# *The conjugacy classes of Chevalley groups of type* $(F_4)$ *over finite fields of characteristic* $p\neq 2$

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Conjugacy classes of Chevalley groups of type $(F_4)$ over finite fields have been determined by K. Shinoda [5] in the case $p=2$. In this paper we deal with the remaining case $p\neq 2$. For the conjugacy classes of $p$-elements, the methods used in [5] are effective in our case, but for the case of $p'$-elements, calculations become more complicated and the ideas in this case are due to K. Mizuno [4].

## §1. Preliminaries

(1.0) We shall follow the notations and the contents §1 of [5]. Let $k$ be a finite fields $F_q$ of characteristic $p$ consisting of $q$ elements, $\bar{k}$ its algebraic closure, $\mathfrak{g}$ a complex Lie algebra of type $(F_4)$, $\bar{G}=G(F_4)_{\bar{k}}$ a Chevalley group associated with $\mathfrak{g}$ and $\bar{k}$. Then $\bar{G}$ is a simply connected semisimple algebraic group defined over a prime field, and we put $G=\bar{G}_k$, the group of $k$-rational points of $\bar{G}$. $\bar{G}$ has several subgroups $\bar{U}$, $\bar{H}$, $\bar{B}$, and corresponding to them, $G$ has $U$, $H$, $B$, etc.

Let $\Phi=\Phi(H, G)$ be a root system relative to $\bar{H}$. We can choose $\Phi^+$, the set of positive roots, and $\Delta$, the set of simple roots, as follows:

$$\Phi^+=\left\{\varepsilon_i\pm\varepsilon_j\ (1\leqq i<j\leqq 4),\ \varepsilon_i\ (1\leqq i\leqq 4),\ \frac{1}{2}(\varepsilon_1\pm\varepsilon_2\pm\varepsilon_3\pm\varepsilon_4)\right\},$$

$$\Delta=\left\{\varepsilon_2-\varepsilon_3,\ \varepsilon_3-\varepsilon_4,\ \varepsilon_4,\ \frac{1}{2}(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4)\right\},$$

and

$$\Phi=\Phi^+\cup(-\Phi^+)\,.$$

In the following, when no confusion arises, we use the notations $i\pm j$, $i$ and $1\pm2\pm3\pm4$, in place of $\varepsilon_i\pm\varepsilon_j$, $\varepsilon_i$ and $\frac{1}{2}(\varepsilon_1\pm\varepsilon_2\pm\varepsilon_3\pm\varepsilon_4)$, respectively.

Put $P_r=\sum_{\alpha\in\Phi}\boldsymbol{Z}\alpha$; then $k^*$ being the multiplicative group of $k$, one has $H\cong\mathrm{Hom}(P_r, k^*)$. Thus for $\chi\in\mathrm{Hom}(P_r, k^*)$, there is associated an element $h=h(\chi)\in H$, which we denote by $h(z_1, z_2, z_3, z_4)$, where $z_1=\chi(2-3)$, $z_2=\chi(3-4)$, $z_3=\chi(4)$, $z_4=\chi(1-2-3-4)$, respectively.

Now, let $\sigma$ be the Frobenius endomorphism of $\bar{G}$ induced by the map $x\mapsto x^q$

in $\bar{k}$. Then the following facts are well known.

(1.1) PROPOSITION ([1], p. 176, 3.4). *Let $C$ be a conjugacy class of $\tilde{G}$ fixed by $\sigma$. Then $G\cap C\neq\emptyset$, and for any $x\in C\cap G$, the classes of $G$ contained in $C\cap G$ correspond bijectively to elements in $H^1(\sigma, Z_{\tilde{G}}(x)/Z_{\tilde{G}}(x)^\circ)$, where $Z_{\tilde{G}}(x)^\circ$ means the connected component of the identity element in $Z_{\tilde{G}}(x)$.*

In fact, above correspondence is obtained explicitly as follows, ([1], p. 174). Let $t$ be an element of $Z_{\tilde{G}}(x)$ which represents an element of $H^1(\sigma, Z_{\tilde{G}}(x)/Z_{\tilde{G}}(x)^\circ)$, then there exists $g\in\tilde{G}$ such that $t=g^{-1}\sigma(g)$, and if we put $y=gxg^{-1}$, then $y\in G\cap C$, and the class of $G$ which contains $y$ is the required.

(1.2) Let $N_{\alpha,\beta}$ $(\alpha, \beta\in\Phi)$ be the structure constants of $\mathfrak{g}$. Now we shall determine these constants for some Chevalley basis. First, for every $\gamma\in\Phi^+-\Delta$, we choose and fix one decomposition of $\gamma$ as a sum of two positive roots, i.e. $\gamma=\alpha+\beta$, $\alpha$, $\beta\in\Phi^+$. Then for this fixed decomposition of $\gamma$, we can take Chevalley basis so as to $N_{\alpha,\beta}>0$. Next, let $\gamma$, $\delta$ be the decomposition of another type of $\alpha+\beta$, i.e. $\alpha+\beta=\gamma+\delta$, $\alpha$, $\beta$, $\gamma$, $\delta\in\Phi^+$, $\alpha\neq\gamma$, $\delta$. Then the following equation is easily verified.

Table 1. The structure constants of $\mathfrak{g}$.

| | | |
|---|---|---|
| $N_{2-3,3-4}=1$ | $N_{3-4,4}=1$ | $N_{2-4,4}=1$ |
| $N_{2-3,3}=1$ | $N_{3,4}=2$ | $N_{2,4}=2$ |
| $N_{2-3,3+4}=1$ | $N_{2,3}=2$ | $N_{2-4,3+4}=-1$ |
| $N_{2+4,3-4}=-1$ | $N_{4,1-2-3-4}=1$ | $N_{3-4,1-2-3+4}=1$ |
| $N_{3,1-2-3-4}=1$ | $N_{2-3,1-2+3-4}=1$ | $N_{2,1-2-3-4}=1$ |
| $N_{2-4,1-2-3+4}=1$ | $N_{4,1-2+3-4}=1$ | $N_{3,1-2-3+4}=-1$ |
| $N_{3+4,1-2-3-4}=-1$ | $N_{4,1+2-3-4}=1$ | $N_{2,1-2-3+4}=-1$ |
| $N_{2+4,1-2-3-4}=-1$ | $N_{2-3,1-2+3+4}=1$ | $N_{3,1+2-3-4}=1$ |
| $N_{2+3,1-2-3-4}=-1$ | $N_{2,1-2+3-4}=-1$ | $N_{2-4,1-2+3+4}=-1$ |
| $N_{3-4,1+2-3+4}=1$ | $N_{4,1+2+3-4}=1$ | $N_{2+4,1-2+3-4}=1$ |
| $N_{2,1-2+3+4}=1$ | $N_{2+3,1-2-3+4}=-1$ | $N_{3,1+2-3+4}=-1$ |
| $N_{3+4,1+2-3-4}=-1$ | $N_{1-2-3-4,1-2+3+4}=2$ | $N_{1-2-3+4,1-2+3-4}=-2$ |
| $N_{1-2-3-4,1+2-3+4}=2$ | $N_{1-2-3+4,1+2-3-4}=-2$ | $N_{1-2,2-3}=-1$ |
| $N_{1-2-3-4,1+2+3-4}=2$ | $N_{1-2+3-4,1+2-3-4}=-2$ | $N_{1-2,2-4}=1$ |
| $N_{1-3,3-4}=-1$ | $N_{1-2-3-4,1+2+3+4}=-1$ | $N_{1-2-3+4,1+2+3-4}=-1$ |
| $N_{1-2+3-4,1+2-3+4}=1$ | $N_{1+2-3-4,1-2+3+4}=-1$ | $N_{1-4,4}=1$ |
| $N_{1-3,3}=-1$ | $N_{1-2,2}=1$ | $N_{1-2-3+4,1+2+3+4}=2$ |
| $N_{1-2+3+4,1+2-3+4}=-2$ | $N_{1,4}=2$ | $N_{1-2,2+4}=1$ |
| $N_{1-3,3+4}=-1$ | $N_{1-2+3-4,1+2+3+4}=2$ | $N_{1-2+3+4,1+2+3-4}=-2$ |
| $N_{1,3}=2$ | $N_{1-2,2+3}=1$ | $N_{1+4,3-4}=-1$ |
| $N_{1-4,3+4}=-1$ | $N_{1+2-3-4,1+2+3+4}=2$ | $N_{1+2+3-4,1+2-3+4}=2$ |
| $N_{1,2}=2$ | $N_{1+4,2-4}=-1$ | $N_{1-4,2+4}=-1$ |
| $N_{1+3,2-3}=-1$ | $N_{1-3,2+3}=1$ | |

$$N_{\alpha,\beta}N_{-\gamma,-\delta}\tilde{\omega}_{\alpha+\beta}+N_{\beta,-\gamma}N_{\alpha,-\delta}\tilde{\omega}_{\beta-\gamma}+N_{-\gamma,\alpha}N_{\beta,-\delta}\tilde{\omega}_{\alpha-\gamma}=0\,,$$

where $\tilde{\omega}_\alpha=2/(\alpha,\alpha)$ for $\alpha\in\Phi$, and $N_{\alpha,\beta}=0$ if $\alpha+\beta\notin\Phi$. Since we know $N_{\alpha,\beta}=-N_{-\alpha,-\beta}$, $N_{\alpha,\beta}N_{-\alpha,\beta+\alpha}\geqq 0$, by utilizing above equations, we can determine all constants inductively with respect to the increasing order of positive roots. The results are given in Table 1.

