Signal Extraction in Economics

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博士論文

Signal Extraction in Economics
（経済学におけるシグナル抽出）

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Contents

Chapter 1

Introduction (In Japanese)

本論文は経済主体が様々な経済環境においてシグナルを発し、また抽出する行動を理論的に分析したものである。分析対象とする経済主体は企業、消費者、募金者、政治家と投票者であり、分析対象とする経済環境は、企業の独占下での時間非整合性と、経済主体間での互いのタイプに関する情報の非対称性である。

時間非整合性とは、Kydland and Prescott(1977) などによって分析された、動学的環境で経済主体が裁量的意思決定を行う場合に発生する非効率性を指す。現在時点において自らの将来の行動をアナウンスする場合、将来時点に至ってこの行動から自ら逸脱するインセンティブがあれば、現在時点でのこのアナウンスは時間非整合的であり、達成不可能となる。市場において企業が、将来の自らの生産量や価格に関して裁量的に行動する場合、時間非整合性に直面することになる。第 1 章では独占企業が時間非整合性に直面している状況を分析し、prize promotion がその解決法になることを証明している。

情報の非対称性とは、経済主体が私的情報を持つことにより、経済主体間で保有する情報に差がある場合を指す。このような状況では、情報が対称的な場合に比べて、社会的な非効率性が発生することになる。この情報の非対称性に対処するため、私的情報を保有する経済主体は、他の経済主体にその私的情報を自ら開示する。これが Spence (1973) などによって分析されたシグナリングである。第 2 章では情報

の非対称性の下で募金者が社会的イメージを含めた自らの利得を最大化するためにシグナリングを行う状況を分析する。第 3 章では同じく非対称情報の下で政治家が自らのタイプが投票者の意向により近いことを示すためにシグナリングを行う状況を分析する。

最後に、第 4 章では家計調査のデータを用いて日本経済の物価・インフレ率に関する実証分析を行っている。以下では各章の概要を記す。

Chapter 1: Prize Promotions as Costless Commitment

本章では独占企業が時間非整合性に直面した状況を分析している。独占企業が将来の財の供給量・価格付けを約束（コミット）できず、裁量的に行動している状況では、企業にとって最大利潤を達成できない場合がある。商品に「くじ」を付ける販売戦略すなわち prize promotion（例えば、購入者の中から抽選で 1000 人に 1 万円が当たるキャンペーン）を行うことで、企業はこのような状況を解決できることを理論的に証明した。すなわち、prize promotion は将来の供給量・価格付けを約束することに等しくなることを示した。

静学的に考えると、くじを付けることは企業にとっても消費者にとっても単なる値下げ以上の意味はない。しかし、理論の核はくじの動学的構造にある。例えば、ある売り手が 2 期間の間くじを販売しているとする。1 期目の時点でくじの賞金額は決まっているが、1 期と 2 期でのくじの合計配布枚数が決まっていない。このとき、2 期目には売り手はくじの配布枚数を増やすインセンティブがある。なぜなら、2 期目にくじの配布枚数を増やすと、2 期目により当選者が生まれやすくなるという意味で 2 期目のくじの価値・売り上げが増える一方、くじのコストである賞金額は一定で変わらないからである。別の言い方をすれば、2 期目に配布枚数を増やすことで、1 期目から 2 期目にくじの価値を移転していることになる。

このような動学的性質を持つくじを、例えば、商品 1 個あたり 1 枚づつ「おまけ」として付けると、くじのついた商品自体の供給を増やそうとするインセンティ

ブが働くことになる。つまり、prize promotion をアナウンスすると、将来の商品の拡大的供給または価格の下落をコミットすることに等しくなる。他のコミットメントの方法（例えば余剰設備投資など）はコストがかかるのに対して、この方法では理論的にコストゼロでコミットメントができ、企業がファーストベストの最大利潤を達成できることを示した。

　本章の貢献としては、まず、prize promotion の新たな役割を提示したこと、また、既存の研究で時間非整合性が問題となっているネットワーク外部性、参入障壁や経験財市場など、様々な文脈で prize promotion がこれまで提案されてきた他の方法より有効かつ最適なコミットメントとして働くことを示したことが挙げられる。

Chapter 2:　Anonymous Giving as a Vice: an Application of Image Motivation

　本章では、個人の利他性に関する情報の非対称性の下で募金者達がシグナリングを行う状況を考え、匿名募金の影響を分析している。Benabou and Tirole (2006) で提示された行動経済学的な image motivation を人々が持っているときに、記名ではなく匿名で募金することは、「陰徳の美」という言葉とは逆に、他人の募金額を減らしてしまい、結果社会全体の募金額を減少させることを理論的に示した。政策的含意として、募金を募る場合、匿名募金者の数を誇張することは望ましくないということを導いた。特にインターネット募金の Web サイトの構造に関して、非効率を生じさせている可能性があることを指摘した。

本研究では、他人から利他的と思われることが自分の効用になるような image motivation を考えている。情報は非対称的で、人々はある人がどれだけ利他的かを、その人の募金行動から予想する。このときシグナリングの状況になるが、重要なのは、実際には利他的ではなくとも、募金する人々がいることである。本来利他的ではないが、社会的イメージが気になる個人は、募金から直接効用を得ることはない

が、そのような彼らの本性は社会的イメージが良くないので、募金をすることによって利他的な人のふり（真似）をする。このようないわば「偽善的行為」は募金総額を大きくしたい資金調達者・NPO にとっては良いことで、いかにしてこのような社会的イメージを気にする人々を募金に導くかということが大切なことになる。ではこのような状況で、他人には見えない匿名募金は誰が行うかというと、今度は逆に、利他的だが自分のイメージを気にしない、いわば pure altruist である (彼らにとって記名・匿名は無差別なはず)。彼らのうち匿名募金を選ぶ割合が増えれば、以下の 2 つの効果が生まれる (1)「偽善者」が真似をしてもあまり彼ら自身の「イメージ」が上がらず (decrease effect)、しかも、(2) 全く募金をしてないように見える人々の中にも匿名募金者がいるかもしれない、という予想は募金をしない人々のイメージを上げてしまう (blend effect)。結果、「偽善者」が募金をするインセンティブを削いでしまう。

本章の貢献としては、まず、Benabou and Tirole (2006) と異なり、image motivation が heterogenous な場合を考察していることが挙げられる。それ故に、「誰が匿名募金をするのか」また、「偽善者」の行動の重要性をはっきりさせることができた。また、これまで外生的に匿名性があたえられた場合の実験的研究はあるが、それらの研究では問題とされなかった、募金者自身自身が匿名性を望んだ場合に、募金者本人以外に負の効果が及ぶことを初めて指摘した研究であると言える。

Chapter 3: A Signaling Explanation for Political Parties and Advertisements

本章では、政治家のタイプに関する情報の非対称性の下で、政治家が投票者に対してシグナリングを行う状況を分析している。Snyder and Ting (2002) のモデル、すなわち政治家が政党に所属することが政治家の隠れたタイプのシグナルとして機能する状況を、選挙が複数回ある場合に応用した理論的研究である。

複数回の選挙がある場合、第一期に当選したとしても、在任期間中に実際の政治家のタイプが投票者に明らかになってしまうため、第二期の選挙では、より投票者の意向に近いタイプの政治家のみ再選が可能である。ここで再選可能な政治家は第一期選挙の時点で、自分と再選不可能な政治家を差別化するインセンティブが出てくる。このため政治家はさらなるシグナルとして追加的に選挙広告を用いると結論付けた。この政党と政治広告があることで投票者ははじめから再選可能な、自身の意向に沿った政治家のみを選挙で選ぶことができる。また、選挙広告は、informative なものでなくとも、純粋に費用の浪費であっても機能することになる。

本章の貢献としては、Snyder and Ting (2002) のモデルを複数回選挙のある動学的状況に応用したこと、また選挙広告の役割と意義に関して、シグナリングの観点から肯定的な結論を導いたことが挙げられる。

Chapter 4: Measuring Quality Changes for Consumer Goods through Quality Engel Curves in the Japanese Economy

本章では日本経済の物価・インフレ率の測定について実証的に研究している。CPI 等の物価指数ではインフレ率の推定を行っているが、それにどの程度の誤差があるのかを従来のヘドニック・アプローチではなく、Bils and Klenow (2001) に基づいた新しい方法で日本経済のデータを用いて計測した。

「粗インフレ率」=「純粋インフレ率」+「品質向上」だが、右辺を分けて観測することはできない。そこで、鍵となるアイデアは、「品質向上」とは相関しているが、「純粋インフレ率」とは相関しない変数を使って、この両者を分離することである。このような特徴を持つ変数として「品質エンゲル曲線」の傾きを用いた。

ある商品の品質が向上すると、はじめにその高品質かつ高価格の商品を購入するのは所得の高い人のことが多い。すると高所得者と低所得者の間で、購入する商

品の価格差が大きくなる。この所得と商品価格の関係を「品質エンゲル曲線」と呼ぶと、以上の議論より、品質の向上は「品質エンゲル曲線」の傾きと相関を持つことになる。この「品質エンゲル曲線」の傾きを財ごとに測定し、それを操作変数として「純粋インフレ率」と「品質向上」を分離、計測した。結果として、日本の CPI の計測したインフレ率の中に 59 – 60% ほどの「品質向上」がいまだ含まれており、上方にかなり大きな誤差が存在することが分かった。

本章の貢献としては、日本の家計調査の中で世界的にも大変細かい分類項目がある所得階層ごと、商品ごとの価格、数量データを利用して「品質エンゲル曲線」を推定し、分析したことで信頼性の高い結果を導いたこと、そして現実に CPI の精度にどれくらい誤差があるか測定することに成功したことが挙げられる。

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Chapter 2

Prize Promotions as Costless Commitment

abstract:

This study theoretically shows that prize promotions work as firms' commitments that resolve their various time-inconsistency problems. Prize promotions are the best commitment tool, since they are equivalent to costless credible announcements on the firm's future policy: the firm can implement the first best commitment equilibrium at no cost. The key lies in the dynamic supply of lottery tickets—the seller has an incentive to distribute more tickets in order to increase the number of winners among the current buyers at the expense of the past buyers. If lottery tickets are attached to products (prize promotions), then the firm can credibly commit to future expansion of production. A network externality model is considered as an example.

2.1 Introduction

Prize promotions have become extremely common in recent times. One such example is the ‘USD 100 present for 100 buyers’ campaign. Such prize promotions reflect a firm’s policy of attaching a lottery ticket to each product. However, in a static economic model, attaching a lottery ticket is meaningless for the firm. It is equivalent to price cuts under risk-neutral preferences, since the firm is essentially paying back money to the consumers in the form of lottery tickets. Thus, if attaching lottery tickets does not play a unique role in the firm’s strategy, then prize promotions can be considered as unprofitable. However, this paper shows that, in a dynamic environment, prize promotions not only imply price cuts, but also serve as a commitment tool for resolving time inconsistency between the firm and the consumers.

In brief, prize promotions work as costless credible commitments to the future expansion of production for the following reasons. First, commitments to the future expansion is the result of the dynamic nature of a lottery. By controlling the number of newly-issued lottery tickets, the lottery seller (the firm) can change the value of not only the newly-issued tickets, but also the old tickets. Thus, the seller has an incentive to print more tickets in the future. Second, prize promotions are costless because, in one aspect, they can be regarded as a pricing strategy. As discussed above, even in a static environment, prize promotions are equivalent to price discounts for both the firm and the consumers. Therefore, prize promotions are not an additional cost, but a costless advertisement for the firms.

There are three contributions of this study. First, this study highlights a new economic role of prize promotions. Prize promotions are equivalent to costless credible announcements. Second, it proposes prize promotions as the best method for resolving a firm’s time inconsistency problems in a general model. While various studies suggest solutions for mitigating the time inconsistency problem and achieving the second-best, including excess capacity (Dixit 1980), coupons (Cremer 1984), and second sourcing (Farrell and Gallini 1988), prize promotions lead to the first-best result at no cost. Third, our model can be regarded as a new case where the lottery is profitable under the risk-neutral preference. Our study indicates how and when prize promotions (lottery) are profitable in a dynamic model, without taking into account asymmetric information (for example, Milgrom and Roberts 1986) and any behavioural preference (for example, Chew and Tan 2005).

There are few works on the economic role of lotteries, sweepstakes, and prize promotions. Selby and Beranek (1981) prove that a lottery is unprofitable under risk-neutral and risk-averse preferences. Chew and Tan (2005) analyse the optimal price of a lottery under behavioural preference and show that it is profitable. However, there is no theoretical study on our case where a firm attaches a lottery ticket to its product. We show that such a lottery can be profitable without behavioural preferences. Kotler and Keller (2012) consider prize promotions as mere sales promotions that include coupons, cash refund offers, free trials, and/or product warranties. They suggest that sales promotions '*stimulate quicker or greater purchase of particular products*'. They do not explain why and how prize promotions stimulate product purchases or the difference between prize promotions and price cuts. However, there are some empirical works on lotteries and prize promotions. Narayana and Raju (1985) empirically test the effect of prize promotions on sales. Kalra and Shi (2010) experimentally show how consumers value them.

The remainder of this study is organised as follows. Section 2 describes the general model where time inconsistency arises. In Section 3, we incorporate prize promotions into the model and solve the inconsistency problem. Section 4 interprets the results of our analysis. Section 5 presents a network externality model as an example where prize promotions work well. Section 6 concludes this study.

2.2 General Model of the Firm's Inconsistency

First, we describe the firm's inconsistency problem in a general model. In this study, the meaning of the term 'inconsistency' is as implied by Kydland and Prescott (1977): inconsistency arises when when the sequential decision is not equal to the simultaneous first-best decision. Consider the following two-period sequential problem. In each period, the monopoly firm supplies the goods and the consumers demand them. The utility function and the budget constraint for consumer i is as follows:

$$U^i(x_1{}^i, x_2{}^i) = u^i(x_1{}^i, x_2{}^i) + M^i, \tag{2.1}$$

$$P_1 x_1{}^i + P_2 x_2{}^i + M^i = w^i, \tag{2.2}$$

where $U^i(\cdot)$ denotes consumer i's utility function, $u^i(\cdot)$ denotes i's utility from the good, $x_t{}^i$ denotes i's demand at time t, P_t denotes the price at time t, M^i denotes i's monetary gain, and w^i denotes i's income. Note that the utility function is quasi-linear in money since we are considering prize promotions where the prize is given to consumers in the form of money.

The monopoly firm and consumers face sequential decisions. They do not make both current and future decisions. They make only current decisions. In the model, At $t = 1$, the firm sets P_1, and consumer i decides $x_1{}^i$ given the current price P_1 and the future expected price P_2[1]. At $t = 2$, the firm sets P_2 and consumer i decides $x_2{}^i$ given the current price P_2 and the past variables P_1 and $x_1{}^i$. This sequential problem can be regarded as the case where the firm and consumers can not credibly commit their future decisions. They make discretionary decisions in each time.

Assuming interior solutions, the total demand functions for the good can be written

[1] At the equilibrium, consumers rationally expect future price P_2.

as[2]

$$X_1 = \sum_i x_1{}^i(P_1, P_2) \tag{2.3}$$

$$X_2 = \sum_i x_2{}^i(P_2 \mid x_1{}^i), \tag{2.4}$$

where X_t denotes the total demand at time t. The profit of the monopoly firm is given by

$$\begin{aligned} \pi &= \pi_1 + \pi_2 \\ &= P_1 X_1 - C_1(X_1) + P_2 X_2 - C_2(X_2), \end{aligned} \tag{2.5}$$

where π denotes the total profit, π_t denotes the profit at time t, and $C_t(\cdot)$ denotes the cost function at time t. We assume that π_1 and π_2 are concave functions of X_1 and X_2, respectively.

