Comparative Concepts
PhilLogMath 2
Seiryo Keikan, Tokyo, 14-03-12

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March 14, 2012
1 Introduction
   - The epistemology of orderings
   - Gärdenfors on natural properties

2 Natural Comparative Concepts
   - Convexity criteria for naturalness
   - From comparative to categorical concepts

3 A Prototype-theoretic Approach
   - Preliminaries
   - Voronoi diagrams I: From prototype type points to areas
   - Voronoi diagrams II: From categorical to comparative concepts

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Comparative Concepts
Overview

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Categorical vs comparative concepts

**Categorical concepts**
- type of concepts expressed by general terms in natural languages, such as “high”, “exactly 10 meters high”, “cat” or “chair”.
- rules of partitioning a set of objects.

**Comparative concepts**
- type of concepts expressed by comparative constructions embedding a general term, such as “is higher than”, “is less tall than” or “to look more red”.
- rules of ordering objects.
Varieties of evidence on ordering behaviour

- explicit comparisons (e.g., of the form “x is F-er than y”).
- orders induced by probabilities of positive categorisation (Hampton [1998, 2007]).
- orders induced by choice probabilities assigned to ordered pairs (the probability that the one item is picked out—as an F—as compared to the other item) (Suppes et al. [1989]).
The Epistemology of Orderings

Questions

- What kind of cognitive structures underly our ability to order objects in certain ways?
- Why do we order objects in certain ways, and not in different ways?
Suppose we examine a sample of colour patches $x_1, \ldots, x_n$, where the series is monotonically increasing in greenness. That is, we have a case where for each $0 < i \leq n$, $x_i$ is *greener* than $x_{i-1}$. Suppose $t$ designates the present point of time. It makes then extensionally no difference to say that we have a case where for each $0 < i \leq n$, $x_i$ is *gruer* than $x_{i-1}$, where this relation is defined as follows: for any pair of colour patches $x$ and $y$, $x$ is gruer than $y$ just in case either (a) $x$ and $y$ are examined by point $t$, and $x$ is greener than $y$, or (b) $x$ and $y$ are both examined after $t$, and $x$ is bluer than $y$. 

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Comparative Concepts
Suppose we examine the colour patches $x_1, \ldots, x_n$ in the temporal order of their mention here, and that there are two other colour patches in the sequence, $x_{n+1}$ and $x_{n+2}$, which are still hidden. Given $n$ is sufficiently high, it would seem only natural to predict that $x_{n+2}$ is greener than $x_{n+1}$. On the other hand, the prediction that $x_{n+2}$ is gruer than $x_{n+1}$ would seem quite bizarre—for it would imply that $x_{n+2}$ is bluer than $x_{n+1}$.
Gradable concepts

- **Gradable concepts**: type of concepts expressed by *gradable terms*, that is, general terms such as “high” or “red” that embed in comparative constructions.

- **Bridge principles**: in order to have a concept of redness, it seems that we need to know that anything redder than something red must be red as well; and also that for something to be distinguishable as red from something else, the former is to be redder than the latter.

  - Put aside delineation based approaches to comparatives (Klein [1980], van Benthem [1982], van Rooij [2009]).
Aim

- Outlining a novel approach to comparative concepts that
  1. supplies means of characterising naturalness for comparative concepts, and
  2. has constraining effects on the theory of gradable concepts.

- Method: Carrying Peter Gärdenfors’ conceptual spaces approach (Conceptual Spaces [2000]), which focusses on ungraded categorisation rules, over to comparative concepts.
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Conceptual spaces

- **Spaces** are sets $D_1, \ldots, D_n$ of *quality dimensions*, i.e., kinds of features with respect to which objects may be judged as more or less similar.

- A **point** in a space is defined by a vector $v = \langle d_1, \ldots, d_n \rangle$ where each index represents a dimension.

- Each dimension has typically a **geometric** structure.

- **Objects** (‘stimuli’) are represented as points in a space.

- **Concepts** are represented as sets in a space.
Conceptual spaces – cont.

examples

- **colours**: a space with the dimensions hue, chromaticness and brightness.