(1.3) Let $W_h$ be the stabilizer of $h\in\bar{H}$ in $W=W(F_4)$. Then the conjugacy classes of $W_h$ in $W$ are characterized by its Dynkin diagram, and for each type $S$ of such a diagram, we fix a representative and denote it by $W_S$. All the representatives $\{W_S\}$ and $\{N_W(W_S)\}$, their nomalizers in $W$, are easily determined and the results are listed in Table 2.

Table 2. The representatives of $W_S$ in $W$.

| $S$ | $\Delta(S)$ | $N_W(W_S)$ |
|---|---|---|
| $F_4$ | $\{2-3,3-4,4,1-2-3-4\}$ | $W_S=W$ |
| $B_4$ | $\{1-2,2-3,3-4,4\}$ | $W_S$ |
| $A_1+C_3$ | $\{1-2,3-4,4,1+2-3-4\}$ | $W_S$ |
| $A_2+\tilde{A}_2$ | $\{1-2,2-3,4,1+2+3-4\}$ | $W_S\times Z$ |
| $A_3+\tilde{A}_1$ | $\{1-2,2-3,3-4,1+2+3+4\}$ | $W_S\times Z$ |
| $C_3$ | $\{3-4,4,1+2-3-4\}$ | $W_S\times Z$ |
| $A_1+\tilde{A}_2$ | $\{1-2,4,1+2+3-4\}$ | $W_S\times Z$ |
| $A_3$ | $\{1-2,2-3,3-4\}$ | $W_S\times W_S^\perp\times Z$ |
| $A_2+\tilde{A}_1$ | $\{1-2,2-3,1+2+3+4\}$ | $W_S\times Z$ |
| $2A_1+\tilde{A}_1$ | $\{1-2,3-4,1+2+3+4\}$ | $\langle w_{1-2-3+4}\rangle W_S\times Z$ |
| $A_1+B_2$ | $\{1-2,3+4,1+2-3-4\}$ | $W_S\times Z$ |
| $B_3$ | $\{2-3,3+4,1-2-3-4\}$ | $W_S\times Z$ |
| $A_2$ | $\{1-2,2-3\}$ | $W_S\times W_S^\perp\times Z$ |
| $\tilde{A}_2$ | $\{4,1+2+3-4\}$ | $W_S\times W_S^\perp\times Z$ |
| $B_2$ | $\{3+4,1+2-3-4\}$ | $W_S\times W_S^\perp$ |
| $2A_1$ | $\{1-2,1+2\}$ | $\langle w_2\rangle W_S\times W_S^\perp$ |
| $A_1+\tilde{A}_1$ | $\{1-2,1+2+3+4\}$ | $W_S\times W_S^\perp$ |
| $A_1$ | $\{1-2\}$ | $W_S\times W_S^\perp$ |
| $\tilde{A}_1$ | $\{1+2+3+4\}$ | $W_S\times W_S^\perp$ |
| $\phi$ | $\phi$ | $W_S^\perp=W$ |

Each entry $S$ of the first column denotes the type of Dynkin diagram, where $\tilde{A}_i$ denotes the diagram of type $A_i$ which consists of short roots, the second column denotes the fundamental system corresponding to $W_S$ (i.e. $W_S$ is the group generated by the reflections $w_\alpha$ such that $\alpha\in\Delta(S)$), and the third column denotes the nomalizer of $W_S$ in $W$, where $W_s^\perp$ is the subgroup of $W$ generated by the reflections which stabilize all roots of $\Delta(S)$, and $Z=Z(W)$ is the center of $W$ consisting of two elements 1, $g$ where $g$ is the unique element of $W$ such that $g(\Delta)=-\Delta$.

From now on, we assume $p\neq 2$.

(1.4) Let $\eta$ be a non-square fixed element of $k^*$, then we can choose $\xi\in k$ such that $X^2+\xi X+\eta$ is an irreducible polynomial in $k[X]$. For, the map $x\mapsto(x^2+\eta)/x$ of $k^*$ to $k$ is not surjective, and $x=0$ is not a root of above polynomial. In the same way, we can choose $\zeta\in k^*$ such that $X^3-X+\zeta$ is an irreducible polynomial in $k[X]$. In the following, we fix $\eta$, $\xi$, and $\zeta$ in this manner. Note if $(q-1, 4)=1$, we can reduce $\xi$ to 0.

## §2. Conjugacy classes of $p$-elements.

(2.1) Let $\Phi(B_3)$, $\Phi(B_4)$ be subsystems of $\Phi$ defined as follows:

$$\Phi(B_3)=\{\pm i\pm j,\ (2\leqq i<j\leqq 4),\ \pm i,\ (2\leqq i\leqq 4)\}\ ,$$

$$\Phi(B_4)=\{\pm i\pm j,\ (1\leqq i<j\leqq 4),\ \pm i,\ (1\leqq i\leqq 4)\}\ .$$

We shall define following subgroups as in [5].

$$G(B_3)=\langle H',\ U_\alpha|\alpha\in\Phi(B_3)\rangle\ ,$$

where

$$H'=\{h(\chi)\in H|\chi(1-2-3-4)=1\}$$

$$G(B_4)=\langle H,\ U_\alpha|\alpha\in\Phi(B_4)\rangle\ .$$

The conjugacy classes of $p$-elements of these groups are easily determined. The representatives and the orders of their centralizers are given in Table 3 and Table 4.

Table 3. Conjugacy classes of $p$-elements of $G(B_3)$

| | |
|---|---|
| $z_0=1$ | $q^9(q^2-1)(q^4-1)(q^6-1)$ |
| $z_1=x_{2+3}(1)$ | $q^9(q^2-1)^2$ |
| $z_2=x_{2-3}(1)x_{2+3}(-1)$ | $2q^7(q^2-1)^2$ |
| $z_3=x_{2-3}(1)x_{2+3}(-\eta)$ | $2q^7(q^4-1)$ |
| $z_4=x_2(1)x_{3+4}(1)$ | $q^7(q^2-1)$ |
| $z_5=x_{2-3}(1)x_4(1)x_{2+3}(1)$ | $2q^6(q-1)$ |
| $z_6=x_{2-3}(1)x_4(1)x_{2+3}(\eta)$ | $2q^6(q+1)$ |
| $z_7=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-1)$ | $2q^4(q-1)$ |
| $z_8=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-\eta)$ | $2q^4(q+1)$ |
| $z_9=x_{2-3}(1)x_{3-4}(1)x_4(1)$ | $q^3$ |

(2.2) By the same way as in [5], we can calculate the conjugacy classes of $p$-elements of $G$ with the aid of above results, and we get

THEOREM 2.1. *Suppose that the characteristic $p\neq 2$, 3 (resp. $p=3$) then $G$ has 26 (resp. 28) conjugacy classes of $p$-elements containing the class of the identity element. Their representatives and the orders of their centralizers*

Table 4. Conjugacy classes of $p$-elements of $G(B_4)$

| | |
|---|---|
| $y_0=1$ | $q^{16}(q^2-1)(q^4-1)(q^6-1)(q^8-1)$ |
| $y_1=x_{2+3}(1)$ | $q^{16}(q^2-1)^2(q^4-1)$ |
| $y_2=x_{2-3}(1)x_{2+3}(-1)$ | $2q^{13}(q^2-1)(q^3-1)(q^4-1)$ |
| $y_3=x_{2-3}(1)x_{2+3}(-\eta)$ | $2q^{13}(q^2-1)(q^3+1)(q^4-1)$ |
| $y_4=x_{2+3}(1)x_{1+4}(1)$ | $2q^{14}(q^2-1)(q^4-1)$ |
| $y_5=x_{2+3}(1)x_{1+4}(\eta)$ | $2q^{14}(q^2-1)(q^4-1)$ |
| $y_6=x_{2+3}(1)x_{1-4}(1)x_{1+4}(-1)$ | $2q^{13}(q-1)(q^2-1)$ |
| $y_7=x_{2+3}(1)x_{1-4}(1)x_{1+4}(-\eta)$ | $2q^{13}(q+1)(q^2-1)$ |
| $y_8=x_{2-3}(1)x_4(1)x_{2+3}(1)$ | $2q^{11}(q-1)(q^2-1)$ |
| $y_9=x_{2-3}(1)x_4(1)x_{2+3}(\eta)$ | $2q^{11}(q+1)(q^2-1)$ |
| $y_{10}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-1)$ | $2q^8(q^2-1)^2$ |
| $y_{11}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-\eta)$ | $2q^8(q^4-1)$ |
| $y_{12}=x_{2-3}(1)x_4(1)x_{1-2}(1)$ | $q^{10}(q^2-1)$ |
| $y_{13}=x_{2-3}(1)x_4(1)x_{1-4}(1)$ | $2q^8(q^2-1)$ |
| $y_{14}=x_{2-3}(1)x_4(1)x_{1-4}(\eta)$ | $2q^8(q^2-1)$ |
| $y_{15}=x_{2-3}(1)x_{3+4}(1)x_{1-2}(1)$ | $2q^8(q^2-1)$ |
| $y_{16}=x_{2-3}(1)x_{3+4}(1)x_{1-2}(\eta)$ | $2q^8(q^2-1)$ |
| $y_{17}=x_{2-3}(1)x_3(1)x_{1-4}(1)x_{1+4}(1)$ | $4q^8$ |
| $y_{18}=x_{2-3}(1)x_3(1)x_{1-4}(1)x_{1+4}(\eta)$ | $4q^8$ |
| $y_{19}=x_{2-3}(1)x_{3+4}(1)x_{1-2}(1)x_{1-4}(\eta)$ | $4q^8$ |
| $y_{20}=x_{2-3}(1)x_4(1)x_{2+3}(\eta)x_{1-2}(1)$ | $4q^8$ |
| $y_{21}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-1)x_{1-2}(1)$ | $2q^5(q-1)$ |
| $y_{22}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-\eta)x_{1-2}(1)$ | $2q^5(q+1)$ |
| $y_{23}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(1)$ | $2q^4$ |
| $y_{24}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(\eta)$ | $2q^4$ |