Next, we analyse the firm's policy. The firm's first-best policy is defined as follows:

Definition (Firm's First-Best Policy)**.** *The firm's first-best policy* $(X_1{}^*, X_2{}^*)$ *solves*

$$\max_{X_1, X_2} \pi. \tag{2.6}$$

Here, the firm maximises the total profit as if it is facing a static problem. This $(X_1{}^*, X_2{}^*)$ can be considered as the *commitment equilibrium* where the firm is able to make a credible commitment (announcement) regarding its current and future decisions. Here, we focus on the decision regarding X_2. For the first-best $X_2{}^*$, the first order condition

[2]The demand does not depend on the past price since the income effect is zero under quasi-linear utility.

(FOC) with respect to X_2 is

$$\frac{\partial \pi}{\partial X_2}({X_1}^*, X_2) = \frac{\partial \pi_1}{\partial X_2}({X_1}^*, X_2) + \frac{\partial \pi_2}{\partial X_2}({X_1}^*, X_2) = 0. \tag{2.7}$$

However, we do not allow the firm to credibly commit to its future decisions. Instead, we consider the case where the firm and the consumers make decisions sequentially. Suppose $X_1 = {X_1}^*$. We focus on the sequential decision regarding X_2 at $t = 2$. The sequential decision solves

$$\max_{X_2} \pi_2, \tag{2.8}$$

For the sequential decision, the FOC with respect to X_2 is

$$\frac{\partial \pi_2}{\partial X_2}({X_1}^*, X_2) = 0. \tag{2.9}$$

Note that the difference between this sequential FOC (2.9) and the first best FOC (2.7) lies in the term $\frac{\partial \pi_1}{\partial X_2}({X_1}^*, X_2)$. In other words, the sequential problem and the first-best problem yield different solutions under the following condition

$$\frac{\partial \pi_1}{\partial X_2}({X_1}^*, {X_2}^*) \neq 0. \tag{2.10}$$

In this case, we say that the first-best policy is the *time inconsistent* policy.

Definition (Time Inconsistency)**.** *If the firm's first-best policy* $({X_1}^*, {X_2}^*)$ *is the same as the solution to the sequential problem, we say that* $({X_1}^*, {X_2}^*)$ *is time consistent. On the other hand, if the firm's first-best policy* $({X_1}^*, {X_2}^*)$ *is not the same as the solution to the* *sequential problem, we say that* $(X_1{}^*, X_2{}^*)$ *is time inconsistent.*

In words, time inconsistency arises when the first-best commitment is not carried out in the sequential decision. Suppose the firm announces the first-best policy $(X_1{}^*, X_2{}^*)$ at $t = 1$. Time inconsistency matters if the firm has an incentive to deviate from the initial announcement $X_2{}^*$ at $t = 2$. In this case, the actual equilibrium is other policy that yields less profit than the first-best policy. In contrast, a *time consistent* policy is one where the firm has no incentive to deviate from the initial announcement; that is, the sequential decision is equal to the first-best.

Time inconsistency arises when the firm cannot credibly commit about the future policy. That is, if there is time inconsistency, then credible commitment is required to implement the first-best policy. In this case, we consider the following question—if credible commitment is available, what kind of credible commitment is required for the first-best policy? The answer depends on the demand function. As seen above, the difference between the first-best FOC (2.7) and the sequential FOC (2.9) is the term $\frac{\partial \pi_1}{\partial X_2}(X_1{}^*, X_2)$ that is rewritten as $\frac{\partial P_1}{\partial X_2}(X_1{}^*, X_2)X_1{}^*$.

Definition (Required Type of Commitment)**.** *If (2.11) holds, then we say that expansionary commitment is required for the first-best.*

$$\frac{\partial \pi_1}{\partial X_2}(X_1{}^*, X_2{}^*) = \frac{\partial P_1}{\partial X_2}(X_1{}^*, X_2{}^*)X_1{}^* > 0, \tag{2.11}$$

Conversely, if (2.12) holds, then we say that contractionary commitment is required for the first-best.

$$\frac{\partial \pi_1}{\partial X_2}(X_1{}^*, X_2{}^*) = \frac{\partial P_1}{\partial X_2}(X_1{}^*, X_2{}^*)X_1{}^* < 0, \tag{2.12}$$

The above definitions are derived from the following. If (2.11) holds, the sequential FOC (2.9) results in a lower X_2 than the first-best FOC (2.7). If (2.12) holds, the sequential FOC (2.9) results in a higher X_2 than the first best FOC (2.7).

In addition, it should be noted that expansionary commitment is equivalent to price reduction commitment. Since the firm is assumed to be a monopoly, quantity expansion implies price declines if the demand functions are decreasing in the current price. Thus, the definition can be restated as follows. *If (2.11) holds, we say that the price reduction commitment is required for the first-best. Conversely, if (2.12) holds, we say that the price rise commitment is required for the first-best.*

This model can be regarded as the generalisation of various other models that consider switching costs (Farrell and Gallini 1988), experience goods (Cremer 1984), and network externality that we discuss in Section 5. For instance, consider a model of switching costs. At times, consumers have to pay switching or adoption costs for using goods; for example, printer/ink or elevator/maintenance fees. Consumers purchase goods at both $t = 1$ and $t = 2$. However, at $t = 1$, they have to additionally pay huge switching (adoption) costs to use and derive utility from the goods. Switching costs serve as entrance fees for the goods. Consumers' entry decision at $t = 1$ clearly depends on the future pricing of the goods by the firm. If consumers expect that the firm will raise prices in the future, then they will not pay entrance fees (switching costs) and not purchase the goods. This situation corresponds to 2.11, and hence, expansionary (price reduction) commitment is required for the first-best. In Section 5, network externality is considered as an example.

2.3 Prize Promotions: A Solution to Inconsistency

We have showed that, the first best is not carried out in the sequential decision. However, in reality, many firms faces the sequential decisions and can not make credible commitments to their future policies. For instance, the firm's announcements themselves are costly, makers are unable to decide on the future retail price, or the announcement can easily be deviated. Then, we suggest prize promotions as a commitment tool in the sequential decision. Though prize promotions are not simple announcements on the surface, they work in the same manner as costless credible announcements.

The firm attaches a lottery ticket to each product at both $t = 1$ and $t = 2$. The lottery prize is money and the total prize amount is announced and fixed at the beginning of $t = 1$[3]. At the end of $t = 2$, one or more winning tickets are randomly selected by draw. It should be noted that the winning tickets are selected from among all the tickets supplied in both time periods. For simplicity, we also assume that the consumers are atomless—for each consumer i, the aggregate variable $X_t \equiv \sum_i {x_t}^i$ is given. The aggregate demand for the good is determined by

$$X_1 = \sum_i {x_1}^i\Big(P_1 - \frac{Prize}{X_1 + X_2},\ P_2 - \frac{Prize}{X_1 + X_2}\Big) \tag{2.13}$$

$$X_2 = \sum_i {x_2}^i\Big(P_2 - \frac{Prize}{X_1 + X_2}\ \Big|\ {x_1}^i\Big), \tag{2.14}$$

where *Prize* denotes the total prize amount. Because $\frac{Prize}{X_1+X_2}$ represents the value of one lottery ticket, $P_t - \frac{Prize}{X_1+X_2}$ can be interpreted as the real price of the good itself. The firm's

[3]We implicitly assume that the firm can announce and commit the prize amount at $t = 1$. This assumption is reasonable because the deviation from the announcement violates commercial laws in many countries.

profit is

$$\begin{aligned}\pi &= \pi_1 + \pi_2 - Prize \\ &= P_1X_1 - C_1(X_1) + P_2X_2 - C_2(X_2) - Prize. \qquad (2.15)\end{aligned}$$

Note that *Prize* is a cost for the firm since it has to pay *Prize* to the consumers at the end of $t = 2$. Here, we derive the main proposition of this paper.

Proposition (Costless Commitment by Prize Promotions)**.** *Consider a two-period sequential model where*

(i) the atomless consumers' demand functions are (2.3) and (2.4),

(ii) the monopoly firm's profit function is 2.5. π_1 *and* π_2 *are concave functions of* X_1 *and* X_2*.*

If expansionary (price reduction) commitment is required for time inconsistent first-best ${X_1}^*$ *and* ${X_2}^*$*, the following firm policy makes* ${X_1}^*$ *and* ${X_2}^*$ *time consistent and yields the first-best profit.*

$$\begin{cases} P_1 = P_1({X_1}^*, {X_2}^* | Prize = 0) + \dfrac{Prize}{{X_1}^* + {X_2}^*}, & (2.16) \\ P_2 = P_2({X_1}^*, {X_2}^* | Prize = 0) + \dfrac{Prize}{{X_1}^* + {X_2}^*}, & (2.17) \\ Prize = ({X_1}^* + {X_2}^*)^2 \dfrac{\partial P_1}{\partial X_2}({X_1}^*, {X_2}^*), & (2.18) \end{cases}$$

where $P_t(X_1, X_2 | Prize = 0)$ *denotes the inverse demand function at time t when* $Prize = 0$.

Proof. Let $P_1(X_1, X_2|Prize = 0)$ and $P_2(X_1, X_2|Prize = 0)$ denote the inverse demand functions at $t = 1$ and $t = 2$ when $Prize = 0$. Thus,

$$P_1(X_1, X_2) = P_1(X_1, X_2|Prize = 0) + \frac{Prize}{X_1 + X_2}, \tag{2.19}$$

$$P_2(X_1, X_2) = P_2(X_1, X_2|Prize = 0) + \frac{Prize}{X_1 + X_2} \tag{2.20}$$

are the inverse demand functions when $Prize > 0$ because the pricing policy $(P_1, P_2) = (P_1(X_1, X_2), P_2(X_1, X_2))$ clearly yields demand X_1 and X_2.

Next, let us check for consistency. Suppose that $X_1 = X_1{}^*$. The first-best FOC (2.7) can be written as

$$\begin{aligned}\frac{\partial \pi}{\partial X_2}(X_1{}^*, X_2) &= \frac{\partial \pi_1}{\partial X_2}(X_1{}^*, X_2) + \frac{\partial \pi_2}{\partial X_2}(X_1{}^*, X_2)\\ &= \frac{\partial P_1(X_1{}^*, X_2|Prize = 0)X_1{}^* - C_1(X_1{}^*)}{\partial X_2} + \frac{\partial P_2(X_1{}^*, X_2|Prize = 0)X_2 - C_2(X_2)}{\partial X_2}\\ &= 0. \end{aligned} \tag{2.21}$$

When $Prize = 0$, the sequential FOC (2.9) is $\frac{\partial \pi_2}{\partial X_2}(X_1{}^*, X_2) = \frac{\partial P_2\left(X_1{}^*, X_2|Prize=0\right)X_2 - C_2(X_2)}{\partial X_2} = 0$. However, when $Prize > 0$, the sequential FOC becomes

$$\begin{aligned}\frac{\partial \pi_2}{\partial X_2}(X_1{}^*, X_2) &= \frac{\partial\left(P_2(X_1{}^*, X_2|Prize = 0) + \frac{Prize}{X_1{}^*+X_2}\right)X_2 - C_2(X_2)}{\partial X_2}\\ &= \frac{\partial P_2(X_1{}^*, X_2|Prize = 0)X_2 - C_2(X_2)}{\partial X_2} + \frac{\partial \frac{Prize}{X_1{}^*+X_2}X_2}{\partial X_2}\\ &= 0. \end{aligned} \tag{2.22}$$

By setting $\frac{\partial \frac{Prize}{X_1{}^*+X_2}X_2}{\partial X_2} = \frac{\partial P_1\left(X_1{}^*, X_2|Prize=0\right)X_1{}^* - C_1(X_1{}^*)}{\partial X_2}$ at $X_2 = X_2{}^*$, equivalently,

$$Prize = (X_1{}^* + X_2{}^*)^2 \frac{\partial P_1}{\partial X_2}(X_1{}^*, X_2{}^*), \tag{2.23}$$

the sequential FOC (2.22) is the same as the first best FOC (2.21). Therefore, the firm's policy

$$\begin{cases} P_1 = P_1({X_1}^*, {X_2}^* | Prize = 0) + \dfrac{Prize}{{X_1}^* + {X_2}^*} & (2.24) \\ P_2 = P_2({X_1}^*, {X_2}^* | Prize = 0) + \dfrac{Prize}{{X_1}^* + {X_2}^*} & (2.25) \\ Prize = ({X_1}^* + {X_2}^*)^2 \dfrac{\partial P_1}{\partial X_2}({X_1}^*, {X_2}^*) & (2.26) \end{cases}$$

yields $X_1 = {X_1}^*$ and $X_2 = {X_2}^*$, which is time consistent. Moreover, if $\frac{\partial P_1}{\partial X_2}({X_1}^*, {X_2}^*) > 0$ (defined earlier as 'expansionary commitment is required'), then there exists a positive value of $Prize = ({X_1}^* + {X_2}^*)^2 \frac{\partial P_1}{\partial X_2}({X_1}^*, {X_2}^*)$.

Finally, we consider the profit resulting from the firm's policy discussed above. The total profit does not depend on *Prize*, but on X_1 and X_2 because

$$\begin{aligned} \pi &= \pi_1 + \pi_2 - Prize \\ &= \Big(P_1(X_1, X_2 | Prize = 0) + \frac{Prize}{X_1 + X_2}\Big)X_1 - C(X_1) \\ &\qquad + \Big(P_2(X_1, X_2 | Prize = 0) + \frac{Prize}{X_1 + X_2}\Big)X_2 - C(X_2) - Prize \\ &= P_1(X_1, X_2 | Prize = 0)X_1 - C(X_1) + P_2(X_1, X_2 | Prize = 0)X_2 - C(X_2) \qquad (2.27) \end{aligned}$$

Therefore, the firm's policy discussed above yields the first-best profit. □

This proposition states that by controlling the amount of *Prize*, the firm can bind itself to not deviate from the initial announcement. In addition, the prize promotions are costless since the firm can make the first-best profit. Thus, the prize promotions are equivalent to costless credible announcements. In the next section, we interpret the above proposition.

2.4 Interpretations

If the firm does not use prize promotions, π_2 is simply represented as $\pi_2 = P_2\big(X_1{}^*, X_2 \mid Prize = 0\big)X_2 - C(X_2)$ in the sequential problem. On the other hand, if the firm uses prize promotions (as seen in 2.22), π_2 becomes

$$\pi_2 = \underbrace{P_2\big(X_1{}^*, X_2 \mid Prize = 0\big)X_2 - C(X_2)}_{Prize=0} + \underbrace{\frac{Prize}{X_1{}^* + X_2}X_2}_{Prize>0}. \tag{2.28}$$

In short, if the firm sets $prize > 0$, the last term appears in the maximisation problem at $t = 2$. Since $\frac{Prize}{X_1{}^*+X_2}$ is the value of one lottery ticket, the last term $\frac{Prize}{X_1{}^*+X_2}X_2$ represents **the value of all the tickets held by the t = 2 consumers**.

- Why is this value an increasing function of X_2?

  The higher the number of lottery tickets distributed by the firm at $t = 2$, the higher the probability of winners at $t = 2$ and lower the probability of winners at $t = 1$. Thus, by printing more tickets at $t = 2$, the firm can shift the prize winners from $t = 1$ to $t = 2$.

- Why does this value appear in π_2?