- **geometric figures**: a space with the dimensions shape, size, and angular orientation.
Gärdenfors on Natural Properties

Conceptual spaces – cont’

a side note

- Stalnaker’s formulation of a bare particular anti-essentialism (in [1979]).
- Lambert’s and van Fraassen’s account of analyticity (in [1970]).
- Churchland’s naturalistic approach to linguistic meaning (in [1986]).
- Bromberger’s realism about types in linguistic theory (in [1992]).
## A geometric approach to similarity

### A metric model of distances

A two-place real-valued function $d$ on a set $M$ is said to be a metric iff:

- **D1** $d(a, b) \leq 0$ and $d(a, b) = 0$ only if $a = b$; (minimality)
- **D2** $d(a, b) = d(b, a)$; (symmetry)
- **D3** $d(a, c) \leq d(a, b) + d(b, c)$. (triangular inequality)

### Similarity and distance

Similarity is inversely related to distance: linear (Tversky [1975]), exponential (Shepard [1987]), Gaussian function (Nosofski [1986]).
A geometric approach to similarity – cont.

**Power metric model**

\[ d(x, y) = \left( \sum_{i=1}^{n} |x_i - y_i|^r \right)^{\frac{1}{r}} \]

- for \( r = 2 \): Euclidean metric.
- for \( r = 1 \), city block or Manhattan metric.
Properties (Gärdenfors [2000])

- **Separable dimensions:** can be perceived/cognised independently from each other
  - e.g., hue, chromaticness and brightness are not separable from each other.

- **Domains:** sets of dimensions that are not pairwise separable, but all separable from other dimensions.

- **Properties:** are concepts that refer to so-called *domains*
  - e.g., compare colour concepts with *apple*, which refers to more than one domain (such as colour, shape or texture).
Criteria for naturalness (Gärdenfors [2000])

1. **connectedness**: A region $X$ is said to be *connected*, if and only if, for all regions $Y$ and $Z$ such that $Y \cup Z = X$, it holds that $C(Y, Z)$. $X$ is *disconnected*, if and only if $X$ is not connected.

2. **star-shapedness**: A subset $C$ of a conceptual space $S$ is said to be *star-shaped with respect to point* $p$, if and only if, for all points $x$ in $C$, all points between $x$ and $p$ are also in $C$.

3. **convexity**: A subset $C$ of a conceptual space $S$ is said to be *convex*, if and only if, for all points $x$ and $y$ in $C$, all points between $x$ and $y$ are also in $C$. 

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Gärdenfors on Natural Properties

Convexity criterion P (Gärdenfors [2000])

A natural property is a convex region of a domain in a conceptual space.
Gärdenfors on Natural Properties

Related discussion

- Oddie [2005] on ‘natural’ value properties.

- evolutionary arguments (from evolutionary psychology: Shepard [1987]; from evolutionary game theory, see Jäger [2009] and Jäger et al. [2009]).

- but see Mormann [1993] for an argument to the effect that the convexity constraint is unnecessarily strong.

- Gärdenfors’ argument from prototype theory ([2000]) (sect. 3).
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Convexity Criteria for Naturalness

Modelling orders of points as orders of sets

- for any partially ordered set $\langle P, \geq \rangle$ and any subset $Q$ of $P$, $Q$ is said to be an order filter (or upward closed set) if, whenever $x \in Q$, $y \in P$ and $y \geq x$, we have $y \in Q$.

- for any arbitrary set $Q$ of $P$, we define:
  - $\uparrow Q := \{ y \in P \mid (\exists x \in Q) \, y \geq x \}$.

- $\uparrow$ is an isomorphism between $\langle P, \geq \rangle$ and $\langle \uparrow P, \subseteq \rangle$. 

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Convexity criteria for naturalness

**Criterion C1**

A strict partial ordering \( > \) referring to one domain in a conceptual space is a natural comparative concept only if for all points \( x \) in the space, the corresponding set \( \{ y \mid y > x \} \) is a convex region.

E.g., criterion C1 implies that for any triple of patches \( x, y \) and \( z \) where both \( x \) and \( y \) are redder than \( z \), any patch in between in colour shade between \( x \) and \( y \) should be redder than \( z \) as well.
Almost-connectedness

- $R$ is almost connected: $x > y \rightarrow (z > y \lor x > z)$.
- (strict) weak orders: (strict) partial orders that are almost connected.
- indifference $(x \not> y \land y \not> x)$ is transitive.
Convexity criteria for naturalness – cont.