Each entry $z_i$ (resp. $y_i$) of the first column of Table 3, (resp. Table 4.) denotes a representative for some conjugacy class of $p$-elements of $G(B_3)$ (resp. $G(B_4)$), the second column denotes the order of its centralizer in $G(B_3)$ (resp. $G(B_4)$), and $\eta$ is a non-square fixed element of $k^*$.

*are given in Table 5. (resp. Table 6).*

PROOF. We shall show Theorem 2.1 in the following steps:

(i) determination of the centralizer of $x_i$,

(ii) no two elements of the $x_i$ are conjugate in $G$,

(iii) $\sum_i |G|/|Z_G(x_i)|=q^{48}$.

(iii) is immediate from (i), and except the cases of $x_{14}$, $x_{15}$, $x_{16}$, $x_{17}$ and $x_{18}$, (i) and (ii) are easily checked, although the calculations are rather long. So, we omit the proof for the remaining cases.

(2.3) The case of $x_{14}, \cdots, x_{18}$.

Put $$x=x_{14}=x_{2-4}(1)x_{3+4}(1)x_{1-2}(-1)x_{1-3}(-1)\ , \quad Z=Z_{\bar{G}}(x)\ ,$$
$\Psi=\langle 2-3, 4, 1-2-3-4\rangle_Z\cap\Phi$, a subsystem of $\Phi$, $\bar{P}$ a parabolic subgroup of $\bar{G}$, relative to $\Psi$, $\bar{V}$ the unipotent radical of $\bar{P}$, i.e.

Table 5. Conjugacy classes of $p$-elements of $G$ (the case $p\neq 2, 3$)

| | |
|---|---|
| $x_0=1$ | $q^{24}(q^2-1)(q^6-1)(q^8-1)(q^{12}-1)$ |
| $x_1=x_{1+2}(1)$ | $q^{24}(q^2-1)(q^4-1)(q^6-1)$ |
| $x_2=x_{1-2}(1)x_{1+2}(-1)$ | $2q^{21}(q^2-1)(q^3-1)(q^4-1)$ |
| $x_3=x_{1-2}(1)x_{1+2}(-\eta)$ | $2q^{21}(q^2-1)(q^3+1)(q^4-1)$ |
| $x_4=x_2(1)x_{3+4}(1)$ | $q^{20}(q^2-1)^2$ |
| $x_5=x_{2-3}(1)x_4(1)x_{2+3}(1)$ | $2q^{17}(q^2-1)(q^3-1)$ |
| $x_6=x_{2-3}(1)x_4(1)x_{2+3}(\eta)$ | $2q^{17}(q^2-1)(q^3+1)$ |
| $x_7=x_2(1)x_{1-2+3+4}(1)$ | $q^{14}(q^2-1)(q^6-1)$ |
| $x_8=x_{2-3}(1)x_4(1)x_{1-2}(1)$ | $q^{16}(q^2-1)$ |
| $x_9=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-1)$ | $2q^{12}(q^2-1)^2$ |
| $x_{10}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-\eta)$ | $2q^{12}(q^4-1)$ |
| $x_{11}=x_{2+3}(1)x_{1+2-3-4}(1)x_{1-2+3+4}(1)$ | $q^{14}(q^2-1)$ |
| $x_{12}=x_{2-3}(1)x_4(1)x_{1-4}(1)$ | $2q^{12}(q^2-1)$ |
| $x_{13}=x_{2-3}(1)x_4(1)x_{1-4}(\eta)$ | $2q^{12}(q^2-1)$ |
| $x_{14}=x_{2-4}(1)x_{3+4}(1)x_{1-2}(-1)x_{1-3}(-1)$ | $24q^{12}$ |
| $x_{15}=x_{2-4}(1)x_{3+4}(1)x_{1-2}(-\eta)x_{1-3}(-1)$ | $8q^{12}$ |
| $x_{16}=x_{2-4}(1)x_{2+4}(-\eta)x_{1-2+3+4}(1)x_{1-3}(-1)$ | $4q^{12}$ |
| $x_{17}=x_{2-4}(1)x_{3+4}(1)x_{1-2-3+4}(1)x_{1-2}(-\eta)x_{1-3}(\xi)$ | $4q^{12}$ |
| $x_{18}=x_2(1)x_{3+4}(1)x_{1-2+3-4}(1)x_{1-2}(-1)x_{1-3}(\zeta)$ | $3q^{12}$ |
| $x_{19}=x_{2-3}(1)x_{3-4}(1)x_4(1)$ | $q^8(q^2-1)$ |
| $x_{20}=x_2(1)x_{3+4}(1)x_{1-2-3-4}(1)$ | $q^8(q^2-1)$ |
| $x_{21}=x_{2-4}(1)x_3(1)x_{2+4}(1)x_{1-2-3+4}(1)$ | $2q^8$ |
| $x_{22}=x_{2-4}(1)x_3(1)x_{2+4}(\eta)x_{1-2-3+4}(1)$ | $2q^8$ |
| $x_{23}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(1)$ | $2q^6$ |
| $x_{24}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(\eta)$ | $2q^6$ |
| $x_{25}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2-3-4}(1)$ | $q^4$ |

$$\bar{P}=\langle \bar{H},\ \bar{V},\ \bar{U}_\alpha \mid \alpha\in\Psi\rangle\ , \qquad \bar{V}=\langle \bar{U}_\alpha \mid \alpha\in\Phi^+-\Psi\rangle\ .$$

Then direct calculations show that $Z=Z_{\bar{P}}(x)$ and $Z^\circ=Z_{\bar{V}}(x)$. Moreover let $M=\{h(1, 1, \pm 1, \pm 1)\}$, a subgroup of $H$, $S=\langle\tau_1, \tau_2\rangle$, a subgroup of $G$ generated by $\tau_1=h(-1, -1, -1, 1)\omega_{2-3}\omega_4$, $\tau_2=x_{2-3}(1)h(-1, -1, 1, -1)\omega_{1-2-3-4}$ where $\omega_\alpha$ means $x_\alpha(1)x_{-\alpha}(-1)x_\alpha(1)$ for $\alpha\in\Phi$. Then $S$ nomalizes $M$, and since $\tau_1^2=\tau_2^2=1$, $(\tau_1\tau_2)^3=1$, $S$ is isomorphic to the symmetric group $\mathfrak{S}_3$. Further M$\cdot$S acts on M$\cdot$S/S faithfully, so we know that M$\cdot$S is isomorphic to the symmetric group $\mathfrak{S}_4$. Since it is easily verified that the elements of M$\cdot$S represent the elements of $Z/Z^\circ$ mod $V$, we have $Z/Z^\circ\cong\mathfrak{S}_4$, and $\sigma$ acts on $Z/Z^\circ$ trivially. Thus, by (1.1), the splitting of the class of $G$ containing $x$ is described by the conjugacy classes of $\mathfrak{S}_4$, and if we note $Z=Z_{\bar{P}}(x)$, we have only to consider the conjugacy in $\bar{P}$.

Let $x'$ be an element of $P$ such that $x'=yxy^{-1}$ for some $y\in\bar{P}$, then $u=y^{-1}\sigma(y)\in Z$ and the following lemma holds.

LEMMA 2.2. *Let $\tilde{u}$ be the image of $u$ in $Z/Z^\circ$, $r$ the order of the central-*

*izer of* $\tilde{u}$ *in* $Z/Z^\circ$. *Then* $|Z_G(x')|=r|Z_V(x')|$.

PROOF. We can choose $\{t_i\}\subset G$ such that $Z=\bigcup_i t_iZ^\circ$, as was mentioned above. Then since $\bar{V}$ is a normal subgroup of $\bar{P}$, $Z_{\bar{G}}(x')=\bigcup_i yt_iy^{-1}Z_{\bar{V}}(x')$, and it is easily checked that $yt_iy^{-1}Z_{\bar{V}}(x')$ is $\sigma$-stable if and only if $\tilde{u}$ and $\tilde{t}_i$ commutes in $Z/Z^\circ$. Therefore the number of $\sigma$-stable cosets of $Z_{\bar{G}}(x')$ is $r$, and for such cosets we can associate $\sigma$-invariant representatives by ([1], p. 173, 2.7). Thus Lemma 2.2 is proved.