  Irrespective of the number of tickets distributed by the firm at $t = 2$, the prize cost (total prize amount) is considered as a sunk cost since it was already announced and fixed at $t = 1$. Thus, at $t = 2$, the firm considers only the benefit from the prize and not the cost; that is, it considers the value of all the tickets held by the $t = 2$ consumers.

To summarize, at $t = 2$, printing more tickets enables the firm to reward the $t = 2$ consumers more and make greater profits without any additional prize payment. Thus, the firm will expand production along with lottery tickets in the future. The firm, through announcements on prize promotions at $t = 1$, informs the consumers that it will be undertaking production expansion in the future in order to distribute more lottery tickets.

Next, we distinguish between effective and ineffective prize promotions. There are two types of prize promotions:

(i) Fixed Prize Funds.

For example,

- USD 100 for 100 buyers,
- 1 automobile for 1 buyer.

(ii) Variable Prize Funds.

For example,

- USD 100 cashback for 1% of the consumers,
- USD 1 cashback for every purchase.

In the type (i) example of ‘USD 100 for 100 buyers’, the prize funds do not depend on sales and are fixed at USD 10,000. In the type (ii) example of ‘USD 100 cashback for 1% of the consumers’, the prize funds depend on sales. We have showed that type (i) is effective and resolves the inconsistency problem. However, is this true for type (ii) as well? We conclude that type (ii) is ineffective and does not resolve the inconsistency problem for the following reason. At $t = 2$, in order to give more prizes to the current consumers, the firm has to distribute more lottery tickets, which leads to the firm’s additional prize payment. The total prize amount for the latter type increases with the number of tickets. In short, the prize fund is not a sunk cost in this type of prize promotion. Therefore, the firm does not have any incentive to distribute more lottery tickets at $t = 2$.

2.5 Network Externality: An Example

If time inconsistency arises in the general sequential model presented in Section 2, the firm implements the first-best by using prize promotions[4]. In this section, we consider a two-generation network externality model that is largely based on Katz and Shapiro's (1986) model as an example where prize promotions work well.

In our study, network externality refers to cases where each user's utility from the good positively depends on the number of users; for example, telecommunications, software, and credit cards.

The structure of the model is as follows. There are two time periods, two generations of consumers, and one monopoly firm that sells a durable good with network externality. The preferences of atomless consumers are quasi-linear in money. Generation 1 lives from time 1 to time 2. If generation 1 buys goods at $t = 1$, it can derive utility at both $t = 1$ and $t = 2$. Generation 2 enters the economy at $t = 2$. Their respective utility functions are

$$\text{Generation 1} : U^i = u^i(x^i, X_1) + u^i(x^i, X_1 + X_2), \tag{2.29}$$

$$\text{Generation 2} : U^i = u^i(x^i, X_1 + X_2). \tag{2.30}$$

Note that the total amount of goods has a positive effect on each good's owner; that is, $u^i(\cdot)$ is an increasing function of X_1 and $X_1 + X_2$.

[4]It should be noted that the proposition holds as long as the demand functions are (2.3) and (2.4). The consumers' problem is not necessarily a one-generation problem as shown in (2.1) and (2.2).

Thus, the demand functions are[5]

$$X_1 = \sum_i x_1\Big(P_1, X_1, X_1 + X_2\Big), \tag{2.31}$$

$$X_2 = \sum_i x_2\Big(P_2, X_1 + X_2 \;\Big|\; X_1\Big). \tag{2.32}$$

Here, X_2 can be rewritten as $X_2 = X_2(P_2 \mid X_1)$; hence, $X_1 = X_1(P_1, P_2)$. These demand functions take the same form as (2.3) and (2.4). The firm's profit is

$$\begin{aligned} \pi &= \pi_1 + \pi_2 \\ &= P_1X_1 - C_1(X_1) + P_2X_2 - C_2(X_2), \end{aligned} \tag{2.33}$$

which is the same as (2.5). Moreover, from (2.31), it is clear that $\frac{\partial \pi_1}{\partial X_2}({X_1}^*, {X_2}^*) = \frac{\partial P_1}{\partial X_2}({X_1}^*, {X_2}^*){X_1}^* > 0$, which was defined earlier as 'expansionary commitment is required'. Intuitively, for the sequential decision at $t = 2$, the firm has an incentive to deviate from the first-best policy; it decreases the supplies because it does not focus on the positive externality from X_2 to the $t = 1$ consumers. Since all the conditions satisfy the assumptions, we can apply the proposition to this model. Prize promotions work to implement the first-best.

2.6 Concluding Remarks

Considering firms' static problems, theoretically, prize promotions are not profitable as they are equivalent to price cuts. Thus, prize promotions are mere sales promotions. However, we show that, in the context of firms' dynamic problems, prize promotions are profitable since they resolve the firm's time inconsistency problems. It is to be emphasised that prize promotions are the best commitment tool for the firm in the sense that they are

[5] We consider the case in which at the equilibrium, generation 1 buys at $t = 1$ and generation 2 buys at $t = 2$.

equivalent to costless credible announcement for the firm's future policy: the firm can implement the first best commitment equilibrium.

To deal with time inconsistency between firms and consumers, various other ways have been suggested to achieve what is known as second-best. For example, Dixit (1980) suggests excess capacity, while Farrell and Gallini (1988) suggest second sourcing. Excess capacity may be an entry barrier. However, the firm has to make an investment in advance and maintain it for a certain period, which is an additional cost for the firm. Farrell and Gallini (1988) describe second sourcing as '*voluntarily inviting competitors into the market, usually by licensing a product at low royalties or by using an "open architecture"*'. At times, second sourcing can increase profits, but the firm must give up its patents, and hence, monopoly power. They only lead to the second-best that is worse than the first best. Thus, prize promotions are costless and better than these methods if the firm needs expansionary (or price reduction) commitment.

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Chapter 3

Anonymous giving as a vice: an application of image motivation

abstract:

While some donors boast about their giving, others give anonymously. A novel feature of this study is that anonymity is controlled by donors themselves, not by fundraisers. Is anonymous giving really a virtue, as is generally recognised? Paradoxically, this study proves that anonymity is a vice for fundraisers even if it is what donors desire. If many altruists (a good type) switch from known to anonymous giving, it relatively lowers the group reputation of known donors and enhances that of non-donors. These effects dilute the incentive for other individuals to become known donors, if they have psychological 'image motivation'. I suggest a practical method to control the expected number of anonymous donors: fundraisers remove the 'check boxes' from their Web sites.

3.1 Introduction

Many studies, including Andreoni and Petrie (2004), Rege and Telle (2004), and Soetevent (2005), show that if experimenters control the anonymity of examinees, the examinees change their prosocial behaviour. The more anonymity experimenters provide, the less altruistic the examinees become. A novel feature of this study is that anonymity is determined by the donors themselves. While some donors boast about their giving, others give anonymously. Is anonymous giving worth the same as known giving to fundraisers? Is anonymous giving really a virtue, as is generally recognised? Paradoxically, this study proves that anonymous giving is a vice, even if the donors themselves desire anonymity.

We extend Benabou and Tirole's image signalling framework to the case in which 'anonymity' is redefined and individuals themselves choose anonymity level. Since people care about their social reputations, even non-altruists donate to conceal their selfishness and enhance their reputations. The key lies in this 'hypocritical' behaviour of donors. Such behaviour is possible because altruism is private information and the donation amount is what others observe; giving enables non-altruists to mimic altruists and achieve recognition as altruists. However, if we allow donors an option to anonymous giving, it is clear that not all donors choose the same anonymity level. It is essential to identify who donates anonymously, because it determines whether anonymous giving is independent of or related to the hypocritical behaviour. Our result is that those who prefer anonymous giving are the best 'target' for the non-altruists to mimic, and the existence of anonymous donors negatively affects the hypocritical behaviour of non-altruists and the total amount of donation.

Practically, fundraisers should control the expected number of anonymous donors. However, we can find one example that exaggerates the existence of anonymous donors. Currently, many fundraisers solicit contributions via the Internet in addition to normal fundraising activities. On their Web sites, some fundraisers place check boxes for donors to select anonymous or known donations. Figure 3.1 shows how a typical such site looks. On the page, donors fill out not only the information about the amount of donation, name, address, and credit card number but also the check box '*I prefer to make this donation anonymously*'. By providing the check boxes, these fundraisers intend to make it convenient to donate anonymously. The problem, however, is that, when facing the choice, donors expect that fundraisers provided the check box in response to many requests from a substantial number of anonymous donors. In other words, the check box exaggerates the number of anonymous donors.

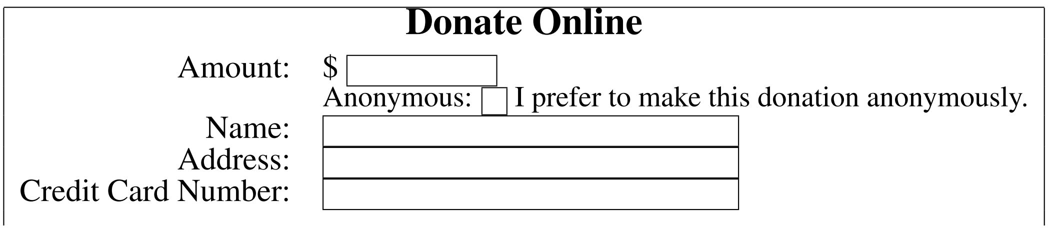

Figure 3.1: Online Donation Page

The contribution of this study is threefold. First and foremost, it suggests how fundraising campaigns can be more efficiently designed: fundraisers should allow anonymous donation only as an exception and should not exaggerate the number of anonymous donors. Specifically, we can identify inefficiency in some fundraising Web sites and suggest an improvement. Such sites can impose a small 'foot cost' on anonymous giving. Second, this is the first study that studies a model in which anonymity is endogenously determined. Here, anonymity is controlled not by the fundraisers but by the donors themselves. Thus, we can study the case in which donors choose perfect anonymity, namely by donating anonymously; almost no economic studies have addressed this case in detail. Third, the study gives weight to the heterogeneity of image motivation among individuals, in contrast to Benabou and Tirole (2006). Thus, we can identify who donates anonymously.

3.2 Structure of the Model

Let us begin our analysis by specifying the utility function of the individuals who have both an image motivation and a warm-glow preference[1]

$$U(c_i, x_i, \hat{\alpha}_i) = (1 - \alpha_i) \ln c_i + \alpha_i \ln x_i + \beta_i \ln I(\hat{\alpha}_i). \tag{3.1}$$

c_i denotes individual i's consumption for private goods, x_i denotes individual i's donation, α_i denotes the altruism parameter, and β_i denotes the self-consciousness of individual i. Individuals are heterogeneous in the parameters α_i and β_i. Because we assume that individual i's parameters α_i and β_i are private information and unobservable to all other individuals, others form beliefs about i's parameters. $\hat{\alpha}_i$ denotes the belief of other individuals about i's altruism, α_i. Individuals derive utility directly from this belief of others, namely, the extent to which others consider an individual altruistic. This corresponds to the concept of self-image or image motivation. The function $I(\cdot)$ represents this image motivation of the individuals. We assume that $I'(\cdot) > 0$ and that $I(0) = 1$. People who are considered socially altruistic have a high $\hat{\alpha}_i$ and hence high utility. The more altruistic they are considered the more utility they gain. Here, the heterogeneity lies in altruism α_i and self-consciousness β_i, which we specify below.

The budget constraint for i is

$$c_i + x_i = y_i, \tag{3.2}$$

$$x_i = D_i x_{ki} + (1 - D_i) x_{ai}. \tag{3.3}$$

y_i is the endowment of individual i. People allocate their income between consumption and donation. x_k is known donation, x_a is anonymous donation, and D_i is a dummy

[1] Theoretical and empirical backgrounds for these two motivations are given in Benabou and Tirole (2006), Ariely et al (2009), Carpenter and Myers (2010), Andreoni (1989), Andreoni (1990), Crumpler and Grossman (2008), and Palfrey and Prisbrey (1997).

variable that takes the value 1 if individual i chooses known over anonymous donation, and takes the value 0 if individual i chooses anonymous over known donation. In short, in this model, when individuals want to donate, they must choose between a known and an anonymous donation. For simplicity, we assume that individuals cannot make both anonymous and known donations at the same time. We must draw attention to the implied assumption that the choice between anonymous and known donation does not directly affect the utility: both choices yield the same warm-glow utility. However, the choice does matter when individuals form beliefs about others' α values: $\hat{\alpha}_i$. $\hat{\alpha}_i$ is formed by

$$\hat{\alpha}_i = E[\alpha_i|\boldsymbol{x_k}]. \tag{3.4}$$

where $\boldsymbol{x_k} \equiv \{x_{k1}, x_{k2}, ..., x_{kn}\}$. In words, $\boldsymbol{x_k}$ denotes amount of known donation for all individuals in the economy. (3.4) claims that beliefs about others' altruism are formed based on known donation. In other words, individual's decision variables except known donation are unobservable to the others. It must be noted that individuals form beliefs about α_i based on x_{ik}, not x_i. This result is owing to the definition of anonymous donation: anonymous donation is unobservable to others, while known donation is observable. It is important that anonymous donors ($x_{ai} \neq 0$ and $x_{ki} = 0$) and non-donors ($x_{ai} = 0$ and $x_{ki} = 0$) are considered the same by others.

Next, we specify the parameter values and the distribution of the individual types in this economy. Table 3.1 below summarises the information about the four individual types of our model.

Type	α	β	Population
Selfish	0	0	N_s
Pure Altruist	$\bar{\alpha}$	0	N_{pa}
Hypocrite	0	$\bar{\beta}$	N_h
Impure Altruist	$\bar{\alpha}$	$\bar{\beta}$	N_{ia}

Table 3.1: Individual Types

3.3 Who Gives Anonymously?

The first task is to determine who donates anonymously. The answer is the 'pure altruists'. There are two reasons for this. First, because anonymous donation, by definition, cannot be observed by others, anonymous donors and non-donors are considered the same (i.e. the same $\hat{\alpha}_i$). Second, the types who are concerned with their own reputation ($\beta_i \neq 0$) want to be seen as altruistic (i.e., as donors); consequently, they never prefer anonymous to known donation. Then, the 'pure altruists' (who are altruistic but not self-conscious) is the only type who may make anonymous donations. Pure altruists must be indifferent to whether a donation is known or anonymous.

We assume that a fraction $A \in (0, 1)$ of the pure altruists choose anonymous over known donation, and thus, $1 - A$ of them choose known over anonymous donation. We also assume that A can be exogeneously controlled by fundraisers by providing the check boxes on their Web sites or not. In the next section, we examine what is the effect of controlling A on the Nash equilibrium.

3.4 Separating Equilibrium

Hypocrites and impure altruists behave strategically. If both types donate different amount at the equilibrium, we say that it is a separating equilibrium. Then, there can be two types of separating equilibria: (1) hypocrite mimic pure altruist and (2) hypocrite mimic selfish. We show that which type of separating equilibrium exist depends on the level of A.

3.4.1 Hypocrite Mimic Pure Altruist

The first type of separating equilibria is that hypocrite mimic pure altruist[2]. In the following, we show beliefs and conditions under which this types of separating equilibria exist.