**Criterion C2**

A strict weak ordering $\succ$ referring to one domain in a conceptual space is a natural comparative concept only if for all points $x$ in the space, the corresponding set $\{y \mid y \succ x \lor (x \not\succ y \land y \not\succ x)\}$ is a convex region.

E.g., criterion C2 implies that for any triple of patches $x$, $y$ and $z$ where both $x$ and $y$ are at least as red as $z$, any patch in between in colour shade between $x$ and $y$ should be at least as red as $z$ as well.
Sivik and Taft [1994]

‘Isosemantic lines’ in the colour space, i.e., areas of colours that test persons tended to categorise as equally red, brown, or other, circumscribed a convex area in space.
From Comparative to Categorical Concepts

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Associatedness

For any given comparative concept $\succ$ and any given categorical concept $F$, $\succ$ and $F$ are said to be associated with each other iff they satisfy:

B1. $x \succ y \rightarrow (F(y) \rightarrow F(x))$.

B2. $(F(x) \land \neg F(y)) \rightarrow x \succ y$.

Note

- $F$ may be interpreted both in terms of binary and in terms of gradable classification criteria.

- On failure of almost-connectedness, the transitive closure of indifference may include pairs of objects that should be treated differently in terms of $F$-ness.

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Comparative Concepts
The no-gap condition

- For any given strictly partially ordered set $\langle P, > \rangle$, a pair $\langle P_1, P_2 \rangle$ is said to be a cut in $\langle P, > \rangle$ iff:
  1. $\{P_1, P_2\}$ is a bipartition in $P$;
  2. if $x \in P_1$ and $y \in P_2$, then $x > y$.

- A strictly partially ordered set $\langle P, > \rangle$ is then said to satisfy the no-gap condition iff for every cut in the set, either $\langle T_1, > \rangle$ has a minimal element or $\langle T_2, > \rangle$ has a maximal element.
Theorem

Let \( \langle P, > \rangle \) be a strict weak ordering that satisfies the no-gap condition, and let \( F \) be a subset in \( P \), where \( > \) and \( F \) are associated with each other. Then for some member \( x \) of \( P \), either

- \( F = \{ y \in P \mid y > x \} \), or
- \( F = \{ y \in P \mid (y > x) \lor (y \not> x \land x \not> y) \} \).
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Introduction
Natural Comparative Concepts

A Prototype-theoretic Approach

Preliminaries

Agenda

- **Approach**: modelling comparative concepts in terms of conceptual space representations of prototypes.

- **Focus** on comparative concepts that:
  - refer to one domain (Euclidean metric).
  - are (strict) weak orderings.
  - which satisfy the no-gap condition.
  - are associated with a categorical concept.

- Optional constraint: prototype points for $F$-ness are maximal elements in $\langle M, >_F \rangle$ (*Maximality*).
Disclaimers – Open issues put aside

- Comparative concepts without prototypes? How about concepts such as *long* or *late*? (Kamp and Partee [1995] vs Hampton [2007]; Tribushinina [2008, 2009])

- Prototypes without comparative concepts? How about *dog*, *apple*, or *city*? (Schwartzchild [2008] vs Sasson [2007])

- Concepts that refer to more than one domain, e.g., plausibly, *grue*/*gruer*.

- Comparative concepts that are less precise: multi-dimensional concepts (*cleverer than*), interval orderings (*later than*), semi-orderings (*definitely larger*) (for the latter types of cases, see Suppes et al. [1989]).
Similarity, typicality, and graded membership

**Naive prototype theory**

- Typicality ($T_F$) is a strictly increasing function of similarity to a prototype.
- Graded membership ($M_F$) is a strictly increasing function of typicality.

**Fuzzy semantics**

Interpretating graded membership as similarity to the closest prototypical element (Ruspini [1991], Dubois and Prade [1997], Dubois et al. [2001]).
## Preliminaries

### Similarity, typicality, and graded membership – cont.

#### Osherson and Smith [1997]

- \( T_{bird}(robin) > T_{bird}(woodpecker) \).
- but:  \( M_{bird}(robin) = M_{bird}(woodpecker) = 1 \).

#### Hampton [2007]

\( M_F \) is a cumulative normal distribution function of \( T_F \), which has 0 as its infimum and 1 as its supremum (i.e., \( M_F(x) := \text{Prob}(X \leq x) \), where the random variable \( X \) takes \( T_F \) values).
Similarity, typicality, and graded membership – cont.’