Since $Z_V(x')$ is easily calculated for $x'\in V$, the centralizers are determined using above results. First, it is easy to see that $|Z_G(x)|=24q^{12}$, and that for $x_{15}$, there exists $y\in\bar{P}$ such that $x_{15}=yxy^{-1}$, and $y^{-1}\sigma(y)$ corresponds to the product of two commutative transpositions in $\mathfrak{S}_4$, thus we have $|Z_G(x_{15})|=8q^{12}$. For $x_{16}$, $y^{-1}\sigma(y)$ corresponds to the transposition in $\mathfrak{S}_4$, therefore we have $|Z_G(x_{16})|=4q^{12}$.

Next, for $x_{17}=x_{2-4}(1)x_{3+4}(1)x_{1-2-3+4}(1)x_{1-2}(-\eta)x_{1-3}(\xi)$, we can take $y=u_1uhn_wv$ such that $x_{17}=yxy^{-1}$ as follows:

$$u=x_{2-3}(t^{-1})x_{1-2-3-4}(-t)\ ,$$

$$v=x_{1-2-3-4}(s)\ ,$$

$$n_w=\omega_{1-2-3-4}\ ,$$

$$h=h((t^2-\eta)/\eta t,\ \eta t/(t^2-\eta),\ \gamma,\ -s(t^2-\eta)/t)\ ,$$

where $t$, $s$, $\gamma$ are elements of $k^*$ which satisfy the following equations:

$$t^2+\xi t+\eta=0\ ,\quad t^2=s^2\eta\ ,\quad \gamma^2=1/t\ ,$$

and $u_1$ is an element of $V$. Then, by the definitions of $\xi$, $\eta$, we have $(t/s)^{q-1}=-1$, $t^{q+1}=\eta$, and from them, we have $y'^{-1}\sigma(y')=x_{2-3}(1)h(-1,-1,\pm1,-1)\omega_{1-2-3-4}$, for $y'=u_1^{-1}y$. This means that $y^{-1}\sigma(y)$ corresponds to the cyclic permutation of order 4 in $\mathfrak{S}_4$. Thus $|Z_G(x_{17})|=4q^{12}$.

Finally, for $x_{18}=x_2(1)x_{3+4}(1)x_{1-2+3-4}(1)x_{1-2}(-1)x_{1-3}(\zeta)$ we can take $y=u_1uhn_wv$ such that $x_{18}=yxy^{-1}$ as follows:

$$u^{-1}=x_{2-3}(s_1)x_4(s_2)x_{1-2-3-4}(s_1)x_{1-2-3+4}(-s_1^2)\ ,$$

$$v=x_{2-3}(t)x_4(t)x_{1-2-3-4}(-1)x_{1-2-3+4}(-1)\ ,$$

$$n_w=\omega_{2-3}\omega_{1-2-3+4}\ ,$$

$$h=h(\alpha,\beta,\gamma,\delta)\ ,$$

where $s_1$ is an elements of $\bar{k}$ such that $s_1^3-s_1+\zeta=0$, and $s_2$, $\alpha$, $\beta$, $\gamma$, $\delta$ are elements of $\bar{k}$ which satisfy the following equations:

$$\alpha\beta\gamma=1\ ,$$

$$\beta\gamma\delta=1\ ,$$

$$\gamma\delta=-3s_1^2+1\ ,$$

$$\gamma=s_1-2s_2\ ,$$

$$\gamma t=s_1+s_2\ ,$$

$$\delta/\gamma=-t^2-t\ ,$$

and $u_1$ is an element of $V$. Then by the definition of $\zeta$ and above equations, we can easily show that $s_2\notin k$, and $y^{-1}\sigma(y)\in BwB$, where $w=w_{2-3}w_4w_{1-2-3-4}$, $(w_\alpha$

Table 6. Conjugacy classes of $p$-elements of $G$ (the case $p=3$)

| | |
|---|---|
| $x_0=1$ | $q^{24}(q^2-1)(q^6-1)(q^8-1)(q^{12}-1)$ |
| $x_1=x_{1+2}(1)$ | $q^{24}(q^2-1)(q^4-1)(q^6-1)$ |
| $x_2=x_{1-2}(1)x_{1+2}(-1)$ | $2q^{21}(q^2-1)(q^3-1)(q^4-1)$ |
| $x_3=x_{1-2}(1)x_{1+2}(-\eta)$ | $2q^{21}(q^2-1)(q^3+1)(q^4-1)$ |
| $x_4=x_2(1)x_{3+4}(1)$ | $q^{20}(q^2-1)^2$ |
| $x_5=x_{2-3}(1)x_4(1)x_{2+3}(1)$ | $2q^{17}(q^2-1)(q^3-1)$ |
| $x_6=x_{2-3}(1)x_4(1)x_{2+3}(\eta)$ | $2q^{17}(q^2-1)(q^3+1)$ |
| $x_7=x_2(1)x_{1-2+3+4}(1)$ | $q^{14}(q^2-1)(q^6-1)$ |
| $x_8=x_{2-3}(1)x_4(1)x_{1-2}(1)$ | $q^{16}(q^2-1)$ |
| $x_9=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-1)$ | $2q^{12}(q^2-1)^2$ |
| $x_{10}=x_{2-3}(1)x_{3-4}(1)x_{3+4}(-\eta)$ | $2q^{12}(q^4-1)$ |
| $x_{11}=x_{2+3}(1)x_{1+2-3-4}(1)x_{1-2+3+4}(1)$ | $q^{14}(q^2-1)$ |
| $x_{12}=x_{2-3}(1)x_4(1)x_{1-4}(1)$ | $2q^{12}(q^2-1)$ |
| $x_{13}=x_{2-3}(1)x_4(1)x_{1-4}(\eta)$ | $2q^{12}(q^2-1)$ |
| $x_{14}=x_{2-4}(1)x_{3+4}(1)x_{1-2}(-1)x_{1-3}(-1)$ | $24q^{12}$ |
| $x_{15}=x_{2-4}(1)x_{3+4}(1)x_{1-2}(-\eta)x_{1-3}(-1)$ | $8q^{12}$ |
| $x_{16}=x_{2-4}(1)x_{2+4}(-\eta)x_{1-2+3+4}(1)x_{1-3}(-1)$ | $4q^{12}$ |
| $x_{17}=x_{2-4}(1)x_{3+4}(1)x_{1-2-3+4}(1)x_{1-2}(-\eta)x_{1-3}(\xi)$ | $4q^{12}$ |
| $x_{18}=x_2(1)x_{3+4}(1)x_{1-2+3-4}(1)x_{1-2}(-1)x_{1-3}(\zeta)$ | $3q^{12}$ |
| $x_{19}=x_{2-3}(1)x_{3-4}(1)x_4(1)$ | $q^8(q^2-1)$ |
| $x_{20}=x_2(1)x_{3+4}(1)x_{1-2-3-4}(1)$ | $q^8(q^2-1)$ |
| $x_{21}=x_{2-4}(1)x_3(1)x_{2+4}(1)x_{1-2-3+4}(1)$ | $2q^8$ |
| $x_{22}=x_{2-4}(1)x_3(1)x_{2+4}(\eta)x_{1-2-3+4}(1)$ | $2q^8$ |
| $x_{23}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(1)$ | $2q^6$ |
| $x_{24}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2}(\eta)$ | $2q^6$ |
| $x_{25}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2-3-4}(1)$ | $3q^4$ |
| $x_{26}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2-3-4}(1)x_{1-2+3+4}(\zeta)$ | $3q^4$ |
| $x_{27}=x_{2-3}(1)x_{3-4}(1)x_4(1)x_{1-2-3-4}(1)x_{1-2+3+4}(-\zeta)$ | $3q^4$ |

Each entry $x_i$ of the first column of Table 5, Table 6 denotes a representative for some conjugacy class of $p$-elements of $G$, the second column denotes the order of its centralizer in $G$, and $\eta$ is a non-square fixed element of $k^*$, $\xi$ is a fixed element of $k$ such that $X^2+\xi X+\eta$ is an irreducible polynomial in $k[X]$, $\zeta$ is a fixed element of $k^*$ such that $X^3-X+\zeta$ is an irreducible polynomial in $k[X]$, respectively.

denotes the reflection with respect to $\alpha\in\Phi$). This means that $y^{-1}\sigma(y)$ corresponds to the cyclic permutation of order 3 in $\mathfrak{S}_4$, and we have $|Z_G(x_{18})|=3q^{12}$. Thus, if we note that the conjugacy classes of $\mathfrak{S}_4$ are represented by above 5 elements including the identity class, we know $x_{14},\cdots,x_{18}$ give all representatives in the class of $G$ containing $x_{14}$.

REMARK. Though the calculations for $x_{18}$ are not independent of the characteristic $p$, a similar arguments hold in the case of $p=3$.