Beliefs

On and Off path beliefs are as follows:

- If $x_{ki} = 0$, it is hypocrite with probability 0, pure altruist with probability $\frac{AN_{pa}}{N_s+AN_{pa}}$ and selfish with probability $\frac{N_s}{N_s+AN_{pa}}$
- If $0 < x_{ki} < \bar{\alpha}y_i$, it is hypocrite with probability 1.
- If $x_{ki} = \bar{\alpha}y_i$, it is hypocrite with probability $\frac{N_h}{N_h+(1-A)N_{pa}}$ and pure altruist with probability $\frac{(1-A)N_{pa}}{N_h+(1-A)N_{pa}}$.
- If $\bar{\alpha}y_i < x_{ki} < \bar{\alpha}y_i + \phi_i$, it is hypocrite with probability 1.
- If $\bar{\alpha}y_i + \phi_i \leq x_{ki}$, it is impure altruist with probability 1.

ϕ_i is defined later.

Selfish

People without self-consciousness, by definition, simply solve their classical optimization problems with respect to donation and private good, but not their images. In short, it is the simple warm-glow setup we see above. It is obvious that the selfish spend all of their income on private goods. For the selfish, $c_i = y_i$ and $x_i = 0$.

[2]For simplicity, we consider the case in which all hypocrites behave as a group. This does not change the result much as long as we focus on the symmetric equilibrium.

Pure Altruist

Pure altruists also face their classical optimisation problems without image motivation. The solution to this problem is $c_i = (1 - \bar{\alpha})y_i$ and $x_i = \bar{\alpha}y_i$. Because $\hat{\alpha}_i$ does not appear in their optimisation problems, pure altruists are indifferent to the choice between x_a and x_k; both options yield the same utility. Here, we assume that a fraction $A \in (0, 1)$ of the pure altruists choose anonymous over known donation, and thus, $1 - A$ of them choose known over anonymous donation. As a result, the introduction of the option of anonymous giving makes some pure altruists switch from known to anonymous giving. We should not overlook that the total donation amount of pure altruists is independent of A.

Hypocrite & Impure Altruist

It is most important to examine the conditions under which even hypocrites make donations in spite of their selfish nature. By making donations, they mimic the pure altruists (a good type) to enhance their reputations.

First, consider the behavior of hypocrite. Note that there are three options for hypocrites to choose, $x_{ki} = 0$, $x_{ki} = \bar{\alpha}y_i$ or $x_{ki} = \bar{\alpha}y_i + \phi_i$. If they choose $x_{ki} = 0$, then they are in the same group as the selfish. If they choose $x_{ki} = \bar{\alpha}y_i$, they are in the same group as the pure altruists. If they choose $\bar{\alpha}y_i + \phi_i$, they are in the same group as the impure altruists. Comparing the three options above, we can derive the conditions for hypocrites to choose $x_{ki} = \bar{\alpha}y_i$ over $x_{ki} = 0$ and $x_{ki} = \bar{\alpha}y_i + \phi_i$:

$$ln\,(y_i - \bar{\alpha}y_i - \phi_i) + \bar{\beta}I(\bar{\alpha}) \leq ln(y_i - \bar{\alpha}y_i) + \bar{\beta}I\left(\frac{(1-A)N_{pa}}{N_h + (1-A)N_{pa}}\bar{\alpha}\right) \tag{3.5}$$

$$ln(y_i) + \bar{\beta}I\left(\frac{AN_{pa}}{N_s + AN_{pa}}\bar{\alpha}\right) \leq ln(y_i - \bar{\alpha}y_i) + \bar{\beta}I\left(\frac{(1-A)N_{pa}}{N_h + (1-A)N_{pa}}\bar{\alpha}\right) \tag{3.6}$$

ϕ_i is defined as the value that makes (3.5) satisfied with equality; and hence, (3.5) is satisfied. In short, if (3.6) is satisfied, hypocrite mimics pure altruist. Next, consider the behavior of impure altruist. It can be easily checked that if (3.5) is satisfied, impure altruist prefers $x_{ki} = \bar{\alpha}y_i + \phi_i$ to $x_{ki} = \bar{\alpha}y_i$.

In summary, if (3.6) is satisfied, beliefs above and hypocrite: $x_{ki} = \bar{\beta}y_i$, impure altruist: $x_{ki} = \bar{\beta}y_i + \phi_i$ is separating equilibrium. Therefore, the question 'when does this separating equilibrium exist' is equivalent to the question 'when is the condition (3.6) satisfied'. We can derive the following proposition.

Proposition (Hypocrite mimic Pure Altruist)**.** *If A is sufficiently small, there exists a separating equilibrium in which hypocrite mimic pure altruist to exist.*

Proof. The left hand side represents the utility when they join the group of non-donors ($x_{ki} = 0$), while the right hand side represents the utility when hypocrites join the group of known donors ($x_{ki} = \bar{\alpha}y_i$). The second term on the left hand side of (3.6) is increasing in A,

$$\frac{\partial \bar{\beta} \ln I\left(\frac{AN_{pa}}{N_s + AN_{pa}}\bar{\alpha}\right)}{\partial A} > 0, \tag{3.7}$$

and the right hand side of (3.6) is decreasing in A,

$$\frac{\partial \bar{\beta} \ln I\left(\frac{(1-A)N_{pa}}{N_h + (1-A)N_{pa}}\bar{\alpha}\right)}{\partial A} < 0. \tag{3.8}$$

Therefore, if A takes a sufficiently large value, (3.6) is difficult to satisfy. In other words, if A takes a sufficiently small value, (3.6) is satisfied and this separating equilibrium exists.

□

The interpretation of these two inequalities is the core of this study. We refer to (3.8) as the 'decrease effect', and (3.7) as the 'blend effect'. We first note that pure altruists are thought to be a 'good' type compared with the selfish and the hypocrites because pure altruists have higher altruism. Next, the group reputation is formed according to the ratio of 'good' (altruistic) group members. To interpret (3.8), suppose that some of the pure altruists (a 'good' type) switch from known ($(x_{ai}, x_{ki}) = (0, \bar{\alpha}y_i)$) to anonymous donation ($(x_{ai}, x_{ki}) = (\bar{\alpha}y_i, 0)$). Such a switch implies a decrease of a 'good' type in the group of known donors ($x_{ki} = \bar{\alpha}y_i$). Then, to the hypocrites, joining the group of known donors ($x_{ki} = \bar{\alpha}y_i$) becomes less attractive. This decrease effect corresponds to inequality (3.8). The first inequality (3.7) represents the blend effect. Because anonymous donation is unobservable, people now think that some perceived non-donors are actually anonymous donors. Some fraction of non-donors is of the 'good' type ($\hat{\alpha} = \frac{AN_{pa}}{N_s+AN_{pa}}\bar{\alpha} > 0$. As a result, the existence of anonymous donors enhances the reputation of non-donors, and joining the group of non-donors ($x_{ki} = 0$) becomes more attractive to the hypocrites. Here, anonymous donors blend into non-donors. This blend effect corresponds to inequality (3.7).

Finally, we should check the existence of other equilibria if this separating equilibrium exists. By intuitive criterion, off-path belief is necessarily "impure altruist with possibility 1 if $x_{ki} > \bar{\alpha}y_i + \phi_i$". Under this belief, pooling equilibrium does not exist if impure altruist at the pooling equilibrium does not gain more than $x_{ki} = \bar{\alpha}y_i + \phi_i$: equivalently, if

$$(1-\bar{\alpha})ln(y_i - \bar{\alpha}y_i) + \bar{\alpha}ln(\bar{\alpha}y_i) + \bar{\beta}I(\frac{(1-A)N_{pa} + N_{ia}}{N_h + (1-A)N_{pa} + N_{ia}}\bar{\alpha})$$
$$< (1-\bar{\alpha})ln(y_i - \bar{\alpha}y_i - \phi_i) + \bar{\alpha}ln(\bar{\alpha}y_i + \phi_i) + \bar{\beta}I(\bar{\alpha}), \quad (3.9)$$

pooling equilibrium does not exist. This paper assumes (3.9) and focus only on separating equilibria.

To sum up, the larger A is, the more difficult it is for this separating equilibrium to exist.

3.4.2 Hypocrite Mimic Selfish

The second type of separating equilibria is that hypocrite mimic selfish. In the following, we show beliefs and conditions under which this types of separating equilibria exist.

Beliefs

On and Off path beliefs are as follows:

- If $x_{ki} = 0$, it is hypocrite with probability $\frac{N_h}{N_s+AN_{pa}+N_h}$, pure altruist with probability $\frac{AN_{pa}}{N_s+AN_{pa}+N_h}$, and selfish with probability $\frac{N_s}{N_s+AN_{pa}+N_h}$.
- If $0 < x_{ki} < \bar{\beta}y_i$, it is hypocrite with probability 1.
- If $x_{ki} = \bar{\beta}y_i$, it is pure altruist with probability 1.
- If $\bar{\beta}y_i < x_{ki} < \bar{\alpha}y_i + \delta_i$, it is hypocrite with probability 1.
- If $\bar{\alpha}y_i + \delta_i \leq x_{ki}$, it is impure altruist with probability 1.

δ_i is defined later.

Selfish & Pure Altruist

The behaviour of the selfish and pure altruist is the same as before. They simply solve their classical optimisation problems with respect to donation and private good, but not their images. For the selfish, $c_i = y_i$ and $x_i = 0$. For the pure altruist, the solution is $c_i = (1 - \bar{\alpha})y_i$ and $x_i = \bar{\alpha}y_i$. A fraction A of the pure altruists choose anonymous over known donation, and thus, $1 - A$ of them choose known over anonymous donation.

Hypocrite & Impure Altruist

First, consider the behaviour of hypocrite. Note that there are three options for hypocrites to choose, $x_{ki} = 0$, $x_{ki} = \bar{\alpha}y_i$ or $x_{ki} = \bar{\alpha}y_i + \delta_i$. If they choose $x_{ki} = 0$, then they are in the same group as the selfish. If they choose $x_{ki} = \bar{\alpha}y_i$, they are in the same group as the pure altruists. If they choose $\bar{\alpha}y_i + \delta_i$, they are in the same group as the impure altruists. Comparing the three options above, we can derive the conditions for hypocrites to choose $x_{ki} = 0$ over $x_{ki} = \bar{\alpha}y_i$ and $x_{ki} = \bar{\alpha}y_i + \delta_i$:

$$ln(y_i - \bar{\alpha}y_i - \delta_i) + \bar{\beta}I(\bar{\alpha}) \leq ln(y_i) + \bar{\beta}I(\frac{AN_{pa}}{N_s + N_h + AN_{pa}}\bar{\alpha}) \tag{3.10}$$

$$ln(y_i - \bar{\alpha}y_i) + \bar{\beta}I(\bar{\alpha}) \leq ln(y_i) + \bar{\beta}I\left(\frac{AN_{pa}}{N_s + N_h + AN_{pa}}\bar{\alpha}\right) \tag{3.11}$$

δ_i is defined as the value that makes (3.10) satisfied with equality; and hence, (3.10) is satisfied. In short, if (3.11) is satisfied, hypocrite mimics pure altruist. Next, consider the behaviour of impure altruist. It can be easily checked that if (3.10) is satisfied, impure altruist prefers $x_{ki} = \bar{\alpha}y_i + \delta_i$ to $x_{ki} = \bar{\alpha}y_i$.

In summary, if (3.11) is satisfied, beliefs above and hypocrite: $x_{ki} = 0$, impure altruist: $x_{ki} = \bar{\beta}y_i + \delta_i$ is separating equilibrium. Therefore, the question 'when does this separating equilibrium exist' is equivalent to the question 'when is the condition (3.11) satisfied'. We can derive the following proposition.

Proposition (Hypocrite mimic Selfish)**.** *If A is sufficiently large, there exists a separating equilibrium in which hypocrite mimic pure altruist.*

Proof. The left hand side represents the utility when hypocrites join the group of known donors ($x_{ki} = \bar{\alpha}y_i$), while the right hand side represents the utility when they join the group of non-donors ($x_{ki} = 0$). The second term on the right hand side of (3.6) is increasing in

A,

$$\frac{\partial I\left(\frac{AN_{pa}}{N_s+N_h+AN_{pa}}\bar{\alpha}\right)}{\partial A} > 0. \tag{3.12}$$

Therefore, if A takes a sufficiently large value, (3.11) is satisfied and this separating equilibrium exists. □

(3.12) represents the blend effect. Because anonymous donation is unobservable, people now think that some perceived non-donors are actually anonymous donors. Some fraction of non-donors is of the 'good' type ($\hat{\alpha} = \frac{AN_{pa}}{N_s+N_h+AN_{pa}}\bar{\alpha} > 0$. As a result, the existence of anonymous donors enhances the reputation of non-donors, and joining the group of non-donors ($x_{ki} = 0$) becomes more attractive to the hypocrites. Here, anonymous donors blend into non-donors.

Finally, we should check the existence of other equilibria if this separating equilibrium exists. By intuitive criterion, off-path belief is necessarily "impure altruist with possibility 1 if $x_{ki} > \bar{\alpha}y_i + \phi_i$". Under this belief, pooling equilibrium does not exist if impure altruist at the pooling equilibrium does not gain more than $x_{ki} = \bar{\alpha}y_i + \phi_i$: equivalently, if

$$\begin{aligned}(1-\bar{\alpha})ln(y_i - \bar{\alpha}y_i) + \bar{\alpha}ln(\bar{\alpha}y_i) + \bar{\beta}I(\frac{(1-A)N_{pa} + N_{ia}}{N_h + (1-A)N_{pa} + N_{ia}}\bar{\alpha}) \\ < (1-\bar{\alpha})ln(y_i - \bar{\alpha}y_i - \phi_i) + \bar{\alpha}ln(\bar{\alpha}y_i + \phi_i) + \bar{\beta}I(\bar{\alpha}), \end{aligned} \tag{3.13}$$

pooling equilibrium does not exist. This paper assumes (3.13) and focus only on separating equilibria.

To sum up, the larger A is, the easier it is for this separating equilibrium to exist.

3.4.3 Summary of the Results

The main results obtained above are as follows: if A is large, a separating equilibrium 'hypocrite mimic selfish' exists. Conversely, if A is small, a separating equilibrium 'hypocrite mimic pure altruist' exists.

In more detail, while impure altruists are averse to be mimicry by hypocrites, pure altruists do not care about it. Thus, the hypocrites mimic the 'target' pure altruists. Then, existence of anonymous donors means 1) the decrease of the best 'target' for the hypocrites to mimic and 2) the increase of the good type in the non-donor group. Therefore, if there are many anonymous donors, hypocrites do not make a giving.

This is because fundraisers have to care about the level of A. To raise funds from hypocrites and maximize the total amount of donation, fundraisers should make A lower.

3.5 Policy Implication and Discussions

The donation amount varies according to an exogenously determined A. This result raises the question, can fundraisers control A and expected A? In this section, we discuss one practical method to control both A and expected A.

Currently, many fundraisers solicit contributions via the Internet in addition to normal fundraising activities. On their Web sites, some fundraisers including Anna Marie's Alliance, the Minnesota Aids Project, and Network for Good place check boxes for donors to select either anonymous or known donations. Figure 3.1 shows how a typical such site looks. The list in table 3.2 shows which fundraiser places check boxes and which does not on their Web sites. Donors fill out not only information about the donation amount, name, address, and credit card number but also a check box, '*I prefer to make this donation anonymously*'. However, some organisations, such as the American Cancer Society, the American Red Cross, and Doctors Without Borders do not provide such a check box on their Web sites. Without a special request, only known donation is available for donors. We see that not a few fundraisers explicitly offer opportunities for anonymous giving. By providing the check boxes, these fundraisers intend to make it convenient to donate anonymously. The problem, however, is that, when facing the choice, donors expect that fundraisers provided the check box in response to many requests from a substantial number of anonymous donors. In other words, the check box exaggerates the number of anonymous donors. Here, A is expected higher than that it actually is. To keep both A

Table 3.2: Fundraisers with Check Boxes

With Check Boxes	Without Check Boxes
Minnesota Aids Project	American Cancer Society
Anna Marie's Alliance	American Red Cross
Akahane Kyodo Bokin	Big Brothers Big Sisters
Mcdonalds' House Charity	Sierra Club
	Doctors Without Borders
	Feed the Children
	Oxfam America
	Unicef

and expected A lower, fundraisers can remove this type of check box. Instead, they can implement some type of small foot cost on anonymous giving and accept anonymous donations only as an exception. For instance, to give anonymously, donors have to send an e-mail to fundraisers in addition to filling out the personal information form. Then, those who require anonymity choose anonymous giving with a small effort, while those who are indifferent to whether the donation is anonymous choose known giving.