Hampton [1998]

Typicality does not always provide a good prediction of graded membership (experiments on artifact concepts).
Open questions

1 What prototypes are relevant?
   1.a non-contrastive accounts: $F$-er is given for a conceptual space by some prototype for $F$-ness in the space.
   1.b contrastive accounts: $F$-er is given for a conceptual space by some set of disjoint prototypes including the prototype for $F$-ness.

2 In what way are prototypes relevant?
   2.a distance infima (suprema): the infimum (or supremum) of distances between a particular point and any point in the prototype area.
   2.b scaling factors: the factor by which the prototype area is to be expanded/contracted in order to reach a particular point.

...
Working hypothesis

Combining [1.b] with [2.a].
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Voronoi diagrams – standard

Two variants

A Given a set of ‘prototypical’ points in a metric space, a Voronoi diagram divides the space into subsets, where each subset contains one and only one ‘prototypical’ point \( p \) and consists of all points with respect to which there is no closer ‘prototypical’ point than \( p \) (Okabe et al. [1992 [2000]]).

B Given a set of ‘prototypical’ points in a metric space, a Voronoi diagram divides the space into subsets, where each subset contains one and only one ‘prototypical’ point \( p \) and consists of all points with respect to which \( p \) is closer than any other ‘prototypical’ point (Aurenhammer and Klein [2000]).
For Euclidean $n$-spaces, Voronoi regions are convex.

Let $\{M, d\}$ be a Euclidean metric space and $P$ be a subset (in that space) of points $p_1, \ldots, p_n$. Then for each $p_i$ in $P$, the Voronoi region associated with $p_i$ relative to $P$ is convex.
How to deal with prototype *areas*?

**Generalised Voronoi Categorisation (for 2D-spaces)**

An object represented as a point in a conceptual space belongs to the category for which the corresponding prototypical circle is the closest (Gärdenfors [2000]).

**Nearest Neighbour Categorisation (for finite sets of prototype points)**

An object represented as a point $x$ in a conceptual space belongs to the category for which the prototype instance that is closest to $x$ is included (cf. Reed [1972]).

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How to deal with prototype areas? – cont.

Average Distance Categorisation

An object represented as a point $x$ in a conceptual space belongs to the category to which $x$ has the smallest average distance (Nosofski [1988]).
How to deal with prototype *areas*? – cont.'

Collated Voronoi Categorisation

An object represented as a point $x$ in a conceptual space belongs to the category for which, for each prototype instance $y$, $x$ at least as close to $y$ as to any prototype instance of any ‘competing’ category.

Collated Voronoi categorisation

Let $R = \{r_1, \ldots, r_n\}$ be a distribution of disjoint prototype areas. The set of prototype point distributions for $R$ is defined as:

$$\Pi(R) := \{P = \langle p_1, \ldots, p_n \rangle \mid p_i \in r_i\}.$$

The Voronoi region associated with a point $p$ relative to $P$, where $P \in \Pi(R)$ and $p \in P$ is defined as

$$v(p, P) := \{q \mid d(q, p) \leq d(q, p'), \text{ with } p' \in P \text{ and } p' \neq p\}.$$

Accordingly, the Voronoi region associated with a set $r_i$ relative to $R$ comes to

$$u(r_i, R) := \bigcap_{P \in \Pi(R)} \{v(p, P) \mid p \in r_i\},$$
Convexity result (Douven et al. [forthcoming: sect. 3])

Let \( \{M, d\} \) a Euclidean metric space and \( P \) be a subset (in that space) of points \( p_1, \ldots, p_n \). Then for each \( p_i \) in \( P \), the collated Voronoi region associated with \( p_i \) relative to \( P \) is convex.
How to deal with *comparative* concepts which are associated with a prototype area?

