43. Some Physical Properties of the Lava of Volcanoes Asama and Mihara.

I. Electric Conductivity and Its Temperature-Coefficient.

By Takesi Nagata.

Earthquake Research Institute

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1. Introduction.

Recently a number of reports\(^1\) have been published of studies of electric and magnetic disturbances accompanying activities of volcanoes, such, for example, as sudden variations in terrestrial magnetism and earth-current. Obviously, continuous and systematic observations of the phenomena, especially of their characteristics in detail, are most important in the present study, as also experimental investigations of the electric and magnetic properties of lava,\(^2\) which may be related directly or indirectly to electric and magnetic disturbances. The writer, therefore, endeavoured to measure the magnetic susceptibility, coercive force, electric conductivity, and the ionization of volcanic lava, and their temperature-coefficients.

First, we measured the electric conductivity and the temperature-coefficient of a number of volcanic rocks ejected from the volcanoes Asama and Mihara. It is well known that conspicuous earth-current disturbances occur in connection with eruptions of volcanoes in the neighbourhood of craters. As possible causes of this phenomenon, there are several facts to be considered in connection with volcanic eruptions, such as electro-motive force due to inequalities in the extent of ionization of the lava in the crater, thermo-electric current produced by inequalities in its neighbourhood, and disturbances in the electric

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field around the crater due to anomalous variation in electric conductivity of lava in the volcano. Although we cannot at present say which of these is the most potent cause of the disturbances, a knowledge of the magnitude of the conductivity, it is believed, might furnish a clue to this problem.

The variation in electric conductivity with temperature is expected to be closely related to that of the viscosity of the lava, as in the case of viscous glass, in which, as is known empirically, the reciprocal of the ionic conductivity varies linearly with the coefficient of viscosity. If this assumption holds in the case of volcanic lava, we can easily obtain the value of the temperature-coefficient of viscosity by measuring its electric conductivity, the absolute value of which, however, must be measured dynamically.


The specimens were crushed fine, the diameter of the particles being about 0·1 mm. A portion of the crushed lava and a pair of electrodes were placed together with a thermo-junction in a Tammun-tube, 10 cm long and 1·8 cm in diameter, arranged as shown in Fig. 1. The electrodes, which were small platinum spheres of 1·7 mm diameter, were connected to the ends of platinum lead-wires 0·5 mm in diameter, inserted in porcelain tubes, except that part of the wires 1 mm from the centre of each sphere. The distance between these two electrodes was 11·5 mm, and the constant for reducing the apparent conductivity into specific conductivity was determined by using fused NaCl and dilute solution of H₂SO₄. In the beginning of this experiment, the conductivity was measured by the so-called Kohlrausch bridge method. Provided a higher order of accuracy is not required, the method of directly measuring the resistance is also applicable, experiment by this method being easier. Consequently in most of the experiments, the conductivity was measured directly. To render the polarization effect as small as possible, alternating-current of 50-cycles was used throughout this experiment. The procedures in these two methods are shown in Figs. 2, and 3.
3. Results of Experiment.

The electric conductivity of Miharaite (basic molten lava of Volcano Mihara) ejected in 1913, and that of olivine-bearing-hypersthene-augite-andesite (the bread-crust bomb) of Volcano Asama ejected in September, 1929, were measured at various temperature ranging from 400°C to 1250°C. The chemical composition of these two rocks are reproduced in Table I from the papers by S. Tsuboi\(^3\) (for Mihara) and by H. Tsuya\(^4\) (for Asama).

