Nonparametric Regression for Complex Data

その他のタイトル	複雑データのためのノンパラメトリック回帰
学位授与年月日	2017-03-23
URL	http://doi.org/10.15083/00075513

論文の内容の要旨

論文題目　　Nonparametric Regression for Complex Data
（複雑データのためのノンパラメトリック回帰）

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A purpose of this thesis is to investigate and propose a methodology for analyzing the complex data using the nonparametric statistics.

The complex data appear in various application fields with modern data science, such as medical analysis and finance data analysis. A methodology for the complex data is still a developing problems. Among several types of the complex data, we mainly focus on the tensor data and the functional data which are typical cases of the complex data.

The approach with nonparametric method is the statistical methodology which allows the statistical models to have an infinite dimensional parameter. Since the nonparametric statistics can reduce the bias from the model misspecification problem, the approach with the nonparametric method is intensively studied. However, it is known that the performance of the nonparametric methods is bad or unknown, since the highly complicated structure of the complex data.

To propose the framework with the nonparametric statistics for the complex data, we mainly work on evaluating and reducing the complexity of the data. Namely, we define an estimator from a less complex hypothesis set to improve the speed of convergence of the estimator, while preserving the misspecification bias small. Especially, we focus on the smoothness property of the complex data or the statistical model. Smoothness often appears in the real data, and it is relatively easy for the statisticians to evaluate the effect theoretically. Our theoretical evaluation shows that the accuracy is improved by the complexity reduction, and experimental results guarantee the theoretical claim.

From Chapter 2 to Chapter 5, we provide some frameworks with the nonparametric statistics for the tensor data and the functional data. Rigorously, Chapter 2 and Chapter 3 propose nonparametric methods for the tensor data, and Chapter 4 and Chapter 5 investigate the nonparametric frameworks for the functional data.

Chapter 2 “Doubly Decomposing Nonparametric Tensor Regression” proposes a non- parametric extension of tensor regression. Nonlinearity in a high-dimensional tensor space is broken into simple local functions by incorporating low-rank tensor decomposition. Compared to naive nonparametric approaches, our formulation considerably improves the convergence rate of estimation while maintaining consistency with the same function class under specific conditions. To estimate local functions, we develop a Bayesian estimator with the Gaussian process prior. Experimental results show its theoretical properties and high performance in terms of predicting a summary statistic of a real complex network.

Chapter 3 “Tensor Decomposition with Smoothness” suggests a new tensor decom- position method with a technique with the nonparametric statistics. Many data in the real world, an observed tensor often has a special property that the adjacent elements are similar or smoothly changing. In Chapter 3, we propose a smoothed Tucker decomposition (STD) that incorporates the smooth property. STD models the smoothness of tensors by a small number of basis functions. By the modelling, STD converts the object of the tensor decomposition into a smaller coefficient tensor. The objective of STD is formulated as a convex problem and, to solve that, an algorithm based on the alternating direction method of multipliers is derived. We theoretically show that, under the smoothness assumption, STD achieves a better error bound by the reduction of the object tensor. The theoretical result and performances of STD are numerically verified.

Chapter 4 “PCA-based Functional Linear Regression with Functional Responses” studies a regression model where both predictor and response variables are random functions. We consider a functional linear model where the conditional mean of the response variable at each time point is given by a linear functional of the predictor variable. The problem is then estimation of the integral kernel $b(s, t)$ of the conditional expectation operator, where s is an output variable while t is a variable that interacts with the predictor variable. This problem is an ill-posed inverse problem, and we consider estimators based on the functional principal component analysis (PCA). We show that under suitable regularity conditions, an estimator based on single truncation attains the convergence rate for the integrated squared error that is characterized by smoothness of the function $b(s, t)$ in t together with the the decay rate of the eigenvalues of the covariance operator, but the rate does not depend on smoothness of $b(s,t)$ in s. This rate is shown to be minimax optimal, and consequently smoothness of $b(s,t)$ in s does not affect difficulty of estimating b. We also consider an alternative estimator based on double truncation, and provide conditions under which the alternative estimator attains the optimal rate. We conduct simulations to verify the performance of PCA-based estimators in the finite sample. Finally, we apply our estimators to investigate the relation between the lifetime pattern of working hours and total income.

In Chapter 5 “Nonlinear Functional Regression with Functional Responses with Derivative Estimation”, a regression model with functional covariates and functional responses is studied. We are interested in the regression model which is possibly nonlinear. Some methods for the nonlinear functional regression model are suggested, however, theoretical properties of the regression is still a developing problem. To clarify the properties for the nonlinear functional regression, we assume that the operator of the regression model has the Frechet derivative and the derivative is a compact operator. With the assumption, we propose an estimator for the regression operator based on the fundamental theorem of calculus. and the functional principal component analysis. Our theoretical analysis shows that a property of the kernel function of the Frechet derivative determines the convergence rate, and the rate is shown to be minimax optimal. Experimentally, the theoretical result is validated, and its prediction performance of the proposed estimator is comparable to the existing nonlinear methods.