Table 7. Conjugacy classes of $p$-elements of $\bar{G}$.

| $c_i$ | rep. element | admissible graph | $Z/Z^\circ$ |
|---|---|---|---|
| $c_0$ | $x_0=1$ | $\emptyset$ | 1 |
| $c_1$ | $x_1$ | $A_1$ | 1 |
| $c_2$ | $x_2,\ x_3$ | $\tilde{A}_1,\ 2A_1$ | $Z_2$ |
| $c_3$ | $x_4$ | $A_1+\tilde{A}_1,\ 3A_1$ | 1 |
| $c_4$ | $x_5,\ x_6$ | $A_2,\ 2A_1+\tilde{A}_1,\ 4A_1$ | $Z_2$ |
| $c_5$ | $x_7$ | $\tilde{A}_2$ | 1 |
| $c_6$ | $x_8$ | $A_2+\tilde{A}_1$ | 1 |
| $c_7$ | $x_9,\ x_{10}$ | $B_2,\ A_3$ | $Z_2$ |
| $c_8$ | $x_{11}$ | $A_1+\tilde{A}_2$ | 1 |
| $c_9$ | $x_{12},\ x_{13}$ | $A_1+B_2$ | $Z_2$ |
| $c_{10}$ | $x_{14},\ x_{15},\ x_{16},\ x_{17},\ x_{18}$ | $A_3+\tilde{A}_1,\ B_2+2A_1,\ A_2+\tilde{A}_2,\ D_4(a_1)$ | $\mathfrak{S}_4$ |
| $c_{11}$ | $x_{19}$ | $B_3,\ D_4$ | 1 |
| $c_{12}$ | $x_{20}$ | $C_3$ | 1 |
| $c_{13}$ | $x_{21},\ x_{22}$ | $C_3+A_1,\ F_4(a_1)$ | $Z_2$ |
| $c_{14}$ | $x_{23},\ x_{24}$ | $B_4$ | $Z_2$ |
| $c_{15}$ | $x_{25},\ (x_{25},\ x_{26},\ x_{27})$ | $F_4$ | $1\ (Z_3)$ |

Each entry of the second column denotes the representative element of $G$ defined in Theorem 2.1 which is in the class $c_i$, the third column denotes the admissible graph associated with each class $c_i$, i.e. the admissible graph of $\{\alpha_1,\cdots,\alpha_n\}\subset\Phi^+$ such that $\{\alpha_i\}$ are linearly independent and $x=\prod_{i=1}^{n} x_{\alpha_i}(1)$ $(1\leqq n\leqq 4)$ is in the class $c_i$, (for the notion of admissible graphs, see [1], p. 298). The fourth column $Z/Z^\circ$ denotes $Z_{\bar{G}}(x)/Z_{\bar{G}}(x)^\circ$ for each $x\in c_i$, where $Z_n$ is the cyclic group of order $n$ and $\mathfrak{S}_4$ is the symmetric group of degree 4. In the last row expressions $(\cdots)$ concerns the case $p=3$, while for the remaining $c_i$, it is the same as in the case $p\neq 3$.

(2.4) We can easily determine the classes of $p$-elements of $\bar{G}$ by making use of Theorem 2.2. In fact, we know already about the class containing $x_{14}$, and for the remaining cases, the calculations are easy. Thus, we have

COROLLARY 2.3. *Suppose that the characteristic $p\neq 2$, then $\bar{G}$ has 16 conjugacy classes of $p$-elements containing the class of the identity element. The*

Table 8. Conjugacy classes of $p$-elements of $G$.

| $h_i$ | type $S$ | graph of $w$ | $M(h_i)$ | $w$ |
|---|---|---|---|---|
| $h_0$ | $F_4$ | $\varnothing$ | 1 | 1 |
| $h_1$ | $A_1+C_3$ | $\varnothing$ | 1 | 1 |
| $h_2$ | $B_4$ | $\varnothing$ | 1 | 1 |
| $h_3$ | $A_2+\tilde{A}_2$ | $\varnothing$ | $(y-1)/2$ | 1 |
| $h_4$ | $A_2+\tilde{A}_2$ | $4A_1$ | $(z-1)/2$ | $g$ |
| $h_5$ | $A_3+\tilde{A}_1$ | $\varnothing$ | $(x-2)/2$ | 1 |
| $h_6$ | $A_3+\tilde{A}_1$ | $4A_1$ | $(4-x)/2$ | $g$ |
| $h_7$ | $C_3$ | $\varnothing$ | $(q-3)/2$ | 1 |
| $h_8$ | $C_3$ | $4A_1$ | $(q-1)/2$ | $g$ |
| $h_9$ | $A_1+\tilde{A}_2$ | $\varnothing$ | $(q-2-y)/2$ | 1 |
| $h_{10}$ | $A_1+\tilde{A}_2$ | $4A_1$ | $(q-z)/2$ | $g$ |
| $h_{11}$ | $A_3$ | $\varnothing$ | $(q-1-x)/4$ | 1 |
| $h_{12}$ | $A_3$ | $4A_1$ | $(q-5+x)/4$ | $g$ |
| $h_{13}$ | $A_3$ | $\tilde{A}_1$ | $(q+3-x)/4$ | $w_{1+2+3+4}$ |
| $h_{14}$ | $A_3$ | $\tilde{A}_1$ | $(q-5+x)/4$ | $w_{1+2-3-4}$ |
| $h_{15}$ | $A_2+\tilde{A}_1$ | $\varnothing$ | $(q-y-x)/2$ | 1 |
| $h_{16}$ | $A_2+\tilde{A}_1$ | $4A_1$ | $(q-4-z+x)/2$ | $g$ |
| $h_{17}$ | $2A_1+\tilde{A}_1$ | $\varnothing$ | $(q-1-x)/4$ | 1 |
| $h_{18}$ | $2A_1+\tilde{A}_1$ | $4A_1$ | $(q-5+x)/4$ | $g$ |
| $h_{19}$ | $2A_1+\tilde{A}_1$ | $\tilde{A}_1$ | $(q-5+x)/4$ | $w_{1-2-3+4}$ |
| $h_{20}$ | $2A_1+\tilde{A}_1$ | $2A_1+\tilde{A}_1$ | $(q+3-x)/4$ | $gw_{1-2-3+4}$ |
| $h_{21}$ | $A_1+B_2$ | $\varnothing$ | $(q-3)/2$ | 1 |
| $h_{22}$ | $A_1+B_2$ | $4A_1$ | $(q-1)/2$ | $g$ |
| $h_{23}$ | $B_3$ | $\varnothing$ | $(q-3)/2$ | 1 |
| $h_{24}$ | $B_3$ | $4A_1$ | $(q-1)/2$ | $g$ |
| $h_{25}$ | $A_2$ | $\varnothing$ | $(q^2-11q+16+2y+3x)/12$ | 1 |
| $h_{26}$ | $A_2$ | $4A_1$ | $(q^2-7q+16+2z-3x)/12$ | $g$ |
| $h_{27}$ | $A_2$ | $\tilde{A}_1$ | $(q^2-3q-2+x)/4$ | $w_4$ |
| $h_{28}$ | $A_2$ | $2A_1+\tilde{A}_1$ | $(q^2-3q+6-x)/4$ | $gw_4$ |
| $h_{29}$ | $A_2$ | $\tilde{A}_2$ | $(q^2+q+1-y)/6$ | $w_4w_{1+2+3-4}$ |
| $h_{30}$ | $A_2$ | $A_1+C_3$ | $(q^2-q+1-z)/6$ | $gw_4w_{1+2+3-4}$ |
| $h_{31}$ | $\tilde{A}_2$ | $\varnothing$ | $(q^2-8q+13+2y)/12$ | 1 |
| $h_{32}$ | $\tilde{A}_2$ | $4A_1$ | $(q^2-4q+1+2z)/12$ | $g$ |
| $h_{33}$ | $\tilde{A}_2$ | $A_1$ | $(q-1)^2/4$ | $w_{1-2}$ |
| $h_{34}$ | $\tilde{A}_2$ | $3A_1$ | $(q-1)^2/4$ | $gw_{1-2}$ |
| $h_{35}$ | $\tilde{A}_2$ | $A_2$ | $(q^2+q+1-y)/6$ | $w_{1-2}w_{2-3}$ |
| $h_{36}$ | $\tilde{A}_2$ | $D_4$ | $(q^2-q+1-z)/6$ | $gw_{1-2}w_{2-3}$ |
| $h_{37}$ | $B_2$ | $\varnothing$ | $(q-3)(q-5)/8$ | 1 |
| $h_{38}$ | $B_2$ | $4A_1$ | $(q-1)(q-3)/8$ | $g$ |