<table>
<thead>
<tr>
<th></th>
<th>Mihara</th>
<th>Asama</th>
</tr>
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<tbody>
<tr>
<td>SiO(_2)</td>
<td>51·45</td>
<td>60·24</td>
</tr>
<tr>
<td>Al(_2)O(_3)</td>
<td>16·84</td>
<td>16·43</td>
</tr>
<tr>
<td>Fe(_2)O(_3)</td>
<td>1·49</td>
<td>1·54</td>
</tr>
<tr>
<td>FeO</td>
<td>10·95</td>
<td>5·26</td>
</tr>
<tr>
<td>MgO</td>
<td>4·48</td>
<td>3·92</td>
</tr>
<tr>
<td>CaO</td>
<td>10·71</td>
<td>7·02</td>
</tr>
<tr>
<td>Na(_2)O</td>
<td></td>
<td>3·00</td>
</tr>
<tr>
<td>Mg(_O)O(_3)</td>
<td>1·23</td>
<td></td>
</tr>
<tr>
<td>K(_2)O</td>
<td>0·37</td>
<td>1·16</td>
</tr>
<tr>
<td>H(_2)O+</td>
<td>(\frac{1}{2})</td>
<td>1·17</td>
</tr>
<tr>
<td>H(_2)O−</td>
<td>(\frac{1}{2})</td>
<td>0·06</td>
</tr>
<tr>
<td>TiO(_2)</td>
<td>1·27</td>
<td>0·70</td>
</tr>
<tr>
<td>P(_2)O(_5)</td>
<td>0·25</td>
<td>0·13</td>
</tr>
<tr>
<td>MnO</td>
<td>0·19</td>
<td>0·11</td>
</tr>
</tbody>
</table>

\(^3\) Y. Okamura, Rep. Geol. Surv., 48 (1914).
The heating and cooling velocities were nearly uniform, and times required for raising the temperature from 300°C to 1250°C and that for lowering from 1200°C to 300°C were about three hours. The relation between the reciprocal of the absolute temperature and the logarithm of the specific electric conductivity was shown in Figs. 4, and 5.

From these figures, it will be seen that the Rasch and Hinrichsen formula for temperature effect on the electric conductivity of a fused salt in amorphous state is applicable to volcanic lava. Thus, the relation between the specific conductivity \( \sigma \) and the absolute temperature \( T \) of lava is expressed by the formula,

\[
\log \sigma = A - \frac{B}{T},
\]

where \( A \) and \( B \) are constants, each of which is significant thermodynamically. Differentiating both sides of equation (1) with respect to \( T \), we have

\[
\frac{d\log \sigma}{dT} = \frac{C}{RT^2}.
\]

This equation is a modified form of van't Hoff's equation referred to by Rasch and Hinrichsen.\(^5\)

In the case of Miharaite, the formula is not so exact as in Asama lava.

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6) loc. cit.
lava. In a similar case of glass, L. L. Holladay\(^7\) proposed theoretically another formula of the form

\[ \log \sigma = A - \frac{1}{2} \log T - B/T, \]  

although there seems to be some arbitrariness in his theoretical considerations. In opposition to Holladay’s theory, H. Schönborn\(^8\) explained the deviation from the linear relation between \(\log \sigma\) and \(1/T\) by pointing out the existence of a critical point where the chemical state of the glass changed discontinuously and the temperature-coefficient in equation (1) also changed discontinuously at this point. Although the writer will not attempt here to criticize this problem, his further studies are expected to provide an explanation of the foregoing deviation from the linear relation.

The curves in Fig. 6 show the variations in intensity of the current that flows through the specimen when the alternating-voltage applied in each case was 10.6, 6.3, and 4.2 volts respectively. Supposing that \(I\) is the current-flow, \(\sigma\) the specific conductivity, \(V\) the applied voltage, and \(C\) a constant depending on the dimensions of the specimen, the following relation is established when the conductivity is measured directly,

\[ I = C\sigma V. \]

From the curves shown in Fig. 6, there is no change in the relation between \(\log \sigma\) and \(1/T\), even if the magnitude of the applied voltage is changed. For the purpose of ascertaining whether the relation does not depend on applied voltage, the conductivity was measured at temperatures of 820°, 960°, 1050°, and 1190°C, at various voltages ranging from 0.01 to 10.0 volts. The results are shown in Fig. 7. The curves in the figure show that current and field intensity are in linear relation, that is, the conductivity is independent of the applied voltage.

\(^7\) L. L. Holladay, Journ. Frank. Inst., 229 (1923), 195.

\(^8\) H. Schönborn, Z. S. f. Phys., 22 (1924), 305.
In heating the specimens, the existence of a critical point was noticed at 1130°C, as shown in Fig. 8. The equation \( \log \sigma = A - B/T \) is applicable within the range of temperature from 400°C to 900°C, whereas at temperature higher than 960°C, coefficient \( B \) gradually diminishing, which however increases discontinuously at 1130°C. The existence of this critical point at 1130°C is a common feature of igneous rocks ejected from Asama and from Mihara. At this critical point some portion of the rocks may begin to melt. Upon heating powdered specimen at room temperature, which will hereafter be called “initial heating,” the curve showing the relation of conductivity to temperature breaks at 1130°C, as shown in Figs. 9, and 10. In these two different states, i.e., the one lower than and the other higher than 1130°C, the decomposition-voltages differ. It was found to be 1.60 volts in the former and 0.95 volts in the latter states respectively, which shows that the ionized molecules governing electric conduction differ in these two states. From these results, the temperature of 1130°C may be regard-
ed as the beginning of the apparent melting of these two rocks, though they are partly ionized at lower temperature than 1130°C. It may be regarded as rather natural that, in cooling, they assume the over-cooled state just as in the case of ordinary glass, with slight recrystal-