Collated Voronoi Categorisation Generalised

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An equivalence result on collated Voronoi categorisation

**Theorem (T1)**

Let \( \langle M, d \rangle \) be a metric space and \( R \) be a set of disjoint subsets \( r_1, \ldots, r_n \) in \( M \). Then Voronoi region associated with a set \( r_i \) relative to \( R \), \( u(r_i, R) \), is given by:

\[
\{ p \in M \mid \sup \{ d(p, x) \mid x \in r_i \} \leq \inf \{ d(p, y) \mid y \in r_j \in R, j \neq i \} \}.
\]

Informally \ldots

**Collated Voronoi Categorisation’**: An object represented as a point \( x \) in a conceptual space belongs to the category \( F \) for which the supremum of distances between \( x \) and any point in the prototype area of \( F \) is no greater than the infimum of distances between \( x \) and any point in any prototype area for any ‘competing’ category.
Collated Voronoi categorisation generalised

Graded Collated Voronoi Categorisation

- For any $\lambda$ where $0 \leq \lambda \leq 1$, call distances scaled by $\lambda$-distances.

- For any $\lambda$ where $0 \leq \lambda \leq 1$, an object represented as a point $x$ in a conceptual space belongs relative to $\lambda$ to the category for which, for each prototype instance $y$, the $\lambda$-distance between $x$ and $y$ is no greater than the $(1 - \lambda)$-distance between $x$ and any prototype instance of any ‘competing’ category.
Collated Voronoi categorisation generalised

**Graded Collated Voronoi Categorisation – more formally**

The *Voronoi region associated with a point* \( p \) *relative to* \( P \) *and a factor* \( \lambda \) where \( P \in \Pi(R) \), \( p \in P \), and \( 0 \leq \lambda \leq 1 \) is defined as

\[
\nu(p, P, \lambda) := \{ q \mid \lambda \cdot d(q, p) \leq (1 - \lambda) \cdot d(q, p'), \text{ with } p' \in P \text{ and } p' \neq p \}.
\]

The *Voronoi region associated with a set* \( r_i \) *relative to a set* \( R \) *and factor* \( \lambda \) *is defined as*

\[
u(r_i, R, \lambda) := \bigcap\{ \nu(p, P, \lambda) \mid p \in r_i \}.\]
Collated Voronoi categorisation generalised – cont.

**limiting case**

For $\lambda = 0.5$, graded collated Voronoi categorisation amounts to collated Voronoi categorisation.
An equivalence result on graded collated Voronoi categorisation

Theorem (T2)

Let $\langle M, d \rangle$ be a metric space and $R$ be a set of disjoint subsets $r_1, \ldots, r_n$ in $M$. Then for any $0 \leq \lambda \leq 1$, the Voronoi region corresponding with $r_i$, $R$ and $\lambda$, $u(r_i, R, \lambda)$, is given by:

$$\{ p \mid \sup \{ \lambda d(p, x) \mid x \in r_i \} \leq \inf \{ (1 - \lambda)d(p, y) \mid y \in r_j \in R, j \neq i \} \}$$

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Comparative Concepts
Informally ... 

For any $\lambda$ with $0 \leq \lambda \leq 1$, an object represented as a point $x$ in a conceptual space belongs relative to $\lambda$ to the category for which the supremum of $\lambda$-distances between $x$ and prototype instances is no greater than the infimum of $(1 - \lambda)$-distances between $x$ and any prototype instances of any ‘competing’ category.
A restricted convexity result

**Theorem (T3)**

Let \( \langle M, d \rangle \) be a Euclidean \( n \)-space, with a prototype set distribution \( R := \{r_1, \ldots, r_n\} \). For any \( r_i \) from \( R \) then, the graded collated Voronoi region \( u(r_i, R, \lambda) \) is convex if \( \lambda \geq .5 \).
Let \( \langle M, d \rangle \) be a metric space and \( R \) be a set of disjoint subsets \( r_1, \ldots, r_n \) in \( M \). Then for any \( 0 \leq \lambda \leq 1 \), for any pair of distinct ‘prototypical points’ \( x \) and \( y \) (where for some \( P \in \Pi(R) \), \( x, y \in P \)), the Voronoi diagram for \( x, y \) and \( \lambda \) is given by the equation:

\[
\Sigma_{1 \leq i \leq n} (\lambda p_i - \lambda x_i)^2 = \Sigma_{1 \leq i \leq n} ((1 - \lambda) p_i - (1 - \lambda) y_i)^2
\]

\( \lambda = .5 \)

Equation of a hyperplane that separates the space into two half-spaces:

\[
\Sigma_{1 \leq i \leq n} (a_i \times p_i) + b_i \leq 0, \text{ where } p_i \text{ is the only variable,}
\]

The half-spaces are (assuming a Euclidean metric) convex.
A restricted convexity result – cont.’’

\[
\lambda \neq .5
\]

- equation of a hypersphere centred on \( a_i \), with the radius being \( \sqrt{c_i} \):

\[
\Sigma_i(p_i - a_i)^2 \leq c_i, \text{ where } p_i \text{ is the only variable and } c_i > 0,
\]

The area circumscribed by the hypersphere is (assuming a Euclidean metric) a convex area, whereas the complement is not convex.

