Algorithmic Studies on Online Knapsack and Related Problems

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論文の内容の要旨

Algorithmic Studies on Online Knapsack and Related Problems

(オンラインナップサックと関連する諸問題に対するアルゴリズム論的研究)

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In classical computational problems such as optimization problems and search problems, we are given entire input at one time, and we then compute a solution for the problem. However, in many practical applications such as routing in communications network, job allocation, and stock trading, we need to choose an action in each step based the current information without knowing the full information which will be completely obtained in the future. Such a problem is called an *online problem* and an algorithm for the problem is called an *online algorithm*. In contrast, a problem with full information on the input is called an *offline problem* and an algorithm for the problem is called an *offline algorithm*. Online algorithms are a natural topic of interest in many fields such as information science, operations research, economics, and learning theory.

Since an online algorithm is forced to make decisions without knowing the entire input, they may later turn out not to be optimal. The quality of an online algorithm is measured by the *competitive ratio*, which is the worst ratio between its performance and an optimal offline algorithm's performance.

The main topic of this thesis is *online knapsack problem*, i.e., online version of the knapsack problem. The *knapsack problem* is one of the most fundamental problems in the field of combinatorial optimization and has a lot of applications in the real world. The knapsack problem is that: given a set of items with values and sizes, we are asked to maximize the total value of selected items in the knapsack satisfying the capacity constraint.

In the online setting of the knapsack problem, the information of the input (i.e., the items) is given gradually, i.e., after a decision is made on the current item, the next item is given. The decisions we have made are irrevocable, i.e., once a decision has been made, it cannot be changed.

In particular, we focus on removable version, i.e., when an item is put into the knapsack, some items in the knapsack are removed if the sum of the sizes of the item and the total size in the current knapsack

exceeds the capacity of the knapsack. It may need some cost to remove some items. Removable online problem with removal cost is studied under the name of *buyback problem*. The problem is motivated by the following real scenario in selling advertisements online. A seller allocates a limited inventory to a sequence of potential buyers. The buyers arrive sequentially, submit bids at their arrival time, and the seller must immediately decide to sell or not for their bid. The seller can cancel earlier allocation decision with some cost. Examples of cancellation costs are compensatory payment, paperwork cost, and shipping charge. Compensatory payment is usually constant rate of the value of canceled bids. On the other hand, paperwork cost and shipping charge usually do not depend on bids values but the number of cancellations.

In this thesis, we provide algorithms for these online problems and conduct competitive analysis. One of the main results of this thesis is on the buyback problem under an unweighted knapsack constraint, where the knapsack problem is called unweighted if the value of each item is proportional to its size. We provide an optimal competitive algorithm for the problem.

Moreover, we introduce optimal composition ordering problem. The input is a set of real-valued functions f_i and a real number c. In maximum total order setting, our goal is to find a composition ordering which maximizes the value of composite function of c. We present a polynomial time algorithm for the problem when all the functions are monotone increasing and linear. We also prove that the problem is NP-hard even if the functions are monotone increasing, convex, and at most 2-piece piecewise linear.