An important experimental finding is reported in Andreoni and Petrie (2004). They show that, if examinees give subjects an option to donate anonymously, subjects increase their giving. Note that the subjects mainly increase known giving, not anonymous giving. Their finding is not inconsistent with our theoretical results, because their finding corresponds to the case of low A in our model. Our model predicts that if A is small, impure altruists increase known giving above their ideal amount. This is what Andreoni and Petrie (2004) observes. We also predict that if we would make A and expected A sufficiently large, then the subjects would decrease their known giving. Fundraisers should keep the actual A and expected A at a low level and consequently maximise the total donation amount.

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Chapter 4

A Signaling Explanation for Political Parties and Advertisements

abstract:

This chapter studies the situation in which politicians make signals to voters under uncertainty of politicians' types. We focus on two types of signaling tools: political parties and political advertisements. Investigating a multi-period version of Snyder and Ting (2002) with possibility of reelection, we show that political parties are not sufficient signals to reveal politicians' types. Political advertisements work as a complementary signal to political parties. With political parties and advertisements, voters can elect their favourable candidates adequately.

4.1 Introduction

Politicians often send signals to voters. Notable examples are political parties and political advertisements. Many politicians and candidates join some political parties which issue manifestos. Almost all political candidates make their posters before the election.

They sometimes spend enormous amounts of money on media advertising. These kinds of political parties and advertisements are often criticised as wasting money. However, this study sheds lights on the positive aspect of them: they function as signals to voters under asymmetric information.

Because in the election candidates' types are hidden and unknown to voters, candidates give signals to differentiate themselves from other candidates. Snyder and Ting (2002) show that if politicians' types are uncertain to voters, parties function as brand names (signals) that ensure the quality of the politicians to some extent. Since belonging to a party implies that his or her policy is close to the party's ideal policy, it conveys information about the hidden types of the politicians. Then, what is the role of the political advertisements? If it also works as a signal, is it substitute or compliment to political parties? This study extends Snyder and Ting (2002) to the dynamic model with possibility of reelection and answers to these questions. The answer is that political parties are not sufficient signals and political advertisements work as a compliment to political parties.

One particular characteristic about reelections is that after one term in office, the hidden type of incumbent is completely revealed to the voters. On the other hand, candidates who run for election for the first time can reveal their types only by joining political parties. Then, the fact that the types of candidates are revealed to the voters can be an advantage for some of them; however, this can serve as a disadvantage for other candidates. This is because some incumbents in the party are less favourable to the voters in reality than expected. In our model, there are three types of candidates: (1) a candidate who cannot win at all; (2) a candidate who wins only once and is not reelected; and (3) a candidate who wins, and after one term in office, gets reelected. Then, there is a possibility that political advertisements would be run even if it is simply a waste of money in the sense of Milgrom and Roberts (1986). Candidates who can be reelected (and hence, ex-post good) have the incentive to differentiate themselves from the candidates who can win office only once. With advertisements, the voters can elect only the ex-post good candidates.

In section 2, we introduce reelection to the basic model of Snyder and Ting (2002), thereby investigating the model to a dynamic environment: there is another election after one term in office. We find that political parties are not sufficient signals in the election. In section 3, we examine the incentive of political advertisements. Political advertisements are complementary to political parties. Section 4 considers what the difference between political parties and advertisements is. Section 5 summarizes the main results.

4.2 The Model with Reelection

4.2.1 Timing of the Model

First, adding reelection to the basic model of Snyder and Ting (2002), we would like to show that political parties are not sufficient signals. Snyder and Ting (2002) prove that parties function as brand names that ensure the quality of politicians to some extent. Since belonging to a party implies that his or her policy is close to the party's ideal policy, it conveys information about the types of politicians. They, however, consider a static model in which there is one election and one term in office. On the other hand, politicians can get reelected only in a dynamic model in which there are more than two elections. Adding the second election to the basic model, we analyze the consequence of reelections and the role of political parties.

The key feature of a reelection is that politicians completely reveal their types after one term in office. Although joining a party partially reveals the politician's type, conducting his ideal policy in his term of office is assumed to reveal his type. Because in Snyder and Ting (2002) and our model, politicians' types directly indicate their ideal policies, this is a natural assumption.

There are three player in our model: candidate, political parties, and voters. The timing of our model is as follows. In short, the last three stages are added to Snyder and Ting (2002).

1. Platform Selection.

2. Candidate Nomination and Selection.

3. 1st Voting. 1st Term of Office.

4. *Decision of Running for Reelection.*

5. *Candidate Nomination and Selection.*

6. *2nd Voting. 2nd Term of Office.*

1. At the beginning, political parties choose their platforms between policy space $[-1, 1]$, which is observed by all players. Each potential candidate and voter has his/her preferable policy that lies between $[-1, 1]$ with uniform distribution. Potential candidates choose to belong to parties or be unaffiliated.

2. In each party, nature randomly select one candidate who chooses to belong to the party for the election. It means that parties can not observe the candidates' hidden types.

3. In the 1st voting, though voters can not also observe the candidates' hidden types, voters can observe whether the candidates belong to the parties or not. The voters elect the winner by plurality rule. The winner of the election holds the office and gains payoff. After one term of office, their types are completely revealed to both parties and voters.

4. In the next period, the incumbent decides whether he/she runs for reelection.

5. The party also decides whether the incumbent runs for reelection or not. If the incumbent does not run for reelection, nature selects a new candidate again.

6. In the 2nd election, the winner is decided by plurality rule. The winner of the 2nd election also holds the office and gains the same payoff.

We should notice that the decision of running for reelection is made by both the politician and the party that he or she belongs to. First, running for reelection must be profitable to the politician who now holds office. Second, for the party, it must nominate such a candidate for reelection who is more likely to win office rather than a newly nominated politician. In other words, a politician runs for reelection only if the probability of his reelection is higher than the probability of another new candidate whose ideal policy is unknown to the public.

The remaining functional setup is as follows. The policy space is between $[-1, 1]$. The payoff of a politician i with an ideal point z_i who is affiliated with party k and wins office is

$$w - c - \alpha(x_k - z_i)^2 \tag{4.1}$$

where w denotes the rent of taking office, $c \in [0, w)$ denotes the cost of taking office, x_k denotes the policy of political party k, and coefficient $\alpha > 0$. On the other hand, if they do not win, their payoff is 0.

Voters lie in $[-1, 1]$ with uniform distribution. The expected payoff of the median voter (who locates at 0) if party k's candidate i wins is

$$E[\,-(0 - z_i)^2] = -(0 - \mu_i)^2 - \sigma_i^2 \tag{4.2}$$

where μ_i denotes the mean and σ_i denotes the variance of the candidate in party i.

4.2.2 The Equilibrium

We deal with the case in which there are two political parties and both of them choose the platform $x_i = 0$[1]. This is natural because the median voter is also at 0. In words, both parties target the median voter and they suggest the same policy.

First, consider the second period. The incumbent who won the first period election has to compete against a candidate of the other party. We call this new candidate of the opponent party 'challenger'. Then, the expected payoff of the median voter at 0 for the challenger is

$$-(0-0)^2 - \frac{\theta^2}{3}, \tag{4.3}$$

where $\theta \equiv \sqrt{\frac{w-c}{\alpha}}$. Note that candidates join a political party if their expected payoff is larger than 0 which is the payoff when they do not run for the election. In short, $w - \alpha(0 - z_i)^2 - c > 0$, and hence, $z_i \in [-\theta, \theta]$. μ_i and σ_i for $z_i \in [-\theta, \theta]$ is 0 and $\frac{1}{3}\theta^2$ respectively. On the other hand, the expected payoff of the median voter at 0 for the incumbent is

$$-(0-z_i)^2 - 0. \tag{4.4}$$

This is because, at the second period, the hidden type of For the incumbent to win,

$$-\frac{\theta^2}{3} < -z_i^2. \tag{4.5}$$

In short, if $z_i \in \left[-\frac{\sqrt{3}}{3}\theta, \frac{\sqrt{3}}{3}\theta\right]$, the challenger loses and the incumbent wins. Otherwise, since the challenger is going to win, the incumbent does not run for reelection.

Next, go back to the first election. Note that, in the first election, candidates who join

[1] This is the equilibrium when $\theta < \sqrt{3}/2$ in Snyder and Ting (2002)

the party (and win) lie in

$$z_i \in [-\theta, \theta]. \tag{4.6}$$

This is the same result as Snyder and Ting (2002). Candidate's nomination and platform selection are independent of introducing the 2nd election.

To sum up, in the first election, candidates $z_i \in [-\theta, \theta]$ join the party. In the second election, candidates $z_i \in \left[-\frac{\sqrt{3}}{3}\theta, \frac{\sqrt{3}}{3}\theta\right]$ run for reelection. In words, incumbents in the narrower range can only run for reelection and win office. There are 3 types of candidates:

Loser: If $z_i \in [-1, -\theta]$ or $z_i \in [\theta, 1]$, he/she does not join a party. We refer to him/her as "loser".

One-time Winner: If $z_i \in [-\theta, -\frac{\sqrt{3}}{3}\theta]$ or $z_i \in [\frac{\sqrt{3}}{3}\theta, \theta]$, he/she joins a party, but does not get reelected. We refer to him/her as "one-time winner".

Two-time Winner: If $z_i \in [-\frac{\sqrt{3}}{3}\theta, \frac{\sqrt{3}}{3}\theta]$, he/she joins a party, and get reelected. We refer to him/her as "two-time winner".

In other words, the incubents $z_i \in [-\theta, -\frac{\sqrt{3}}{3}\theta]$ or $z_i \in [\frac{\sqrt{3}}{3}\theta, \theta]$ are not favourable to the voters in reality even though they are elected in the first election.

Because each party selects one-time winner or two-time winner as a candidate, there are three types of situations in the election[2].:

- **One-time Winner vs One-time Winner**

  Both of them join parties, one of them gets elected with 50% in the first election, and does not get reelected.

- **Two-time Winner vs One-time Winner**

  Both of them join parties, one of them gets elected with 50% in the first election, and only two-time winner gets reelected.

[2]For simplicity, we omit Loser from the following list.

- **Two-time Winner vs Two-time Winner**

  Both of them join parties, one of them gets elected with 50% in the first election, and gets reelected.

Note that if promising candidates (two-time winners) compete against non-promising candidates (one-time winners), promising candidates' possibility of winning is only 50%. This implies that political parties are not sufficient signals. Then, the main result is as follows: incumbents in a narrower range can run for reelection and win office. However, joining a political party does not differentiate between ex-post good politicians (who can get reelected) and ex-post bad politicians (who can not get reelected).

4.3 The Model with Reelection and Advertisement

4.3.1 Timing of the Model

As is seen in the previous section, political parties are not sufficient signals. Next, by introducing political advertisements in the above-mentioned model, we would like to answer the question: What is the role of political parties? The main result is that political advertisements work as a complementary signal to political parties. Promising (ex-post good) candidates design political advertisements to differentiate themselves from other (ex-ante good) candidates even if it is simply a waste of money.

The timing of the model is as follows. Stage 3 is added to the above-mentioned model; in other words, before the election, candidates including incumbents can design political advertisements.

1. Platform Selection.

2. Candidate Nomination and Selection.

3. *Advertisement*

4. 1st Voting. 1st Term of Office.

5. *Decision of Running for Reelection.*

6. *Candidate Nomination and Selection.*

7. *2nd Voting. 2nd Term of Office.*

The important point to note is that advertisements neither affect the payoff of the voters nor convey any information except their quantity; therefore, political advertisements are simply a waste of money in the same sense as in Milgrom and Roberts (1986).

Promising candidates who can win office twice want to design political advertisements to send more signals to the public before the first election. This is because, as seen in the previous section, in the first election, simply joining a party does not differentiate a two-time winner from a one-time winner, while it differentiates a winner from a loser in the first election. To be more precise, in the two parties case, in the equilibrium, each candidate can win the first election with 50%. However, if the candidate were known to be a two-time winner, he/she would win the election with 100%. Then, he/she has the incentive to reveal his/her hidden type by designing advertisements.

4.3.2 The Equilibrium

Whether or not candidates make political advertisements depends on the opponent's type and policy. We use the same definition of 'loser', 'one-time winner' and 'two-time winner' as before: a candidate is called loser if $z \in [-1, -\theta]$ or $z \in [\theta, 1]$, a candidate is called one-time winner if $z \in [-\theta, -\frac{\sqrt{3}}{3}\theta]$ or $z \in [\frac{\sqrt{3}}{3}\theta, \theta]$, and a candidate is called two-time winner if $z \in [-\frac{\sqrt{3}}{3}\theta, \frac{\sqrt{3}}{3}\theta]$. As the same as before, one-time winner can not win the reelection and two-time winner can win the reelection.

Because each party selects one-time winner or two-time winner as a candidate, there are three types of situations:

- **One-time Winner vs One-time Winner**
- **Two-time Winner vs One-time Winner**
- **Two-time Winner vs Two-time Winner**

As is claimed above, promising candidate have incentive to differentiate themselves from other candidates. In the two parties case, in the equilibrium, each candidate can win the first election with 50%. However, with political advertisements, if the candidate were known to be a two-time winner, he/she would win the election with 100%.

Here, let me briefly describe the equilibrium in words. In all cases, all candidates join parties. Whether candidates make advertisement or not depends on the relative positions of them. If they are located far enough (e.g. two-time winner vs one-time winner), the promising candidate joins a party and additionally make advertisements. If they are located nearly, the promising candidate only join a party but does not make advertisements. To be precise,

- One-time Winner vs One-time Winner
  - if $\theta^2 < 2|z_j|^2 - |z_i|^2$, the promising candidate uses both a party and advertisements.
  - if $\theta^2 > 2|z_j|^2 - |z_i|^2$, the promising candidate uses only a party.
- Two-time Winner vs One-time Winner
  - the promising candidate uses both a party and advertisements.
- Two-time Winner vs Two-time Winner
  - if $\theta^2 < |z_j|^2 - \frac{1}{2}|z_i|^2$, the promising candidate uses both a party and advertisements.
  - if $\theta^2 > |z_j|^2 - \frac{1}{2}|z_i|^2$, the promising candidate uses only a party.

In the following, we investigate each equilibrium in more detail.

Equilibrium: One-time Winner vs One-time Winner

The first case is that both candidates i and j are one-time winners. We assume that i is closer to the median voter than j. As a result, at the equilibrium, if $\theta^2 < 2|z_j|^2 - |z_i|^2$,

- 'Loser' does not join a party, does not make advertisements, and cannot win at all.
- 'One-time Winner' j joins a party, does not make advertisements, and can not win the first election.
- 'One-time Winner' i joins a party, make advertisements $ad_i = w - c - \alpha z_j{}^2$, and win the first election with 100%, but does not get reelected.

