論文の内容の要旨

Theoretical study on a renormalization-group limit cycle in Efimov physics
(エフィモフ物理における繰り込み群のリミットサイクルの理論的研究)

氏名　堀之内裕理

In this thesis, we address a relationship between renormalization-group flows and universalities in quantum few-body physics. Universality in physics often refers to a situation in which apparently different physical systems exhibit the same low-energy behavior. A prominent example is the critical phenomena in which the same power-law singularity of observables is shared by microscopically different systems. A modern understanding of the universality is given by Wilson's renormalization group (RG), which investigates how a given Hamiltonian changes under a continuous coarse-graining RG transformation. In particular, the universal nature of critical phenomena is described by an attraction of different RG flows onto a fixed point of the RG transformation. In general, the universal nature of a phenomenon is described by an attraction of RG flows onto a small sub-theory space, which is called a renormalized trajectory by Wilson and Kogut, since the attraction signifies that microscopically distinct Hamiltonians will arrive at the same Hamiltonian by the RG transformation to exhibit the same low-energy behavior. The general motivation of this thesis is to reveal a relationship between a renormalized trajectory and universality in quantum few-body physics.

A renowned universal phenomenon in quantum few-body physics is the Efimov effect, in which three identical bosons with a resonant interaction form an infinite series of three-body self-similar bound states. In particular, the self-similarity leads to a discrete-scale-invariant energy spectrum of Efimov trimers with a constant scaling factor of 22.694 at the unitarity limit where the scattering length diverges. Compared with ordinary critical phenomena, the Efimov effect exhibits, due to the discrete scale invariance, an RG limit-cycle behavior which refers to a periodic RG flow. Due to its universality and uniqueness, much theoretical effort has been devoted to reveal an emergence of the Efimov effect in a wide range of systems including nucleons, atoms, and magnons. Experimentally, since a resonant interaction is rare in nature, an observation of the Efimov effect had been elusive until a major breakthrough achieved in ultracold atoms, in which the scattering length can be tuned to its resonance via a magnetic Feshbach resonance. By making use of the high tunability of the interaction strength, a signature of the Efimov effect was observed from an enhanced three-body inelastic scattering due to the existence of the trimer states.

The experimental observation has provided a renewed interest for the Efimov physics. In particular, motivated by the experiment, universality of the Efimov effect in various extended systems, such as dipoles and particles in mixed dimensions, has been investigated recently. One of them is a fundamental question of an N-body extension of the Efimov effect, which investigates the phenomena when N particles interact resonantly. Among all, a four-body extension of the Efimov effect has been energetically investigated to reveal universal nature of four identical bosons with a resonant interaction. In particular, a calculation by von Stecher et al. of the four-body Schrödinger equation has revealed the existence of two tetramers accompanying every Efimov trimer, as originally conjectured by Bedaque et al. by an effective field-theory technique. Furthermore, the energies of the tetramer states are found to be universally related to that of an Efimov trimer. Experimentally, signatures of the two tetramers have been observed in ultracold cesium atoms by measuring an enhanced inelastic four-body scattering due to the existence of the two tetramers. Contrary to the established universality in four-body physics, there is a remaining a major missing link in this subject: the RG counterpart for the four-body universality. In particular, it has been elusive whether or not the RG limit cycle emerging in the three-body Efimov effect describes the four-body universality. In this thesis, we therefore address the relation between the RG limit cycle and the universality in four-body physics by performing a non-perturbative RG calculation of the three- and the four-body physics.

To investigate the RG counterpart of the four-body universality, we first address the question of what is the renormalized trajectory that represents the universality of the Efimov effect. Although there is an expectation that the RG limit cycle is a renormalized trajectory, there has been no evidence due to the lack of computational methods. To verify the naïve expectation in an unbiased manner, we perform an exact RG calculation to demonstrate that the RG flows of microscopically different systems are attracted to the RG limit cycle. To this end, we employ the functional renormalization-group (FRG) method together with a separable approximation of realistic interaction potentials, where the separable approximation was applied to the Efimov physics by Naidon et al. and reproduced trimer energies for various realistic interparticle interactions within 10% deviations. Concerning the one- and the two-body sectors, we demonstrate that the one- and the two-body system parameters will arrive universally at a zero-dimensional sub-theory space at the unitarity limit. Concerning the three-body sector, we demonstrate that the RG flows of a three-body coupling constant universally exhibits the RG limit-cycle behavior at sufficiently low energy, while at high energy, the RG flows behave individually depending on the short-range details of each system. The results provide a numerical evidence that the renormalized trajectory of the Efimov effect is the RG limit cycle since the results suggests that the one-, two-, and three-body coupling constants will arrive at a one-dimensional RG limit cycle at sufficiently low energy.