Table 8. (Continued)

| $h_i$ | type $S$ | graph of $w$ | $M(h_i)$ | $w$ |
|---|---|---|---|---|
| $h_{39}$ | $B_2$ | $A_1$ | $(q-1)(q-3)/8$ | $w_{3-4}$ |
| $h_{40}$ | $B_2$ | $\tilde{A}_1$ | $(q-1)^2$ | $w_{1-2-3+4}$ |
| $h_{41}$ | $B_2$ | $B_2$ | $(q-1)(q+1)/4$ | $w_{1-2-3+4}w_{3-4}$ |
| $h_{42}$ | $2A_1$ | $\emptyset$ | $(q^2-10q+17+2x)/16$ | $1$ |
| $h_{43}$ | $2A_1$ | $4A_1$ | $(q^2-6q+13-2x)/16$ | $g$ |
| $h_{44}$ | $2A_1$ | $A_1$ | $(q-1)(q-3)/8$ | $w_{3-4}$ |
| $h_{45}$ | $2A_1$ | $\tilde{A}_1$ | $(q-1)(q-3)/8$ | $w_4$ |
| $h_{46}$ | $2A_1$ | $B_2$ | $(q^2-1)/8$ | $w_{3-4}w_4$ |
| $h_{47}$ | $2A_1$ | $\tilde{A}_1$ | $(q^2-6q+13-2x)/16$ | $w_2$ |
| $h_{48}$ | $2A_1$ | $2A_1+\tilde{A}_1$ | $(q^2-2q-7+2x)/16$ | $gw_2$ |
| $h_{49}$ | $2A_1$ | $A_1+\tilde{A}_1$ | $(q^2-1)/8$ | $w_2w_{3-4}$ |
| $h_{50}$ | $2A_1$ | $2A_1$ | $(q-1)(q-3)/8$ | $w_2w_4$ |
| $h_{51}$ | $2A_1$ | $A_3$ | $(q^2-1)/8$ | $w_2w_{3-4}w_4$ |
| $h_{52}$ | $A_1+\tilde{A}_1$ | $\emptyset$ | $(q^2-10q+15+2y+2x)/4$ | $1$ |
| $h_{53}$ | $A_1+\tilde{A}_1$ | $4A_1$ | $(q^2-6q+11+2z-2x)/4$ | $g$ |
| $h_{54}$ | $A_1+\tilde{A}_1$ | $A_1$ | $(q-1)(q-3)/4$ | $w_{3-4}$ |
| $h_{55}$ | $A_1+\tilde{A}_1$ | $\tilde{A}_1$ | $(q-1)(q-3)/4$ | $w_{1+2-3-4}$ |
| $h_{56}$ | $A_1$ | $\emptyset$ | $(q^3-19q^2+115q-169-8y-12x)/48$ | $1$ |
| $h_{57}$ | $A_1$ | $4A_1$ | $(q^3-13q^2+51q-79-8z+12x)/48$ | $g$ |
| $h_{58}$ | $A_1$ | $A_1$ | $(q-1)(q-3)(q-5)/16$ | $w_{3-4}$ |
| $h_{59}$ | $A_1$ | $2A_1$ | $(q-1)(q-3)^2/16$ | $w_{1+2}w_{3-4}$ |
| $h_{60}$ | $A_1$ | $\tilde{A}_1$ | $(q^3-7q^2+13q+1-2x)/8$ | $w_4$ |
| $h_{61}$ | $A_1$ | $B_2$ | $(q-1)(q+1)(q-3)/8$ | $w_{3-4}w_4$ |
| $h_{62}$ | $A_1$ | $A_1+\tilde{A}_1$ | $(q^3-5q^2+9q-13+2x)/8$ | $w_{3-4}w_{1+2-3-4}$ |
| $h_{63}$ | $A_1$ | $A_1+B_2$ | $(q-1)^2(q+1)/8$ | $w_{1+2}w_{3-4}w_4$ |
| $h_{64}$ | $A_1$ | $\tilde{A}_2$ | $(q^3-q^2-2q-1+y)/6$ | $w_4w_{1+2-3-4}$ |
| $h_{65}$ | $A_1$ | $C_3$ | $(q^3-q^2-1+z)/6$ | $w_{3-4}w_4w_{1+2-3-4}$ |
| $h_{66}$ | $\tilde{A}_1$ | $\emptyset$ | $(q^3-16q^2+85q-118-8y-6x)/48$ | $1$ |
| $h_{67}$ | $\tilde{A}_1$ | $4A_1$ | $(q^3-10q^2+33q-40-8z+6x)/48$ | $g$ |
| $h_{68}$ | $\tilde{A}_1$ | $\tilde{A}_1$ | $(q^3-6q^2+9q+4-2x)/16$ | $w_{1-2-3+4}$ |
| $h_{69}$ | $\tilde{A}_1$ | $2A_1$ | $(q^3-4q^2+5q-10+2x)/16$ | $w_{1-3}w_{2-4}$ |
| $h_{70}$ | $\tilde{A}_1$ | $A_1$ | $(q-1)(q-2)(q-3)/8$ | $w_{1-2}$ |
| $h_{71}$ | $\tilde{A}_1$ | $A_1+\tilde{A}_1$ | $(q-1)(q^2-3q+4)/8$ | $w_{1-2}w_{1+2-3-4}$ |
| $h_{72}$ | $\tilde{A}_1$ | $B_2$ | $(q-1)(q+1)(q-2)/8$ | $w_{2-3}w_{1-2+3-4}$ |
| $h_{73}$ | $\tilde{A}_1$ | $A_3$ | $q(q-1)(q+1)/8$ | $w_{2-3}w_{3-4}w_{1-2}$ |
| $h_{74}$ | $\tilde{A}_1$ | $A_2$ | $(q^3-q^2-2q-1+y)/6$ | $w_{1-2}w_{2-3}$ |
| $h_{75}$ | $\tilde{A}_1$ | $B_3$ | $(q^3-q^2-1+z)/6$ | $w_{2-3}w_{3-4}w_{1-2-3+4}$ |
| $h_{76}$ | $\emptyset$ | $\emptyset$ | $(q^4-28q^3+286q^2-1260q+1673+64y+72x)/1152$ | $1$ |
| $h_{77}$ | $\emptyset$ | $4A_1$ | $(q^4-20q^3+142q^2-420q+521+64z-72x)/1152$ | $g$ |

Table 8. (Continued)

| $h_i$ | tpye $S$ | graph of $w$ | $M(h_i)$ | $w$ |
|---|---|---|---|---|
| $h_{78}$ | $\emptyset$ | $A_1$ | $(q-1)(q-3)^2(q-5)/96$ | $w_{1-2}$ |
| $h_{79}$ | $\emptyset$ | $\tilde{A}_1$ | $(q^4-12q^3+44q^2-48q-33+12x)/96$ | $w_4$ |
| $h_{80}$ | $\emptyset$ | $2A_1$ | $(q-1)^2(q-3)^2/64$ | $w_{1-2}w_{3-4}$ |
| $h_{81}$ | $\emptyset$ | $A_1+\tilde{A}_1$ | $(q-1)(q-3)(q^2+1)/16$ | $w_{1-2}w_4$ |
| $h_{82}$ | $\emptyset$ | $A_2$ | $(q^4-4q^3+q^2+6q+2-2y)/36$ | $w_{1-2}w_{2-3}$ |
| $h_{83}$ | $\emptyset$ | $\tilde{A}_2$ | $(q^4-4q^3+q^2+6q+2-2y)/36$ | $w_4w_{1+2+3-4}$ |
| $h_{84}$ | $\emptyset$ | $B_2$ | $(q-1)(q+1)(q-3)^2/32$ | $w_{1-2}w_2$ |
| $h_{85}$ | $\emptyset$ | $3A_1$ | $(q-1)(q-3)(q^2-4q+7)/96$ | $w_1w_2w_{3-4}$ |
| $h_{86}$ | $\emptyset$ | $2A_1+\tilde{A}_1$ | $(q^4-8q^3+20q^2-28q+63-12x)/96$ | $w_1w_2w_3$ |
| $h_{87}$ | $\emptyset$ | $A_3$ | $(q-1)(q+1)(q^2-2q-1)/16$ | $w_{2-3}w_{1-2}w_{3-4}$ |
| $h_{88}$ | $\emptyset$ | $B_2+A_1$ | $(q-1)^3(q+1)/16$ | $w_{1-2}w_2w_{3-4}$ |
| $h_{89}$ | $\emptyset$ | $C_3$ | $q(q-1)(q+1)(q-2)/12$ | $w_4w_{3-4}w_{1+2-3-4}$ |
| $h_{90}$ | $\emptyset$ | $B_3$ | $q(q-1)(q+1)(q-2)/12$ | $w_{1-2}w_{2-3}w_3$ |
| $h_{91}$ | $\emptyset$ | $A_1+\tilde{A}_2$ | $q^2(q-1)(q+1)/12$ | $w_{1-2}w_4w_{1+2+3-4}$ |
| $h_{92}$ | $\emptyset$ | $A_2+\tilde{A}_1$ | $q^2(q-1)(q+1)/12$ | $w_{1-2}w_{2-3}w_4$ |
| $h_{93}$ | $\emptyset$ | $A_2+\tilde{A}_2$ | $(q^4+2q^3-5q^2-6q-4+4y)/72$ | $w_{1-2}w_{2-3}w_4w_{1+2+3-4}$ |
| $h_{94}$ | $\emptyset$ | $A_3+\tilde{A}_1$ | $(q-1)^3(q+1)/32$ | $w_{1-2}w_{2-3}w_{3-4}w_{1+2+3+4}$ |
| $h_{95}$ | $\emptyset$ | $C_3+A_1$ | $(q^4-2q^3+q^2+2-2z)/36$ | $w_{3-4}w_4w_{1+2-3-4}w_{1-2}$ |
| $h_{96}$ | $\emptyset$ | $D_4$ | $(q^4-2q^3+q^2+2-2z)/36$ | $w_3w_4w_{1-2}w_{2-3}$ |
| $h_{97}$ | $\emptyset$ | $D_4(a_1)$ | $(q-1)(q+1)(q-3)(q+3)/96$ | $w_{1-2}w_{3+4}w_2w_4$ |
| $h_{98}$ | $\emptyset$ | $B_4$ | $(q-1)(q+1)(q^2+1)/8$ | $w_{1-2}w_{2-3}w_{3-4}w_4$ |
| $h_{99}$ | $\emptyset$ | $F_4$ | $q^2(q-1)(q+1)/12$ | $w_{2-3}w_{3-4}w_4w_{1-2-3-4}$ |
| $h_{100}$ | $\emptyset$ | $F_4(a_1)$ | $(q^4-2q^3-5q^2+6q-4+4z)/72$ | $w_{2+3}w_4w_{3-4}w_{1-2-3-4}$ |

Each entry of the second column denotes the type $S$ for $h_i=\Gamma(w, S)$, and the third column denotes the admissible graph of $w$ ([1], p. 298), the fourth column denotes $M(h_i)$ $=M(w, S)$, where $x=(4, q-1)$, $y=(3, q-1)$, $z=(3, q+1)$, respectively, and the last column denotes $w$ such that $h_i=\Gamma(w, S)$.