tallization or transition, the rock being, as a whole, in the amorphous
state. The conductivity obtained at the temperature of our experiment
may not be exactly the same as that in the final chemical state of the
rocks at the same temperature, seeing that the extraordinary slowness
in the velocity of chemical reaction in these rocks should be taken
into the consideration. In other words, each state in the heating or
cooling process in our experiments is not in heterogeneous equilibrium.
Notwithstanding this defect, petrologically, the results of our experi-
ments are satisfactory for our purpose of estimating the order of
magnitude of earth-current disturbances due to anomalies in the con-
ductivity of heated lava, or of securing a general idea of viscosity-
variation due to temperature.

4. Earth-Current and Electric Conductivity of Lava.

As mentioned in §1, the earth-current varies when the electric
conductivity of a part of the earth’s crust undergoes marked changes.
From our experiments, at temperatures higher than 1200°C\(^9\) the

\(^9\) Approximate values of temperature of the heated lava in craters have been
reported as follows. 1000°C (Mihara), 1000°C (Asama), 1100~1300°C (Kilauea),
1150~1180°C (Vesuvius), 1100~1150°C (Storomboli).
specific resistivity of andesite and basalt is smaller than 5 ohm cm, while at room temperature, that of igneous rocks, sand-stones, and other soils is of the order of $10^8 \sim 10^5$ ohm/cm.\(^{10}\) We shall now estimate the magnitude of the anomaly just mentioned. In the case of a volcano, we could consider a simple model like the following: for the sake of simplicity, we shall deal with a two-dimensional problem. In Fig. 11, the circle represents the boundary of the crater, in the outer region of which the electric conductivity $\sigma_2$ of the medium is homogeneous, and in the inner region of which the conductivity $\sigma_1$ is also uniform. We assume that the electric field $E$ is constant at points infinitely distant from the circle. With these assumptions, we get for the field of the stationary current,

$$
\begin{align*}
\mathbf{i} &= -\sigma \text{grad} \varphi \\
\sigma_1 \nabla^2 \varphi_1 &= 0 \\
\sigma_2 \nabla^2 \varphi_2 &= 0
\end{align*}
$$

where $\varphi_1$ and $\varphi_2$ are the potentials in the inner and outer regions of the boundary-circle of radius $R$.

The boundary-conditions (contact electro-motive force at the boundary not taken into consideration) are

$$
\begin{align*}
(\varphi_1)_{r=R} - (\varphi_2)_{r=R} &= 0 \\
\sigma_1 \frac{\partial \varphi_1}{\partial r} \bigg|_{r=R} - \sigma_2 \frac{\partial \varphi_2}{\partial r} \bigg|_{r=R} &= 0 \\
\frac{\partial \varphi_2}{\partial x} \bigg|_{r=\infty} &= -E, \quad \frac{\partial \varphi_2}{\partial y} \bigg|_{r=\infty} = 0
\end{align*}
$$

From these equations and boundary-conditions, we get the following solutions in polar-coordinate, the origin of which has been taken at the centre of the circle, and azimuth $\theta$ is measured with reference to the X-axis,

$$
\begin{align*}
\varphi_1 &= C - \frac{2E \sigma_2}{\sigma_1 + \sigma_2} \ r \cos \theta \\
\varphi_2 &= C - E r \cos \theta + \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} E \ r^2 \cos \theta
\end{align*}
$$

Outside the circle, the intensity of the electric field is expressed by

\[
E_\phi = -\frac{\partial \phi_2}{\partial x} = E \left\{ 1 + \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \left( \frac{R}{r} \right)^2 \cos 2\theta \right\}
\]

\[
E_\theta = -\frac{\partial \phi_2}{\partial y} = E \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \left( \frac{R}{r} \right)^2 \sin 2\theta
\]

(4)

If the results of this calculation can be applied to the problem of disturbances in the earth-current in the vicinity of the crater, the magnitude of the disturbances in the electric field due to anomalies in the conductivity of lava in the crater, is given by the formula,

\[
\Delta E = E \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \left( \frac{R}{r} \right)^2.
\]

(5)

As we have dealt with a stationary state in the foregoing considerations, the results cannot be used in explaining the case of a sudden commencement of disturbances due to abrupt change in conductivity. It can be used only in explaining a gradual change in the electric field.