If $\theta^2 > 2|z_j|^2 - |z_i|^2$,

- 'Loser' does not join a party, does not make advertisements, and cannot win at all.
- 'One-time Winner' i and j join parties, do not make advertisements, and can win the first election with 50%, but do not get reelected.

Note that the belief of voters is formed rationally[3] as in Snyder and Ting (2002).

Let us verify that the above is the equilibrium. Suppose that the candidate j's strategy is joining a party and making no advertisement. In the following, we check whether or not the candidate i's best response is the above strategy. Note that there are three options for the candidates to choose: (1) only join a party, (2) only make advertisements, and (3) both party and advertisements.

For the candidate i, the option (1) gives the gain

$$\frac{1}{2}[(w - c - \alpha z_i^2)]. \tag{4.7}$$

[3]To be precise, the belief is as follows: (i) no party joining and $ad < w - c$: then he/she is z_j or loser. (ii) no party joining and $ad \geq w - c$, then he/she is z_i. (iii) party joining and $ad < w - c - \alpha z_j^2$, then he/she is z_j. (iv) party joining and $ad \geq w - c - \alpha z_j^2$, then he/she is z_i.

This is because both of i and j are seen as in the same group and the candidate i's probability of winning is $\frac{1}{2}$.

Next, in the option (2), the candidate i only makes advertisements. What amount of advertisements does the candidate i pay? The answer is

$$ad_i = w - c. \tag{4.8}$$

The candidate i has to pay $w-c$ as advertisements because he/she has to differentiate from the candidate j and non-party-affiliated candidate. If they would win, both of them would gain $w - c$ in the 1st term of office. Therefore, the candidate i pays the larger amount $w - c$ to differentiate himself from the other candidates and win the election. Then, for the candidate i, the option (2) gives the gain

$$w - c - ad_i = (w - c) - (w - c) \tag{4.9}$$

$$= 0. \tag{4.10}$$

In the option (3), candidate i joins a party and additionally make advertisements. Though joining a party works to differentiate himself/herself from the unaffiliated candidate, advertisements work to differentiate himself/herself from the other party-affiliated candidate. Therefore, the amount of advertisements is

$$ad_i = w - c - \alpha z_j^2, \tag{4.11}$$

which is equal to the gain of candidate j, since j can not pay that amount of money even if he/she can win the election. Then the voters can elect the better candidate even in the first election. For the candidate i, the option (3) gives the gain

$$(w - c - \alpha z_i^2) - (w - c - \alpha z_j^2) \tag{4.12}$$

$$= \alpha z_j^2 - \alpha z_i^2. \tag{4.13}$$

Comparing these three options (1), (2) and (3), we can say that candidates' best response depend on the relative positions of them. if $\theta^2 < 2|z_j|^2 - |z_i|^2$, the candidate i chooses the option (3) and get elected. If $\theta^2 > 2|z_j|^2 - |z_i|^2$, the candidate i chooses the option (1) and get elected with probability $\frac{1}{2}$. On the other hand, it is clear that in both cases, the candidate j's best response is also joining a party and making no advertisement. Therefore, we can conclude that these i and j's strategy is the equilibrium.

Equilibrium: Two-time Winner vs One-time Winner

The next case is that one candidate i is two-time winner and the other candidate j is one-time winner. We assume that i is closer to the median voter than j. At the equilibrium,

- 'Loser' does not join a party, does not make advertisements, and cannot win at all.
- 'One-time Winner' j joins a party, does not make advertisements, and can not win the first election.
- 'Two-time Winner' i joins a party, make advertisements $ad_i = w - c - \alpha {z_j}^2$, and win the first election with 100%, and get reelected.

Note that the belief of voters is formed rationally[4] as in Snyder and Ting (2002).

Let us verify that the above is the equilibrium. Suppose that the candidate j's strategy is joining a party and making no advertisement. In the following, we check whether or

[4]To be precise, the belief is as follows: (i) no party joining and $ad < w - c$: then he/she is z_j or loser. (ii) no party joining and $ad \geq w - c$, then he/she is z_i. (iii) party joining and $ad < w - c - \alpha z_j^2$, then he/she is z_j. (iv) party joining and $ad \geq w - c - \alpha z_j^2$, then he/she is z_i.

not the candidate i's best response is the above strategy. Note that there are three options for the candidates to choose: (1) only join a party, (2) only make advertisements, and (3) both party and advertisements.

For the candidate i, the option (1) gives the gain

$$\frac{1}{2}[(w - c - \alpha z_i^2) + (w - c)]. \tag{4.14}$$

This is because both of i and j are seen as in the same group and the candidate i's probability of winning is $\frac{1}{2}$.

Next, in the option (2), the candidate i only makes advertisements. What amount of advertisements does the candidate i pay? The answer is

$$ad_i = w - c. \tag{4.15}$$

The candidate i has to pay $w - c$ as advertisements because he/she has to differentiate from the candidate j and non-party-affiliated candidate. If they would win, both of them would gain $w - c$ in the 1st term of office. Therefore, the candidate i pays the larger amount $w - c$ to differentiate himself from the other candidates and win the election. Then, for the candidate i, the option (2) gives the gain

$$2(w - c) - ad_i = 2(w - c) - (w - c) \tag{4.16}$$

$$= w - c. \tag{4.17}$$

In the option (3), candidate i joins a party and make advertisements. Though joining a party works to differentiate himself/herself from the unaffiliated candidate, advertisements work to differentiate himself/herself from the other party-affiliated candidate. Therefore, the amount of advertisements is

$$ad_i = w - c - \alpha z_j^2, \tag{4.18}$$

which is equal to the gain of the other affiliated candidate, since j can not pay that amount of money even if he/she can win the election. The voters can then distinguish between better and worse candidate even in the first election by observing whether the candidates have advertised or not. For the candidate i, the option (3) gives

$$(w - c - \alpha z_i^2) - (w - c - \alpha z_j^2) + (w - c) \tag{4.19}$$

$$= w - c + \alpha z_j^2 - \alpha z_i^2. \tag{4.20}$$

Comparing these three options (1), (2) and (3), we can say that the candidate i's best strategy is (3) joining a party and making advertisements. As a result the candidate i gets elected. On the other hand, it is clear that the candidate j's best response is also joining a party and making no advertisement. Therefore, we can conclude that these i and j's strategy is the equilibrium.

Equilibrium: Two-time Winner vs Two-time Winner

The last case is that both candidates i and j are two-time winners. We assume that i is closer to the median voter than j. As a result, at the equilibrium, if $\theta^2 < |z_j|^2 - \frac{1}{2}|z_i|^2$,

- 'Loser' does not join a party, does not make advertisements, and cannot win at all.
- 'Two-time Winner' j joins a party, does not make advertisements, and can not win the first election.
- 'Two-time Winner' i joins a party, make advertisements $ad_i = 2(w - c) - \alpha {z_j}^2$, and win the first election with 100%, and get reelected.

If $\theta^2 > |z_j|^2 - \frac{1}{2}|z_i|^2$,

- 'Loser' does not join a party, does not make advertisements, and cannot win at all.
- 'One-time Winner' i and j join parties, do not make advertisements, and can win the first election with 50%, and get reelected.

Note that the belief of voters is formed rationally[5] as in Snyder and Ting (2002).

Let us verify that the above is the equilibrium. Suppose that the candidate j's strategy is joining a party and making no advertisement. In the following, we check whether or not the candidate i's best response is the above strategy. Note that there are three options for the candidates to choose: (1) only join a party, (2) only make advertisements, and (3) both party and advertisements.

For the candidate i, the option (1) gives the gain

$$\frac{1}{2}[(w - c - \alpha z_i^2) + (w - c)]. \tag{4.21}$$

This is because both of i and j are seen as in the same group and the probability of winning is $\frac{1}{2}$.

Next, in the option (2), the candidate i only makes advertisements. What amount of advertisements does the candidate i pay? The answer is

$$ad_i = 2(w - c). \tag{4.22}$$

The candidate i has to pay $w - c$ as advertisements because he/she has to differentiate from the candidate j and non-party-affiliated candidate. If they would win, the candidate j would gain $2(w - c)$ and non-party-affiliated candidate would gain $w - c$in the 1st term of

[5]To be precise, the belief is as follows: (i) no party joining and $ad < 2(w - c)$: then he/she is z_j or loser. (ii) no party joining and $ad \geq 2(w - c)$, then he/she is z_i. (iii) party joining and $ad < 2(w - c) - \alpha z_j^2$, then he/she is z_j. (iv) party joining and $ad \geq 2(w - c) - \alpha z_j^2$, then he/she is z_i.

office. Therefore, the candidate i pays the larger amount $2(w-c)$ to differentiate himself from the other candidates and win the election. Then, for the candidate i, the option (2) gives the gain

$$2(w-c) - ad_i = 2(w-c) - 2(w-c) \tag{4.23}$$

$$= 0. \tag{4.24}$$

In the option (3), candidate i joins a party and make advertisements. Though joining a party works to differentiate himself/herself from the unaffiliated candidate, advertisements work to differentiate himself/herself from the other party-affiliated candidate. Therefore, the amount of advertisements is

$$ad_i = (w - c - \alpha z_j^2) + (w - c), \tag{4.25}$$

which is equal to the gain of the other affiliated candidate, since j can not pay that amount of money even if he/she can win the election. The voters can then distinguish between better and worse candidate even in the first election by observing whether the candidates have advertised or not. For the candidate i, the option (3) gives

$$\{(w - c - \alpha z_i^2) - [(w - c - \alpha z_j^2) + (w - c)]\} + w - c \tag{4.26}$$

$$= \alpha z_j^2 - \alpha z_i^2. \tag{4.27}$$

Comparing these three options (1), (2) and (3), we can say that candidates' best response depend on the relative positions of them. If $\theta^2 < |z_j|^2 - \frac{1}{2}|z_i|^2$, the candidate i chooses the option (3) and get elected. If $\theta^2 > |z_j|^2 - \frac{1}{2}|z_i|^2$, the candidate i chooses the option (1) and get elected with probability $\frac{1}{2}$. On the other hand, it is clear that in both cases, the candidate j's best response is also joining a party and making no advertisement.

Therefore, we can conclude that these i and j's strategy is the equilibrium.

4.4 Political Party and Political Advertisement

We have found that both political parties and political advertisements work as signals to voters. In this section, we would like to explain in words about the difference between political parties and political advertisements.

Basically, political parties work more efficient signals than advertisements. This is because joining parties costs candidates heterogeneously, while making advertisements costs candidates homogeneously. First, consider the political parties. Candidates who are located further to the parties' ideal policy pay larger cost to join the parties than candidates who are located nearer to the parties' ideal policy. This is seen in (4.1). Next, consider the political advertisements. When candidates make political advertisements, all candidates pay the same cost to make the same amount of advertisements. In short, joining parties is a better signal for promising candidates than advertisements because it costs less.

Therefore, in all cases, promising candidates who are located nearer to the parties' ideal policy firstly join political parties. However, joining parties is sometimes not enough to differentiate themselves from the other candidates. They need additional signal, political advertisements, though it is less efficient for promising candidates. When is the first signal, joining a party, not enough? The answer is mainly when the gain of winning an election is very different among candidates (e.g. two-time winner vs one-time winner). Then, candidates have stronger incentives to differentiate themselves from other candidates. They make less efficient political advertisements in addition to joining political parties.

In short, political advertisements work as a complement signal to political advertisements. With the two signals, the promising candidates can make voters distinguish them from the other candidates. Voters can choose adequate candidates even in the first election.

4.5 Summary

Let us summarize the main points of this paper. Investigating a multi-period version of Snyder and Ting (2002), we study the signaling aspect of political parties and political advertisements. Political advertisements work as a complementary signal to political parties: political parties are basically more efficient signals. However, candidates who have strong incentives to win and need more signals make advertisements in addition to joining parties.

We can conclude that both political parties and advertisements help voters to adequately choose their ex-post good politicians in the election. Though political advertisements can be regarded as social cost in a sense that they do not directly yield any utility or gain, they signaling tool that conveys correct information about politicians types.

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Chapter 5

Measuring Quality Changes for Consumer Goods through Quality Engel Curves in the Japanese Economy

abstract:

This study examines the extent to which the Japanese Consumer Price Index (CPI) controls for quality changes, without employing the hedonic approach. Unit price changes can be divided into two components: quality change and pure price change. To distinguish between the two, we use the slopes of quality Engel curves that are correlated with quality change and uncorrelated with pure price change. Applying this basic idea to our Instrumental variable estimation, we measure the extent to which the CPI controls for quality changes. The results of our estimation reveal that the Japanese CPI inflation rate captures only 41.3% of quality change rate with respect to consumer goods, with the exception of food-related goods. Our approach is suitable to the Japanese data because the Japanese Family Income and Expenditure Survey has noticeable features for estimating quality Engel curves.

5.1 Introduction

New goods of high quality typically have high unit prices[1]. Such unit price changes are not equal to pure price change because quality changes are in fact a fraction of the real output changes as far as quality changes affect consumers' utility. Statistics concerning pure price change (and inflation in the whole economy) aims to control for quality changes and, thus, report 'quality-adjusted prices'. In Japan, the prices used in constructing the consumer price index (CPI) are quality-adjusted prices. However, measuring the quality of goods is difficult, and statistics only obtain a round estimate through baseless procedures. It is unclear whether these quality-adjusted prices accurately control for quality changes. If there is quality upgrading, but the CPI prices do not entirely capture it, the CPI inflation rate is biased upwards.

This study examines the extent to which the Japanese CPI controls for quality changes. The basic idea is as follows: we assume that price changes can be divided into two components: quality change and pure price change[2]. Generally, it is difficult to draw a distinction between the two components, because they are unmeasurable. To measure the pure price change, it is necessary to find variables that are correlated with quality change but uncorrelated with pure price change. We propose that the slopes of 'quality Engel curves' are legitimate candidates for such variables. Using the slopes of quality Engel curves, we can divide the two components. By applying this basic idea to our estimation in which the slopes of quality Engel curves work as an an instrumental variable, we can estimate the extent to which the CPI controls for quality changes. We employ the following two types of price data: quality-adjusted prices in CPI list and unit prices. The unit price for each good is obtained from the Family Income and Expenditure Survey (FIES).

A quality Engel curve refers to the relationship between household income and the

[1] We refer to the market price as 'unit price'.

[2] In this study, the terms 'pure price change' is used for each good while the term 'inflation' is used for the whole economy.

unit prices of certain goods. For instance, according to the cross-sectional data, wealthy households purchase more expensive automobiles than do non-wealthy households, whereas the price difference is not so wide with respect to the types of socks that both types of households purchase. Thus, we regard the slope of the quality Engel curve of automobiles to be steeper than that of socks. Note that a well-known Engel curve refers to the relationship between income and expenditure with respect to certain goods. However, the quality Engel curve is the relationship between income and unit price. Our finding reveals that the slopes of quality Engel curves display a highly positive correlation with unit price changes. In the previous example, the price of an automobile increases more rapidly than that of socks. Since such property is reflected in the slopes of the quality Engel curve, the slope can be regarded as a plausible candidate for an instrument for our study.