- for \( \lambda > .5 \) (\( \lambda < .5 \)), the hypersphere is centred on \( x \) (\( y \)).
Nestedness Lemma

For any metric space $\langle M, d \rangle$, with a prototype set distribution $R := \{r_1, \ldots, r_n\}$, for any $\lambda \in [0, 1]$ and $\lambda' \in [0, 1]$, if $\lambda \geq \lambda'$, then $u(r_i, R, \lambda) \subseteq u(r_i, R, \lambda')$. 
Collated Voronoi orderings: definition

For any $n$-space with a metric $d$, $\langle M, d \rangle$, with a prototype area distribution $R := \{r_1, \ldots, r_n\}$, for any $\lambda$ where $0 \leq \lambda \leq 1$, let $u(r_i, R, \lambda)$ be the category corresponding to $r_i$, $R$ and $\lambda$. For any set $r \in R$, for any $x$ and $y$ in $\langle M, d \rangle$ then:

$$x >^{c_{\langle R, r \rangle}}_{\langle R, r \rangle} y \iff df$$

$$(\exists \lambda : 0 \leq \lambda \leq 1) \ (x \in u(r_i, R, \lambda) \land y \notin u(r_i, R, \lambda)).$$
Collated Voronoi orderings: features

- If the metric is **Euclidean**, then $\succ_{\langle R,r \rangle}^{cV}$ validates $C_1$ and $C_2$ only restrictedly— with respect to any Voronoi region $u(r_i, R, \lambda)$, where $\lambda \geq .5$.

- $\succ_{\langle R,r \rangle}^{cV}$ is a **strict weak** ordering.

- If the metric is **Euclidean**, then any categorical concept that is associated with $\succ_{\langle R,r \rangle}^{cV}$ is convex, if it is identical with $\{y \mid y \succ_{\langle R,r \rangle}^{cV} x\}$, or identical with $\{y \mid (y \succ_{\langle R,r \rangle}^{cV} x) \lor (y \nsucc_{\langle R,r \rangle}^{cV} x \land x \nsucc_{\langle R,r \rangle}^{cV} y)\}$, for some member $x$ of $u(r, R, \lambda)$ where $\lambda \geq .5$. 

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Comparative Concepts
Collated Voronoi orderings: features – cont.

- \( >^c_{\langle R,r \rangle} \) does not satisfy **Maximality**. E.g.:

  **Suppose** \( R = \{ p_1, \cdots p_3 \} \),
  where \( p_1 = [0, 1], p_2 = [2, 3], p_3 = [5, 6] \).
  Then for \( x = 2 \) and \( y = 3 \), \( x, y \in p_2 \),
  but \( y >^c_{\langle R, p_2 \rangle} x \).

  **Suppose** \( R = \{ q_1, \cdots q_3 \} \),
  where \( q_1 = [0, 2] \times [0, 1], q_2 = [3, 5] \times [0, 1], q_3 = [0, 2] \times [2, 3] \).
  Then for \( x = \{2, 0\} \) and \( y = \{1, 1\} \), \( x, y \in q_2 \),
  but \( y >^c_{\langle R, q_2 \rangle} x \).
The collated Voronoi tessellation method in Douven et al. [2009], which accommodates prototype areas, can be furthermore generalised for graded cases of categorisation.

Gärdenfors’ convexity criterion P for natural properties may be recovered in terms of the convexity criteria C1 and C2 for order filters.

C1 and C2 supply even more sufficient means of motivating a generalisation of the convexity criterion P for graded categorisation.

The criteria C1 and C2 are logically independent from P, and they have intuitive force of their own.

Food for thought: More general models which still have some psychological reality (concepts more than one domain; doing without prototypes; doing without geometric criteria in the first instance).
References


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References – cont.’

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References – cont.”

Thank you!