalies. Assume further that the surface densities along \( BB \), and consequently the gravity anomalies along \( AA \), do not change in the direction that extends from \(+\infty\) to \(-\infty\) perpendicularly to the plane of this paper, so that they can be expressed by \( \rho(y) \) and \( Jg(x) \) respectively, \( x \) and \( y \) being the abscissae along \( AA \) and \( BB \), with line \( OO \) as the common origin. It was shown in the preceding paper\(^1\) that if

\[
\rho(y) = \rho_0 \cos ny,
\]

then

\[
Jg(x) = 2\pi k^2 \rho_0 e^{-ny} \cos nx.
\]

The horizontal component of attraction due to the same mass is given by

\[
\frac{\partial V}{\partial x} = 2k^2 \rho_0 \int_{-\infty}^{+\infty} \frac{(y-x) \cos ny}{(y-x)^2 + a^2} \, dy.
\]

Putting

\[
p = y - x,
\]

we get

\[
\frac{\partial V}{\partial x} = 2k^2 \rho_0 \int_{-\infty}^{+\infty} \frac{p \cos n(p+x)}{p^2 + a^2} \, dp
\]

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\[ = 2k^2 \rho_\eta \left\{ \cos nx \int_{-\infty}^{\infty} \frac{p \cos np}{p^2 + \sigma^2} \, dp - \sin nx \int_{-\infty}^{\infty} \frac{p \sin np}{p^2 + \sigma^2} \, dp \right\} \]

\[ = -2\pi k^2 \rho_\eta e^{-\eta^2} \sin nx, \]

making use of the integrals

\[ \int_{-\infty}^{\infty} \frac{p \cos np}{p^2 + \sigma^2} \, dp = 0 \]

and

\[ \int_{-\infty}^{\infty} \frac{p \sin np}{p^2 + \sigma^2} \, dp = \pi e^{-\eta^2}. \]

The deflection of the vertical \( \Delta \theta \) at \( x=x \) is given by

\[ \Delta \theta = \frac{1}{g} \frac{\partial V}{\partial x} \]

\[ = -\frac{2\pi k^2 \rho_\eta}{g} e^{-\eta^2} \sin nx. \]

The height of the geoid \( h \) at \( x=x \) relative to that at \( x=x_0 \) is obtained by integrating the above expression with respect to \( x \) from \( x_0 \) to \( x \), thus

\[ h = \int_{x_0}^{x} \Delta \theta \, dx \]

\[ = -\frac{2\pi k^2 \rho_\eta}{g} e^{-\eta^2} \cos nx - h_0. \]

Comparing this expression with that for \( \Delta g \), we see that \( h \) and \( \Delta g \) are in phase with each other, a positive gravity anomaly being associated with up-warping of the geoid and a negative anomaly with down-warping.

If we omit the \( h_0 \) in the expression for \( h \), which is a constant and combine it with that for \( \Delta g \), we get

\[ h = \frac{\Delta g}{ng}. \]

It is to be noted that \( h \) can be calculated from \( \Delta g \) with the aid of this expression without any previous knowledge of the depth of the subterranean mass \( \rho (y) \).
Since
\[ n = \frac{2\pi}{\lambda_n}, \]
we have
\[ h = \frac{\lambda_n}{2\pi} \frac{\Delta g}{g}. \]
If, for instance, we put
\[ \lambda_n = 100 \text{ km}, \]
\[ |\Delta g| = 0.100 \text{ gal}, \]
then we get
\[ |h| = \frac{10^7 \times 0.1}{2\pi \times 10^3} = 170 \text{ cm}. \]

The above relation between \( h \) and \( \Delta g \) can be extended to more general and complicated cases in which there is more than one harmonic component. Thus, if
\[ \Delta g(x) = \sum A_n \cos nx + \sum B_n \sin nx, \]
then
\[ h = \frac{a}{2\pi g} \left\{ \sum \frac{A_n}{n} \cos nx + \sum \frac{B_n}{n} \sin nx \right\}, \]
where \( a \) is the extent of the range in which \( \Delta g \) is expressed in the Fourier series. It is not necessary that all the harmonic components of \( \Delta g \) should be due to the mass at one and the same depth, the only requirement being that each component shall be due to the mass that is respectively horizontal.

Because each of the Fourier coefficients for \( h \) is \( n \) times smaller than the corresponding one for \( \Delta g \), the relative variation in the height of the geoid will be much smaller than that in \( \Delta g \).

The gravity anomalies found by Vening Meinesz along his profile No. 21 have already been discussed in the preceding paper. Utilizing the Fourier coefficients for \( \Delta g \) calculated in that paper, those for \( h \) were found to be as follows:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \sin )</th>
<th>( \cos )</th>
<th>( n )</th>
<th>( \sin )</th>
<th>( \cos )</th>
<th>( n )</th>
<th>( \sin )</th>
<th>( \cos )</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>44.6</td>
<td>4</td>
<td>1.9</td>
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<td>7</td>
<td>-0.1</td>
<td>-0.2</td>
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<tr>
<td>2</td>
<td>-11.5</td>
<td>7.3</td>
<td>5</td>
<td>0.4</td>
<td>0.9</td>
<td>8</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>-2.3</td>
<td>-4.5</td>
<td>6</td>
<td>-0.4</td>
<td>0.3</td>
<td></td>
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</tr>
</tbody>
</table>
The form of the geoid along this profile, which was obtained by synthesis of the harmonic components given in the table, is shown in Fig. 2.

It is hoped that the method here proposed, which corresponds to that of Stokes, except that the latter applies to spherical surfaces, will find many applications in practical problems.

41. 鉛直線偏倚，ジオイドの凹凸と重力異常

地質研究所 坪井 忠二

適當な假定を設けること，表題の三者の間に簡単な関係がある事を示す，特に重力異常が與へられればジオイドの凹凸が解る事云う事は應用が深いと思へられる。本文中には関領印度諸島附近に於けるマイネスの測定について應用例が示してある。