Applied Conformal Bootstrap

論文の内容の要旨

論文題目　Applied Conformal Bootstrap
(応用共形ブートストラップ)

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In this thesis, I will discuss the application of the bootstrap program to conformal field theories (CFTs) in $d = 3$ Euclid space-time dimensions, with particular emphasis on the hypothetical models with $O(n)\times O(2)$-symmetry, the existence of which have been somewhat controversial.

These hypothetical CFTs correspond to the scale invariant fixed points, if any, of $O(n) \times O(2)$-symmetric Landau-Ginzburg-Wilson (LGW) models formulated in $d = 3$ Euclid space-time, and are candidates for the thermal effective theories of various physical systems at their criticality. These models are of great physical interest, including geometrically frustrated spin systems on triangular lattice and massless two-flavor quantum chromodynamics in the scenario where the axial $U(1)_A$ is restored above the chiral phase transition temperature. The phenomenological relevance of these hypothetical CFTs then follows: if they are absent, the corresponding systems must undergo discontinuous (first order) phase transitions, while the presence allows continuous transition with the prediction of universal critical exponents.

Despite their importance, however, these models are notoriously hard to analyze. The serious problem is that the answer is method-dependent. Notwithstanding their common origin, while ε-expansion based scheme for dealing with β-functions and functional renormalization group analysis predict the absence of these CFTs (first order transitions), resummed β-functions indicate the presence of such models.

We employ a technique called the *conformal bootstrap program* to study CFTs. The benefit of the method is the rigorous bounds (with easily controllable errors) on the parameters (including various critical exponents)

characterizing CFTs, and these bounds must be met by all the unitary CFTs. What is quite intriguing is that these numerically derived bounds often seem to be saturated by the actual models, which have been located so far by other methods, with characteristic behaviors called "kinks". The models cornered in this way include nontrivial ones in $d = 3$ dimensions, like the Ising, XY, Heisenberg models and their $O(n)$ descendants.

Given the success of the bootstrap program for these simple LGW-models, we here carry out the bootstrap studies for $O(n) \times O(2)$-symmetric general CFTs following the state of the art bootstrap technologies to obtain any information about the controversial fixed points. We numerically compute the bounds for various operators contained in these models, and it will turn out that some of these bounds too are strong enough to be saturated by the hypothetical CFTs stated above, with some characteristic behaviors observed as in the previous examples of simple LGW models. We will also provide non-trivial checks for our scenario, i.e., the saturation of the bootstrap bounds by these hypothetical CFTs, existence of which we are led to believe in.

As a by-product, we will simply argue that the ramification of $U(1)_A$-restoration scenarios are required to conclude the order of massless two-flavor QCD chiral phase transition.