Based on these results, we then investigate the relationship between the RG limit cycle and the universality in four-body physics. Since the exact four-body FRG calculation is believed to be impossible due to a large number of degrees of freedom, we first develop a simple effective field theory that exactly reproduces the three-body Efimov effect. In the effective field theory, we reduce the Yukawa-type interaction among two particles and a dimerized molecule in the effective field theory of Bedaque et al. to a particle-exchange interaction between a particle and a dimer. We show first that the reduction does not alter the universal observables of the two- and the three-body sectors. Based on the FRG equation of the effective field theory, we then deal with the four-body sector by developing a simple approximation to the three-body-sub amplitude in the entire four-body scattering process. In the approximation, which we call the separable pole approximation, the position and the residue of the three-body bound-state pole of the three-body sub amplitude are respected since the pole structure of the sub amplitude often dominates the loop momentum integral in the four-body scattering process. Based on the approximation, we numerically obtain for the first time the RG limit cycle of the four-body coupling constant which reproduces a qualitative aspects of the four-body universality. In particular, the obtained RG limit cycle is found to reproduce the numbers and the order of the energies of the tetramers accompanying every Efimov trimer. To establish the relationship between the RG limit cycle and the four-body universality, we then systematically improve the separable pole approximation by making use of the Hilbert-Schmidt expansion of a self-adjoint operator. The improved approximation leads to the RG limit cycle of the four-body coupling constant which reproduces the energies of the tetramers within 7%. With the results, we demonstrate that the RG limit cycle is closely connected to the four-body universality and, in particular, that the RG limit cycle indeed contains the information of the observables of the four-body physics. Based on the results, we make a conjecture that the universal one-to-two ratio of the numbers of the trimers and the tetramers has a topological origin in terms of the RG limit cycle. Namely, if we compactify the two-dimensional theory space of the three- and the four-body coupling constants to make it a torus, the RG limit cycle forms a closed loop which winds onto a torus once in the direction of the three-body coupling constant, while it winds twice in the direction of the four-body coupling constant. To establish the conjecture, however, we should justify the compactification of the theory space, which remains an outstanding issue.

We organize the thesis as follows. First in Chap. 1, we discuss our motivation of relating a renormalized trajectory and universal observables in the Efimov physics. A brief history of the Efimov physics is also reviewed to put this work in a proper historical context. In Chap. 2, the Efimov effect in ultracold atoms is reviewed in detail to summarize the current status of the subject. We first review Efimov's original discussion on identical bosons together with the emergence of an RG limit cycle in the three-body sector. An experimental observation of the Efimov effect is then reviewed before we review the universality of the Efimov effect in extended systems such as mass-imbalanced systems and particles in mixed dimensions. We finally review an *N*-body extension of the Efimov effect by focusing on the four-body extension. In Chap. 3, we review the FRG method by following the convenient formalism developed by Wetterich. A formulation suitable for the quantum few-body physics is then presented to deal with *N*-body correlations separately for different *N*. In Chap. 4, we discuss the renormalized trajectory in Efimov physics. In particular, we provide a numerical evidence that the renormalized trajectory is the RG limit cycle by demonstrating that the RG flows universally exhibit RG limit-cycle behavior at low energy. An RG counterpart of the one- and two-body universality is also presented in some detail. In Chap. 5, we investigate the relation between the RG limit cycle and the four-body universality. By developing a simple effective field theory and a separable pole approximation, we first obtain an RG limit cycle which reproduces the four-body universality qualitatively. We then systematically improve the separable pole approximation and obtain the RG limit cycle which reproduces the four-body universality quantitatively. A conjecture concerning the limit-cycle topology is also made with a discussion about its justification.

In conclusion, we investigate a close relationship between the RG limit cycle and universality in quantum few-body physics. In particular, we provide a numerical evidence that the RG limit cycle is the renormalized trajectory in Efimov physics and connects a major missing link in the subject of Efimov physics by obtaining the RG limit cycle that is found to be closely connected to the four-body universality. As an outlook, we consider to justify the conjecture concerning the limit-cycle topology. To this end, we will investigate the stability of the topology against a small perturbation such as a small variation of dimension and we will justify the compactification of the theory space. An investigation of a topological phase transition concerning the limit-cycle topology may be an interesting subject which merits further investigation.