*representatives for each class* $c_i$ *and the structure of* $Z_{\bar{G}}(x)/Z_{\bar{G}}(x)^\circ$ *for* $x\in c_i$ *are given in Table 7.*

REMARK. It is known ([1], p. 246, [2]) that for sufficiently large $p$, $\bar{G}$ has 16 classes of $p$-elements. Corollary 2.3 shows that this is true for all $p\geqq 3$. But in the case of $p=2$, this is not true since $\bar{G}$ has 20 classes by [5].

## § 3. Conjugacy classes of $p'$-elements.

(3.1) Let $\Gamma(w, S)=\{h\in\bar{H}\,|\,h^q=w(h),\ W_h=W_S\}$, $\Gamma(W)=\cup\,\Gamma(w, S)$ ($w\in W$, $S$; as in Table 2). Then it is known ([1], p. 197, 3.11) that the conjugacy classes of $p'$-elements of $G$ correspond bijectively to elements of $\Gamma(W)$ up to $W$-conjugacy, and it is easily checked that if $\Gamma(w, S)\neq\emptyset$, then $w\in N_W(W_S)$, and that for $h_i\in\Gamma(w_i, S)$ $(i=1, 2)$, if they are conjugate in $W$, $w_1$ and $w_2$ are conjugate in $N_W(W_S)$ mod $W_S$. Thus we have only to consider all representatives of $N_W(W_S)/W_S$. Put $\tilde{W}_S=N_W(W_S)/W_S$, and $\tilde{w}$ the image of $w$ in $\tilde{W}_S$, $M(w, S)$ the number of the classes which intersect $\Gamma(w, S)$, then

LEMMA 3.1. *Let* $N=|Z_{\tilde{W}_S}(\tilde{w})|$, *then* $M(w, S)=N^{-1}|\Gamma(w, S)|$.

The proof is immediate from the facts mentioned above.

Let $\Delta(S)\subset\Phi^+$ be the fundamental system corresponding to $W_S$, then we can replace $w\in N_W(W_S)$ by $w'$ such that $w'(\Delta(S))\subset\Delta(S)$. Put $\sigma'=\mathrm{Int}\, n_{w'}^{-1}\circ\sigma$, $t$ an element of $G$ which corresponds to $h\in\Gamma(w, S)$, the following facts are known ([1], p. 201, 4.1)

$$Z_G(t)\cong T(w')\cdot G_1\ ,$$

where $G_1=\langle\bar{U}_\alpha, \bar{U}_{-\alpha}|\alpha\in\Delta(s)\rangle_{\sigma'}$, the group of fixed points by $\sigma'$, $T(w')=\{h\in\bar{H}|h^q=w'(h)\}$. Since $w'(\Delta(S))\subset\Delta(S)$, $G_1$ is a (twisted) Chevalley group of type $S$. Using these facts, we can determine the classes of $p'$-elements of $G$, and the orders of their centralizers. Thus we have the following theorem. (See Theorem 4.1 for the order of the centralizer.)

THEOREM 3.1. *All the representatives of $p$-elements $\Gamma(w, S)$, which we shall denote by $\{h_i\}$, and the number of the classes for each $\Gamma(w, S)$, i.e. $M(w, S)=$ $=M(h_i)$ are given in Table 8.*

## §4. Conjugacy classes of $p$-singular elements.

(4.1) Let $x$ be a $p$-singular element of $G$, then we have the so-called Jordan decomposition $x=x_sx_u=x_ux_s$, where $x_s$ is a semisimple element, and $x_u$ is a unipotent element of $G$. By the uniqueness of above decomposition of $x$, we have $Z_G(x)=Z_{Z_G(x_s)}(x_u)$. Thus the problem of determining the conjugacy classes of $p$-singular elements is reduced to determining the classes of $p$-elements of the centralizers of $p'$-elements of $G$, ([5], §3). Since the structure of $Z_G(x)$ for $p'$-element $x\in G$ is known by §3, we can easily determine all the representatives and their centralizers of $p$-elements of $Z_G(x)$ except for the case of $x=h_0$. The latter case is given in Tables 5, 6. Therefore we only write the number of the classes of $p$-elements of $Z_G(x)$.

THEOREM 4.1. *Let $t_i$ be an element of $G$ corresponding to $h_i\in\bar{H}$. The order of the centralizer of $t_i$ in $G$ and the number of the classes of $p$-elements of $Z_G(t_i)$ are given in Table 9.*

Let $m_i$ be the number of the classes of $p$-elements of $Z_G(t_i)$, then it is clear

Table 9. Centralizers of $p'$-elements.

| $h_i$ | $m_i$ | $Z_G(t_i)$ | type of $G_1$ |
|---|---|---|---|
| $h_0$ | 26 | $q^{24}(q^2-1)(q^6-1)(q^8-1)(q^{12}-1)$ | $F_4$ |
| $h_1$ | 26 | $q^{10}(q^2-1)^2(q^4-1)(q^6-1)$ | $A_1+C_3$ |
| $h_2$ | 25 | $q^{16}(q^2-1)(q^4-1)(q^6-1)(q^8-1)$ | $B_4$ |
| $h_3$ | 11 | $q^6(q^2-1)^2(q^3-1)^2$ | $A_2+\tilde{A}_2$ |
| $h_4$ | 11 | $q^6(q^2-1)^2(q^3+1)^2$ | ${}^2A_2+{}^2\tilde{A}_2$ |
| $h_5$ | 16 | $q^7(q^2-1)^2(q^3-1)(q^4-1)$ | $A_3+\tilde{A}_1$ |
| $h_6$ | 16 | $q^7(q^2-1)^2(q^3+1)(q^4-1)$ | ${}^2A_3+\tilde{A}_1$ |
| $h_7$ | 10 | $q^9(q-1)(q^2-1)(q^4-1)(q^6-1)$ | $C_3$ |
| $h_8$ | 10 | $q^9(q+1)(q^2-1)(q^4-1)(q^6-1)$ | $C_3$ |
| $h_9$ | 6 | $q^4(q-1)(q^2-1)^2(q^3-1)$ | $A_1+\tilde{A}_2$ |
| $h_{10}$ | 6 | $q^4(q+1)(q^2-1)^2(q^3+1)$ | $A_1+{}^2\tilde{A}_2$ |
| $h_{11}$ | 7 | $q^6(q-1)(q^2-1)(q^3-1)(q^4-1)$ | $A_3$ |
| $h_{12}$ | 7 | $q^6(q+1)(q^2-1)(q^3+1)(q^4-1)$ | ${}^2A_3$ |
| $h_{13}$ | 7 | $q^6(q+1)(q^2-1)(q^3-1)(q^4-1)$ | $A_3$ |
| $h_{14}$ | 7 | $q^6(q-1)(q^2-1)(q^3+1)(q^4-1)$ | ${}^2A_3$ |
| $h_{15}$ | 6 | $q^4(q-1)(q^2-1)^2(q^3-1)$ | $A_2+\tilde{A}_1$ |
| $h_{16}$ | 6 | $q^4(q+1)(q^2-1)^2(q^3+1)$ | ${}^2A_2+\tilde{A}_1$ |
| $h_{17}$ | 10 | $q^3(q-1)(q^2-1)^3$ | $2A_1+\tilde{A}_1$ |
| $h_{18}$ | 10 | $q^3(q+1)(q^2-1)^3$ | $2A_1+\tilde{A}_1$ |
| $h_{19}$ | 6 | $q^3(q-1)(q^2-1)(q^4-1)$ | ${}^2(2A_1)+\tilde{A}_1$ |
| $h_{20}$ | 6 | $q^3(q+1)(q^2-1)(q^4-1)$ | ${}^2(2A_1)+\tilde{A}_1$ |
| $h_{21}$ | 12 | $q^5(q-1)(q^2-1)^2(q^4-1)$ | $A_1+B_2$ |
| $h_{22}$ | 12 | $q^5(q+1)(q^2-1)^2(q^4-1)$ | $A_1+B_2$ |
| $h_{23}$ | 10 | $q^9(q-1)(q^2-1)(q^4-1)(q^6-1)$ | $B_3$ |
| $h_{24}$ | 10 | $q^9(q+1)(q^2-1)(q^4-1)(q^6-1)$ | $B_3$ |
| $h_{25}$ | 3 | $q^3(q-1)^2(q^2-1)(q^3-1)$ | $A_2$ |
| $h_{26}$ | 3 | $q^3(q+1)^2(q^2-1)(q^3+1)$ | ${}^2A_2$ |
| $h_{27}$ | 3 | $q^3(q^2-1)^2(q^3-1)$ | $A_2$ |
| $h_{28}$ | 3 | $q^3(q^2-1)^2(q^3+1)$ | ${}^2A_2$ |
| $h_{29}$ | 3 | $q^3(q+1)(q^3-1)^2$ | $A_2$ |
| $h_{30}$ | 3 | $q^3(q-1)(q^3+1)^2$ | ${}^2A_2$ |
| $h_{31}$ | 3 | $q^3(q-1)^2(q^2-1)(q^3-1)$ | $\tilde{A}_2$ |
| $h_{32}$ | 3 | $q^3(q+1)^2(q^2-1)(q^3+1)$ | ${}^2\tilde{A}_2$ |
| $h_{33}$ | 3 | $q^3(q^2-1)^2(q^3-1)$ | $\tilde{A}_2$ |
| $h_{34}$ | 3 | $q^3(q^2-1)^2(q^3+1)$ | ${}^2\tilde{A}_2$ |
| $h_{35}$ | 3 | $q^3(q+1)(q^3-1)^2$ | $\tilde{A}_2$ |
| $h_{36}$ | 3 | $q^3(q-1)(q^3+1)^2$ | ${}^2\tilde{A}_2$ |
| $h_{37}$ | 5 | $q^4(q-1)^2(q^2-1)(q^4-1)$ | $B_2$ |
| $h_{38}$ | 5 | $q^4(q+1)^2(q^2-1)(q^4-1)$ | $B_2$ |
| $h_{39}$ | 5 | $q^4(q^2-1)^2(q^4-1)$ | $B_2$ |