Our approach is based on Bils and Klenow (2001) and very different from the hedonic approach, which is a popular method for measuring quality changes. The hedonic approach is often applied to cars, computers and other durable goods, as surveyed in Shiratsuka (1998). However, we can only apply this approach to a limited number of goods because the quality and performances of most types of goods are unclear and unmeasurable. For instance, the performances and quality of clothes, beds or haircut is difficult to define and measure. Few studies estimate quality changes for a wide range of goods simultaneously. In the U.S., the Boskin Commission Report (Michael J. Boskin et al., 1996) roughly estimates that unmeasured quality changes bias the U.S. CPI inflation rate upwards by 0.6% per year. Bils and Klenow (2001) were the first to suggest the concept of the quality Engel curve. Our strategy is based on this, and it is compatible with Japanese data. The U.S. household cross-section data obtained from the Consumer Expenditure Survey (CEX) does not report the prices and quantities of purchased goods separately. Consequently, to estimate the quality Engel curves, Bils and Klenow (2001) had to separate the price and quantity data from the expenditure data. Therefore, they could only focus on durable goods. On the other hand, in Japan, the FIES reports the prices and quantities separately for a wide range of goods. Thus, we can deal with a wider range of goods in the context of the Japanese economy.

This chapter is organized as follows. Section 2 provides the equation that we would like to estimate. Our estimation equation examines to what extent the CPI controls for quality changes. In section 3, for our estimation, we introduce our instrumental variables, namely, the slopes of the quality Engel curves. In section 4, we provide an overview of the data—obtained from the FIES and CPI —on which our estimation is based. Section 5 presents the results of our estimation and compare them with preceding literatures. In section 6, we see the difference between our approach and existing hedonic approach. In section 7, we provide a microeconomic foundation of quality Engel curves based on product cycle theory. Section 8 summarizes the conclusions of this study.

5.2 Estimation Equation

Our goal is to measure the extent to which the Japanese CPI controls for (or omits) the quality changes of many consumer goods. In this section, we describe our estimation equation. First, we divide unit prices into two components—quality and other factors.

$$x_{it} = q_{it} z_{it} \tag{5.1}$$

where x_{it} denotes the unit price for good i at time t, q_{it} denotes the quality component and z_{it} denotes the other residual factors. We can define z_{it} as the other factors than quality change that affect unit price. Examples of the factors include oil price, monetary policy and competition environment. Note that x_{it} is the market price without any quality-adjustments. x_i is a measurable variable, while q_i and z_i are unmeasurable variables. Next, the change rates of both sides become

$$\overline{\Delta x_i} = \overline{\Delta q_i} + \overline{\Delta z_i}. \tag{5.2}$$

where Δ denotes the change rate, and overbar denotes the time average. The only variation remaining is that occurring across goods i. $\overline{\Delta z_i}$ can be interpreted as the pure price change rate of good i. In contrast to unit prices, quality-adjusted prices aim to measure pure price change by holding quality constant, that is, quality-adjusted price change rate should be equal to $\overline{\Delta z_i}$. In this chapter, we refer to quality-adjusted prices in CPI list simply as 'quality-adjusted prices', and denote the quality-adjusted price for good i at time t as $p_{i\,t}$. Ideally, $\overline{\Delta p_i} = \overline{\Delta z_i}$.

However, we presume that the CPI inflation rate is inaccurate in the sense that it still includes quality changes to a certain extent. We assume that the quality-adjusted price change rate $\overline{\Delta p_i}$ in goods prices in the CPI still includes quality change rate at the constant rate of μ across goods:

$$\overline{\Delta p_i} = \overline{\Delta z_i} + \mu\overline{\Delta q_i} + \epsilon_i \tag{5.3}$$

Here, quality-adjusted price p_i is a measurable variable in the CPI list. μ can be interpreted as the fraction of quality growth rate that incorrectly shows up as inflation in the CPI price deflators. In other words, the CPI inflation rate can eliminate only a fraction $(1 - \mu)$ of the quality growth rate. If quality-adjusted prices completely capture and exclude quality changes, $\mu = 0$ and $\overline{\Delta p_i} = \overline{\Delta z_i}$. Notice that μ does not depend on i. μ can be seen as the overall performance of quality-adjusted prices. Finally, ϵ_i denotes the measurement error. While quality-adjusted price p_i measures price of one specific good chosen in category i, z_i, q_i and x_i measures average price of many goods in category i. Here, the index i does not exactly refer to the same goods. Therefore, measurement error ϵ_i is included in the equation (5.3). In words, we assume that CPI samples randomly depart from the average samples in the category. We also assume that

$$E(\epsilon_i \mid \overline{\Delta z_i}, \overline{\Delta q_i}) = 0, \tag{5.4}$$

and ϵ_i is independent of $\overline{\Delta z_i}$, $\overline{\Delta q_i}$ and hence $\overline{\Delta x_i}$. This assumption is considered to be reasonable because the selection error of CPI basket seems to be determined independently of market price change. Note that we admit that market price change affects the selection of CPI basket. This does not contradict the independency of ϵ_i. The independency of ϵ_i only claims that market price change does not affect selection error of CPI basket.

Combining (5.2) and (5.3) yields the following relation between the quality-adjusted price and unit price inflation (substituting out the unmeasurable variable $\overline{\Delta q_i}$):

$$\overline{\Delta p_i} = \mu\overline{\Delta x_i} + (1-\mu)\overline{\Delta z_i} + \epsilon_i \tag{5.5}$$

Equation (5.5) provides our estimation equation. Using this equation, we estimate μ. Δp_i (i.e. the price change rate of good i in CPI list) and Δx_i (i.e. the price change rate of good i in FIES) are both measurable variables in the data. In our estimation, we regard the last term $(1-\mu)\overline{\Delta z_i} + \epsilon_i$ as the error term.

However, $\overline{\Delta x_i}$ and $\overline{\Delta z_i}$ are clearly positively correlated. This correlation can be easily verified in equation (5.2). In conclusion, this regression has the endogenous regressor $\overline{\Delta z_i}$. To estimate μ consistently we require some instrumental variables which satisfy the two conditions given below:

(1) correlated with $\overline{\Delta q_i}$ and

(2) uncorrelated with $\overline{\Delta z_i}$.

In the next section, we introduce the slopes of quality Engel curves as good candidates for an instrument which satisfies these two conditions.

5.3 Instrumental Variables

We make use of the slopes of quality Engel curves as an instrument to estimate μ in equation (5.5). In subsection 1, we explain the concept of quality Engel curves and estimate their slopes for 44 consumer goods in the Japanese economy based on the panel data of FIES. In subsection 2, we demonstrate why the slopes of quality Engel curves are good candidates for an instrument to estimate equation (5.5).

5.3.1 Quality Engel Curves

The concept of quality Engel curves was first proposed by Bils and Klenow (2001). According to the cross-sectional household data, wealthy consumers tend to purchase more expensive automobiles than non-wealthy consumers do. We refer to this relationship between income and the prices of automobiles as the quality Engel curve for automobiles (see Figure 1). There are two lines: the upper line represents the quality Engel curve for automobiles, while the lower line represents that for socks. The slopes of the quality Engel curves vary across goods. In Figure 1, the slope for automobiles is steeper than that for socks. This is because wealthy households purchase more expensive automobiles than do non-wealthy households, while the socks that wealthy households purchase are only moderately more expensive. This is consistent with our intuitions: though, there are very expensive automobiles that normal people cannot afford, there are not very expensive socks in the market. The number of slopes of quality Engel curves that we can obtain is equal to the number of goods. Note that the well-known Engel curves represent the relationships between income and expenditure with respect to certain goods; on the other hand, quality Engel curves focus on the prices of goods and not on consumers' expenditure.

To estimate the quality Engel curve for each good, we use data obtained from the FIES conducted in Japan. The FIES reports the prices and quantities separately for many consumer goods with respect to 18 household income classes. Therefore, for each good i, we obtain the panel data of households (j) at time (t). This is one of the advantages of using Japanese economy data. FIES reports the prices of wide variety of goods that each consumer faces at the market. The following is our estimation equation for the slopes of the quality Engel curves:

$$\log x_{ijt} = \alpha_{it} + \theta_i \log Y_j \tag{5.6}$$

where x_{ijt} is the unit price for good i consumed by income class j at time t, Y_j is the average income for income class j and the coefficient θ_i is the slope of quality Engel curve for good i. Note that Y_j does not depend on t because the income classifications in the FIES are constant over time. For example, in every year, the FIES reports the prices and quantities purchased by the annual income class 1-1.5 millions Yen, as well as 1.5-2 and 2-2.5 millions yen respectively. We assume that the constant coefficient α_{it} can vary over time, on the other hand, the slope of the quality Engel curve θ_i remains constant over time. In brief, there is a fixed effect at any given time. One reason for this is that certain types of shocks such as monetary disturbances affect the unit price of each good equally. Therefore, our estimator for the slopes of the quality Engel curves θ_i (and also for α_{it}) is the 'fixed-effects estimator', which is based on the panel data. The results are presented in Table 4.1. Goods such as 'beds', 'automobiles' and 'handbags' display steeper slopes in their quality Engel curves. On the other hand, goods such as 'refrigerators', 'blankets' and 'haircut charges' display flatter slopes. We use the results of θ_i as an instrument in our estimation equation (5.5).

With respect to the quality Engel curves, we focus not only on the level of the slopes that are averaged over time but also on the time series movements of the slopes. Quality Engel curves move dynamically over time. If the slopes of quality Engel curves become flatter, it means that wealthy households become to purchase moderately expensive goods or non-wealthy households become to purchase more expensive goods. To observe the dynamic movements of their slopes, we obtain $\theta_{i\,t}$ from the cross-sectional household data, not from the panel data. The estimation equation is

$$\log x_{i\,j\,t} = \alpha_{i\,t} + \theta_{i\,t} \log Y_j \tag{5.7}$$

For each time t and good i, we regress this equation over households j through OLS (Ordinary Least Squares) regressions. The change rate of the slopes are defined by

$$\Delta\theta_{i\,t} = \frac{\theta_{i\,t} - \theta_{i\,t-1}}{\theta_{i\,t-1}}. \tag{5.8}$$

Averaging $\Delta\theta_{i\,t}$ across time t, we get another instrument $\overline{\Delta\theta_i}$.

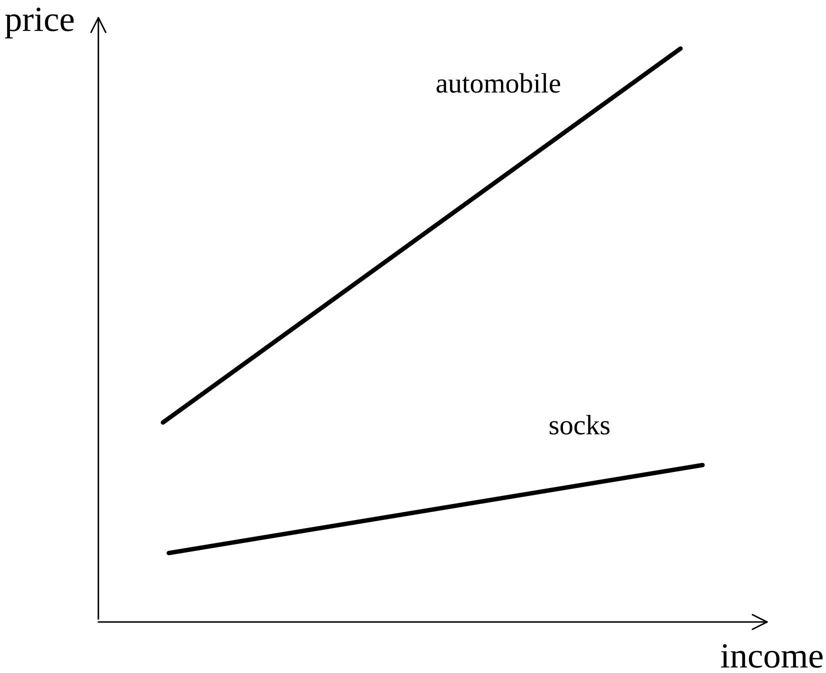

Figure 5.1: Quality Engel Curves

5.3.2 Quality Engel Curves as an Instrumental Variable

As is evident in section 2, in order for the slopes of quality Engel curvesθ_i to be good candidates for an instrument in estimation equation (5.5), θ_i needs to satisfy the following two conditions:

$$Cov\ (\theta_i\,,\ \overline{\Delta x_i}) \neq 0 \tag{5.9}$$

$$Cov\ (\theta_i\,,\ \overline{\Delta z_i}) = 0 \tag{5.10}$$

We can verify condition (5.9) using FIES data because both θ_i and Δx_i are measurable variables. We find there is a positively high correlation, with a correlation coefficient of 0.455 (see Figure 5.2). ‘Sideboards’ and ‘TV sets’, which have the highest inflation rates,

Good	Slope of Quality Engel Curve (θ_i)
Microwave Ovens	0.061
Refrigerators	0.008
Vacuum Cleaners	0.175
Washing Machines	0.174
Sewing Machines	0.117
Room Air Conditioners	0.099
Stoves and Fan Heaters	0.110
Kotatsu	0.226
Chests of Drawers	0.204
Sideboards	0.370
Beds	0.318
Quilts	0.140
Blankets	0.098
Bowls and Dishes	0.219
Thermos bottles	0.049
Pans and Kettles	0.197
Men's Suits	0.210
Men's Slacks	0.269
Skirts	0.284
Women's Slacks	0.238
Men's Business Shirts	0.152
Men's Sweaters	0.27
Women's Blouses	0.176
Women's Sweaters	0.225
Children's Shirts	0.227
Children's Sweaters	0.267
Neckties	0.213
Men's Socks	0.123
Women's Socks	0.11
Canvas Shoes	0.183
Men's Shoes	0.338
Women's Shoes	0.250
Children's Shoes	0.130
Automobiles	0.324
TV Sets	0.162
Stereo Phonograph Sets	0.041
Tape Recorders	0.210
Video Cameras	0.044
Haircut Charges	0.096
Toilet Soap	0.090
Umbrellas	0.145
Handbags	0.340
Suitcases	0.250

Table 5.1: Slopes of Quality Engel Curves

Notes: The sample is from cross sections of households in the 2001–2004 FIES.

are on the extreme right. 'Video cameras', which have the lowest inflation rate, are on the extreme left. The unit price inflation ($\overline{\Delta x_i}$) data is presented in Table 4.2 in the next section. Even if these three goods are excluded (i.e. 'sideboards', 'TV sets' and 'video

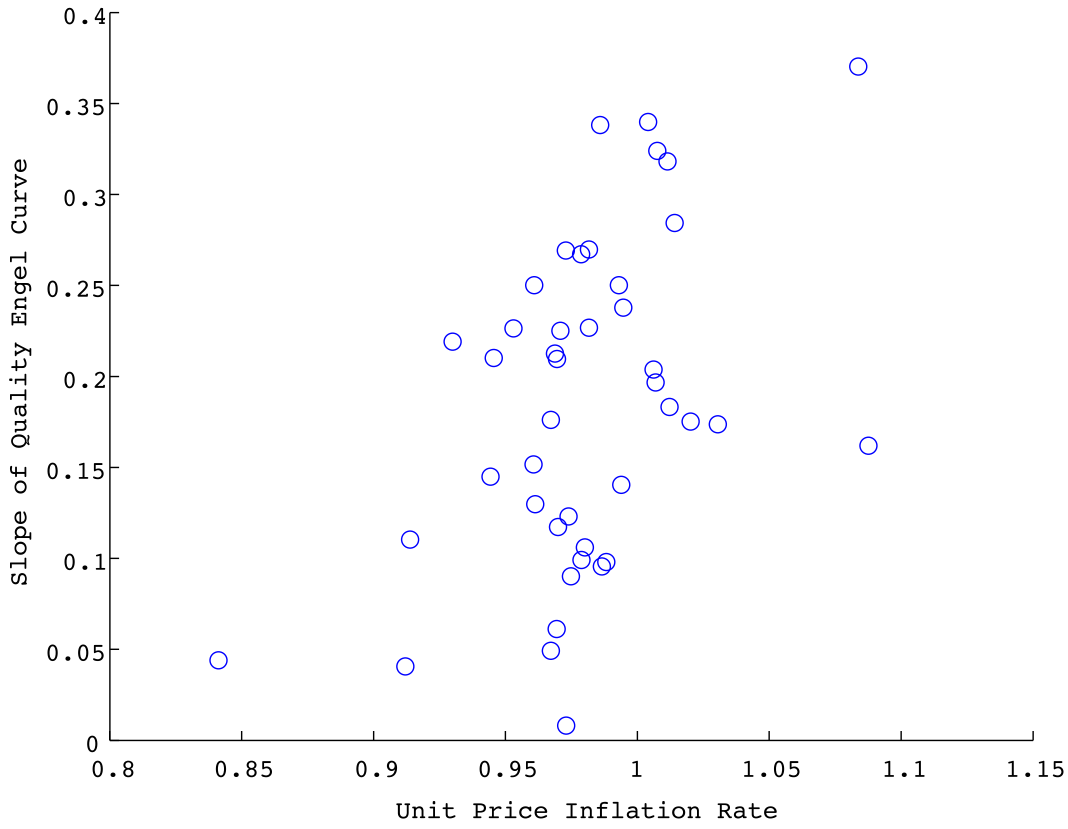

Figure 5.2: Unit Price Change Rates and the Quality Slopes

However, it is natural that we cannot verify condition (5.10) from the data. Therefore, condition (5.10) is held as an assumption. However, as will be seen in section 5.5, our instruments pass the orthogonality tests with $H = 1.61, 1.73$ respectively. Therefore, we can conclude that the condition 5.10 is satisfied in our analysis.