When the lava in the crater is in the molten state, the ratio \( \sigma_1/\sigma_2 \) amounts to \( 10^3 \sim 10^5 \) and \( \Delta E \approx E (R/r)^2 \), whereas if the interior of the crater were approximately hollow, \( \Delta E = -E (R/r)^2 \). It is possible for the electric field actually to vary between these two extreme cases. Further detailed discussion must be withheld until the survey of the electric field in the vicinity of the volcano is completed.

### 5. Viscosity and Electric Conductivity of Lava.

In our experiment, the electric conductivity of lava in the amorphous state satisfies approximately the equation

\[
\log \sigma = A - B/T, \quad \therefore \sigma = \alpha e^{-\frac{B}{T}}.
\]

This means that the velocity of the ions in electric conduction are distributed microscopically after Maxwell's law. As in the fluid state, Stokes' law may be invoked in considering the effect of viscosity on the motion of these ions, provided Maxwell's distribution is assumed. Then

\[
u = \frac{k}{6\pi \eta r},
\]

(1)

where \( u \) is the velocity of the ions, \( \eta \) the coefficient of viscosity, \( K \) the
intensity of the electric field, and \( r \) the radius of an ion. On the other hand, the specific conductivity \( \sigma \) is given by

\[
\sigma = NuzF,
\]

where \( N \) is the number of ions in unit volume, \( z \) the electric charge of an ion, and \( F \) Faraday's constant. From equations (1) and (2), we get

\[
\sigma = \frac{NzFK}{6\pi r} \cdot \frac{1}{\eta}.
\]

Taking \((NzFK)/(6\pi r) = \text{constant} = C\), and \(1/\sigma = \text{specific resistivity} = w\), we have

\[
w = C\eta.
\]

Although this relation does not exactly hold in the case of vitreous substances, it may be used in estimating the values of the coefficient of viscosity, as measurement of conductivity is not so difficult compared with measurement of viscosity by dynamical methods.

In glass engineering, F. F. S. Bryson\(^{11}\) showed that an approximate parallelism between \( w \) and \( \eta \) holds in the case of ordinary glass. Seeing that both glass and volcanic lava are silicates, it would seem that in the case of volcanic lava, it would also be practicable to estimate the variation in viscosity by measuring its electric conductivity. This problem is now receiving the attention of the writer.

6. Conclusion.

This report is a preliminary study on the electric and magnetic properties of volcanic lava, in which the general aspects of the relation between electric conductivity and temperature is discussed, details of which appear in a following paper.

In conclusion, the writer wishes to express here his sincere thanks to Prof. M. Ishimoto and Dr. R. Takahasi for their encouragement in this study. His cordial thanks are also due to Dr. H. Tsuya for his interest in the present work.

43. 浅間及び三原火山熔岩の物理的性質
I. 電気伝導度及び共の温度変化

地震研究所 永 田 武

最近火山活動に伴う地殻の電磁気的要素の変化の研究が盛んになったが、一方に於いては火山熔岩の電磁気的性質を実験的に調べる事も、此の問題に関連して興味ある事と想ふ。

従来この種の実験的理論は、二三研究に報告されたものがあるが、筆者はいま少しく組織的に浅間及び三原兩火山の代表的噴出熔岩に就いて、共の電磁気的性質を調べる事にした。

本報告では、先づ熔岩の電気伝導度の温度による変化を調べ、1130℃附近に変異点が存在して、之以上の温度に於いては、電気伝導度が急激に増加する事、又硬度の変化を保たしめて冷却する時には、\( \log \sigma = A - B/T \)（\( \sigma \)は比電気伝導度、\( T \)は絶対温度、\( A, B \)は常数）の関係が適用される事が分った。

この実験結果の應用として、地電流の変化、赤熱熔岩の粘度の温度変化等に対して、若干の議論を行った。