Table 9. (Continued)

| $h_i$ | $m_i$ | $Z_G(t_i)$ | type of $G_1$ |
|---|---|---|---|
| $h_{40}$ | 5 | $q^4(q^2-1)^2(q^4-1)$ | $B_2$ |
| $h_{41}$ | 5 | $q^4(q^4-1)^2$ | $B_2$ |
| $h_{42}$ | 5 | $q^2(q-1)^2(q^2-1)^2$ | $2A_1$ |
| $h_{43}$ | 5 | $q^2(q+1)^2(q^2-1)^2$ | $2A_1$ |
| $h_{44}$ | 5 | $q^2(q^2-1)^3$ | $2A_1$ |
| $h_{45}$ | 5 | $q^2(q^2-1)^3$ | $2A_1$ |
| $h_{46}$ | 5 | $q^2(q^2-1)(q^4-1)$ | $2A_1$ |
| $h_{47}$ | 3 | $q^2(q-1)^2(q^4-1)$ | ${}^2(2A_1)$ |
| $h_{48}$ | 3 | $q^2(q+1)^2(q^4-1)$ | ${}^2(2A_1)$ |
| $h_{49}$ | 3 | $q^2(q^2-1)(q^4-1)$ | ${}^2(2A_1)$ |
| $h_{50}$ | 3 | $q^2(q^2-1)(q^4-1)$ | ${}^2(2A_1)$ |
| $h_{51}$ | 3 | $q^2(q^2+1)(q^4-1)$ | ${}^2(2A_1)$ |
| $h_{52}$ | 4 | $q^2(q-1)^2(q^2-1)^2$ | $A_1+\tilde{A}_1$ |
| $h_{53}$ | 4 | $q^2(q+1)^2(q^2-1)^2$ | $A_1+\tilde{A}_1$ |
| $h_{54}$ | 4 | $q^2(q^2-1)^3$ | $A_1+\tilde{A}_1$ |
| $h_{55}$ | 4 | $q^2(q^2-1)^3$ | $A_1+\tilde{A}_1$ |
| $h_{56}$ | 2 | $q(q-1)^3(q^2-1)$ | $A_1$ |
| $h_{57}$ | 2 | $q(q+1)^3(q^2-1)$ | $A_1$ |
| $h_{58}$ | 2 | $q(q-1)(q^2-1)^2$ | $A_1$ |
| $h_{59}$ | 2 | $q(q+1)(q^2-1)^2$ | $A_1$ |
| $h_{60}$ | 2 | $q(q-1)(q^2-1)^2$ | $A_1$ |
| $h_{61}$ | 2 | $q(q-1)(q^4-1)$ | $A_1$ |
| $h_{62}$ | 2 | $q(q+1)(q^2-1)^2$ | $A_1$ |
| $h_{63}$ | 2 | $q(q+1)(q^4-1)$ | $A_1$ |
| $h_{64}$ | 2 | $q(q^2-1)(q^3-1)$ | $A_1$ |
| $h_{65}$ | 2 | $q(q^2-1)(q^3+1)$ | $A_1$ |
| $h_{66}$ | 2 | $q(q-1)^3(q^2-1)$ | $\tilde{A}_1$ |
| $h_{67}$ | 2 | $q(q+1)^3(q^2-1)$ | $\tilde{A}_1$ |
| $h_{68}$ | 2 | $q(q-1)(q^2-1)^2$ | $\tilde{A}_1$ |
| $h_{69}$ | 2 | $q(q+1)(q^2-1)^2$ | $\tilde{A}_1$ |
| $h_{70}$ | 2 | $q(q-1)(q^2-1)^2$ | $\tilde{A}_1$ |
| $h_{71}$ | 2 | $q(q+1)(q^2-1)^2$ | $\tilde{A}_1$ |
| $h_{72}$ | 2 | $q(q-1)(q^4-1)$ | $\tilde{A}_1$ |
| $h_{73}$ | 2 | $q(q+1)(q^4-1)$ | $\tilde{A}_1$ |
| $h_{74}$ | 2 | $q(q^2-1)(q^3-1)$ | $\tilde{A}_1$ |
| $h_{75}$ | 2 | $q(q^2-1)(q^3+1)$ | $\tilde{A}_1$ |
| $h_{76}$ | 1 | $(q-1)^4$ | $\varnothing$ |
| $h_{77}$ | 1 | $(q+1)^4$ | $\varnothing$ |
| $h_{78}$ | 1 | $(q-1)^2(q^2-1)$ | $\varnothing$ |
| $h_{79}$ | 1 | $(q-1)^2(q^2-1)$ | $\varnothing$ |

Table 9. (Continued)

| $h_i$ | $m_i$ | $Z_G(t_i)$ | type of $G_1$ |
|---|---|---|---|
| $h_{80}$ | 1 | $(q^2-1)^2$ | $\emptyset$ |
| $h_{81}$ | 1 | $(q^2-1)^2$ | $\emptyset$ |
| $h_{82}$ | 1 | $(q-1)(q^3-1)$ | $\emptyset$ |
| $h_{83}$ | 1 | $(q-1)(q^3-1)$ | $\emptyset$ |
| $h_{84}$ | 1 | $(q-1)^2(q^2+1)$ | $\emptyset$ |
| $h_{85}$ | 1 | $(q+1)^2(q^2-1)$ | $\emptyset$ |
| $h_{86}$ | 1 | $(q+1)^2(q^2-1)$ | $\emptyset$ |
| $h_{87}$ | 1 | $(q^4-1)$ | $\emptyset$ |
| $h_{88}$ | 1 | $(q^4-1)$ | $\emptyset$ |
| $h_{89}$ | 1 | $(q-1)(q^3+1)$ | $\emptyset$ |
| $h_{90}$ | 1 | $(q-1)(q^3+1)$ | $\emptyset$ |
| $h_{91}$ | 1 | $(q+1)(q^3-1)$ | $\emptyset$ |
| $h_{92}$ | 1 | $(q+1)(q^3-1)$ | $\emptyset$ |
| $h_{93}$ | 1 | $(q^2+q+1)^2$ | $\emptyset$ |
| $h_{94}$ | 1 | $(q+1)^2(q^2+1)$ | $\emptyset$ |
| $h_{95}$ | 1 | $(q+1)(q^3+1)$ | $\emptyset$ |
| $h_{96}$ | 1 | $(q+1)(q^3+1)$ | $\emptyset$ |
| $h_{97}$ | 1 | $(q^2+1)^2$ | $\emptyset$ |
| $h_{98}$ | 1 | $(q^4+1)$ | $\emptyset$ |
| $h_{99}$ | 1 | $(q^4-q^2+1)$ | $\emptyset$ |
| $h_{100}$ | 1 | $(q^2-q+1)^2$ | $\emptyset$ |

The quantity $m_i$ in the second column denotes the number of the conjugacy classes of $p$-elements of $Z_G(t_i)$, where $t_i$ is an element of $G$ corresponding to $h_i$, and the last column denotes the type of the (twisted) Chevalley group $G_1$ contained in $Z_G(t_i)$.

that the number of the conjugacy classes in $G$ is given by $\sum_i m_i M(h_i)$. Thus, by Theorem 4.1, we have the following theorem.

THEOREM 4.2. *Suppose* $p\neq 2.3$ (*resp.* $p=3$). *Then the number of conjugacy classes of* $G$ *is* $q^4+2q^3+7q^2+15q+30$ (*resp.* $q^4+2q^3+7q^2+15q+27$).

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