Why do unit prices and the slopes of quality Engel curves display such a positively high correlation? One answer to this question is provided in Aoki and Yoshikawa (2002) and Matsuyama (2002) which describes how new goods are spread to the whole economy. Though the detailed discussions are left for the later section, here, we briefly describe the main points. Initially, when new and high-quality goods enter the market, only wealthy households purchase these high-priced goods. Gradually, through the competition and learning-by-doing effect, the prices of these goods decline and non-wealthy households become able to buy these high-quality goods. Thus, in the first stage, only wealthy households purchase new and high-priced goods, and hence estimated quality Engel curves get steeper. In the second stage, non-wealthyer households become to purchase the goods because the price has declined, and hence estimated quality Engel curves get flatter. Through these product cycles, θ_{it} dynamically moves over time and the goods with steeper quality Engel curves display higher price increases. For example, LCD televisions actually seem to display this type of product cycle from 2001 to 2006 in Japan. In 2001, when LCD televisions come into the market, the quality Engel curve of TV got steeper. We observe that, until 2006, the quality Engel curve became flatter because of the fierce competitions and price decrease.

We can also verify the correlation between the instrument $\overline{\Delta\theta_i}$ and $\overline{\Delta x_i}$. The correlation coefficient is 0.343. If we assume $Cov(\theta_i, \overline{\Delta z_i}) = 0$ and $Cov(\overline{\Delta\theta_i}, \overline{\Delta z_i}) = 0$, we can use both θ_i and $\overline{\Delta\theta_i}$ as instruments in equation (5.5).

5.4 Data on Prices

This section provides an overview of the price data we use in estimation equations (5.5) and (5.6). The unit prices are obtained from the FIES, and the quality-adjusted prices, from the CPI.

5.4.1 Unit Prices

We require two types of unit price data, namely, x_{ijt} in equation (5.6) and $\overline{\Delta x_i}$ in equation (5.5).

The FIES reports the unit prices and quantities separately for many consumer goods with respect to 18 household income classes. These unit prices are those that households encounter in the market and are assumed to reflect fully their quality. Therefore, we construct x_{ijt} in equation (5.6) based on FIES data. In contrast to the CEX conducted in the U.S., the FIES reports the prices and quantities of goods separately.

For instance, it reports that people in each income class purchase how many quantities at how much price. This is our advantage to Bils and Klenow (2001) that employ CEX data. Bils and Klenow (2001) were required to estimate quality changes by extracting the unit prices of purchased goods from the expenditure data; thus, they could only estimate μ for durable goods. In contrast, we have price data on a wide range of goods in the Japanese economy including service goods, clothes, and durable goods. FIES data is better suited for our approach than CEX data. It should be noticed that we exclude date on food goods from the samples for two reasons. First, Bils and Klenow (2001) and other related literature employ date on durable goods. While Bils and Klwnow (2001) faces difficulty in estimating quantity in food goods, other related literature that mainly employs the hedonic approach faces the problem of the definition of quality of food goods. Second, there are not much quality upgrading on food production, comparing to durable goods including TV, fan heaters, and video cameras.

Moreover, the FIES reports the unit price of each good averaged across households. These unit prices are also not quality-adjusted. In addition, in estimation equation (5.5), we construct the unit price inflation for each consumer good ($\overline{\Delta x_i}$) based on the FIES. We employ the data of 'Two-or-more-person households', not 'total households' because the data on all households does not report prices pertaining to the narrower income classes. The resulting inflation rates are presented in the first of Table 4.2. The most extreme increases are for 'washing machines' (3.05%), 'sideboards' (8.38%) and 'TV sets' (8.76%). The most extreme declines are for 'stoves and fan heaters' (−8.62%), 'stereo phonograph sets' (−8.8%) and 'video cameras' (−15.9%).

5.4.2 Quality-Adjusted Prices

We construct quality-adjusted prices $\overline{\Delta p_i}$ from the CPI, which reports the prices in each prefecture but not the average price of a country as a whole. Therefore, we calculate the average of all the prefecture prices and regard it as the average price for the entire country. The resulting quality-adjusted price change rates are presented in the second column of Table 4.2. The most extreme increases are for ‘washing machines’ (23.66%), ‘room air conditioners’ (9.40%) and ‘men’s shoes’ (11.3%). The most extreme declines are for ‘vacuum cleaners’ (−17.92%), ‘Kotatsu’ (−11.64%) and ‘stereo phonograph sets’ (−10.95%).

The Japanese CPI reports the quality-adjusted price for each consumer good. These prices are constructed on the basis of certain representative goods in the market. The quality-adjusted price is required to continuously substitute the appropriate representative goods over time in accordance with changes in product cycles. In such cases, to capture the pure price change, The quality-adjusted price must exclude quality changes from nominal price changes. These adjustments are called ‘specification changes’.

According to Shiratsuka (1999), the current Japanese CPI primarily employs the following three methods to determine specification changes.

1. Direct Comparison Method:

   If there is no quality and price change, then the specification is directly taken over by the new good.

2. Price Link Method:

   If there is an apparent quality improvement accompanying a price increase, the CPI conducts a specification change on the assumption that there is no pure price change.

3. No Quality and Price Change:

> If there is no quality change accompanying with a price increase, the CPI regards this price change as a pure price change.

These methods for determining specification changes are clearly inaccurate. For example, the price link method is true only if $\Delta q_i = \Delta x_i$, or equally $\Delta z_i = 0$, in equation (5.2). When $\Delta q_i > \Delta x_i$, price link method estimates lower quality changes than the true value. Conversely, When $\Delta q_i < \Delta x_i$, price link method estimates higher quality changes than the true value. Moreover, in the first and third methods, the meaning of quality change is unclear. For these reasons, the CPI inflation rate is often considered to be biased.

To make quality-adjusted price more accurate for pure price change, we should employ some other ways. One approach is the hedonic analysis, which is surveyed in the section (5.6). This study can be regarded as an alternative approach to measure pure price change, or equivalently quality change.

Good	Unit Price Change Rate	Quality-Adjusted Price Change Rate
Microwave Ovens	-3.06 %	-6.35 %
Refrigerators	2.69 %	0.42 %
Vacuum Cleaners	2.02 %	-17.92 %
Washing Machines	3.05 %	23.66 %
Sewing Machines	-3.01 %	-1.54 %
Room Air Conditioners	-2.12 %	9.40 %
Stoves and Fan Heaters	-8.62 %	-7.19 %
Kotatsu	-4.70 %	-11.64 %
Chests of Drawers	0.61 %	0.35 %
Sideboards	8.38 %	1.18 %
Beds	1.15 %	-1.90 %
Quilts	-0.61 %	-3.10 %
Blankets	-1.17 %	-1.23 %
Bowls and Dishes	-7.00 %	2.99 %
Thermos bottles	-3.28 %	-3.60 %
Pans and Kettles	0.70 %	-0.70 %
Men's Suits	-3.05 %	-3.54 %
Men's Slacks	-2.71 %	-1.31 %
Skirts	1.41 %	-2.78 %
Women's Slacks	-0.52 %	-0.85 %
Men's Business Shirts	-3.93 %	-3.01 %
Men's Sweaters	-1.83 %	-1.86 %
Women's Blouses	-3.28 %	-4.96 %
Women's Sweaters	-2.92 %	-4.28 %
Children's Shirts	-1.83 %	-0.78 %
Children's Sweaters	-2.13 %	-1.07 %
Neckties	-3.13 %	-1.56 %
Men's Socks	-2.60 %	-1.84 %
Women's Socks	-1.98 %	0.52 %
Men's Shoes	-1.41 %	11.3 %
Women's Shoes	-0.70 %	-1.44 %
Children's Shoes	-3.87 %	0.47 %
Automobiles	0.76 %	1.09 %
TV Sets	8.76 %	-7.16 %
Stereo Phonograph Sets	-8.79 %	-10.95 %
Tape Recorders	-5.44 %	-7.94 %
Video Cameras	-15.88 %	-8.56 %
Haircut Charges	-1.34 %	-0.12 %
Toilet Soap	-2.51 %	-1.72 %
Umbrellas	-5.57 %	-2.74 %
Handbags	0.42 %	9.36 %
Suitcases	-3.91 %	-3.67 %

Table 5.2: Unit Price and Quality-Adjusted Price for Consumer Goods
Notes: The sample is obtained from FIES and quality-adjusted prices in the CPI list in 2001-2004.

5.5 Results and Discussions

This section presents the results of our estimation. The fraction of the mismeasurement of quality change μ in equation (5.5) is estimated by the generalized method of moment (GMM). Finally, we compare our results with related literature and discuss the difference of them.

5.5.1 Results

To begin with, we estimate μ in equation (5.5) using only the instrument θ_i. The regressors $\overline{\Delta p_i}$ are based on quality-adjusted prices, and $\overline{\Delta x_i}$ and θ_i are based on the FIES, as described in the previous section. The sample size is 43; the list of samples are presented in Tables 4.1 and 4.2. We select those samples from the FIES that have corresponding goods listed in the CPI. We exclude food-related goods from the samples because these goods do not satisfy condition (5.9). The following is the sample correlation coefficient:

$$Correlation\ (\theta_{food}\,,\ \overline{\Delta x_{food}}) = 0.0832. \tag{5.11}$$

This is because there is relatively less unit price change and quality changes in food-related goods.

Therefore, we estimate equation (5.5) based on 43 samples, excluding food-related goods. The result is presented in Table 4.3. We obtain the value of the mismeasurement ratio as $\mu = 0.608$, that is, the Japanese CPI inflation rate omits 60.8% of all quality change rate. In other words, the Japanese CPI inflation rate capture only 39.2% of all quality change rate.This clearly rejects the hypothesis that $\mu = 0$.

Next, using the change rate of quality Engel curves $\overline{\Delta\theta_i}$ as another instrumental variable, we estimate μ by the GMM. The result is also presented in Table 4.3. The Japanese CPI inflation rate omits 58.7% of all quality change rate. In addition, our instruments pass the orthogonality tests with $H = 1.61, 1.73$ respectively.

instrument set	μ	adjusted R^2
θ_i	0.608 (0.134) $t = 4.70$	0.561
θ_i, $\overline{\Delta\theta_i}$	0.587 (0.126) $t = 5.01$	0.602

Table 5.3: Estimates of μ

Notes: The number of observations is 43. μ is the fraction of quality change rate that incorrectly shows up as pure price change rate. θ_i is the quality slope for good i. The estimation equation is $\overline{\Delta p_i} = \mu\overline{\Delta x_i} + (1-\mu)\overline{\Delta z_i}$. Here μ is estimated by the GMM.

5.5.2 Discussions on the Results

Here, we compare our results with related literature and discuss the difference of them. Bils and Klenow (2001) adopts the similar approach to ours and estimates the accuracy of the U.S. CPI. Shiratsuka (1995a), (1995b) and Shiratsuka and Kuroda (1995) adopts hedonic approach and estimates the accuracy of the Japanese CPI.

First of all, Bils and Klenow (2001) which is the basis of our approach estimates mismeasurement of the U.S. CPI. It claims that the U.S. CPI inflation rate still includes the quality change rate about 0.601% to 0.618%. In other words, the U.S. CPI inflation rate only excludes 0.382% to 0.399%. Our results that estimate the Japanese CPI gives similar results. In our estimation, $\mu = 0.587$ or $\mu = 0.608$, which means that the Japanese CPI inflation rate still includes about 60% of quality change. In other words, the Japanese CPI inflation rate only controls 0.392% to 0.413% of all quality change rate. Because our approach is based on Bils and Klenow (2001), we can directly compare our results with theirs. We can say that Japanese CPI has almost similar accuracy in controlling quality changes to the U.S. CPI.

Next, Shiratsuka (1995a), (1995b) and Shiratsuka and Kuroda (1995) employ the hedonic approach to estimate the quality changes of some durable goods such as automobiles, camcorders, and computers. For example, the Japanese CPI inflation rate (i.e.

quality-adjusted price change rate) is 0.1% for automobiles, and −4.0% for camcorders. However, the hedonic approach estimates −4.0% and −9.6% respectively. In words, the Japanese CPI inflation rate is upward biased when the economy falls into deflation. The accuracy of quality change control is below 50% in these samples. This result corresponds to our result that claims that the accuracy is about 40% as presented above.

To sum up, comparing our results with existing literatures, we can claim as follows:

- Accuracy of the Japanese CPI in quality adjustment is at the similar level to the U.S. CPI.
- Our estimations are in line with the estimations by the hedonic approach.

In the next section, we review hedonic approach that is referred above and our approach.

5.6 Preceding Approach and Our Approach

We estimate quality changes by using quality Engel curves. This approach was first employed by Bils and Klenow (2001). In contrast, the preceding approach in quality measurement is called as hedonic approach. In this section, we overview this hedonic approach and discuss the difference between hedonic approach and our approach.

5.6.1 Preceding Approach: Hedonic Approach

One typical way to measure quality changes is the hedonic approach whose theoretical background was first provided by Lancaster (1966). Nowadays, the hedonic approach is widely used by many economists and government institutions. The basic idea of hedonic approach is as follows: utility is derived by quality of the good, and quality is composed of many kinds of visible features of the good. Then, the value of the good is the summation of the values of these features. Then, quality upgrading can be measured as the value of the upgrading of these features.

Lancaster (1966) first laid the theoretical foundation of the hedonic approach. Consumer's utility function is

$$U = U(c_1, c_2, ..., c_J).$$

where c_j is the total amount of characteristic j. There are J characters that yield utility. Next, characteristic j is included in each good:

$$c_j = \sum_i V_{ij} a_i.$$

where V_{ij} is the amount of the characteristic j in good i, and a_i is the amount of good i. Then the consumer's maximization problem is

$$\max_{\mathbf{c},\mathbf{a}} \; U(\mathbf{c}) \tag{5.12}$$

$$s.t. \;\; \mathbf{c} = \mathbf{V}\mathbf{a} \tag{5.13}$$

$$Y \geq \mathbf{p}\mathbf{a}, \tag{5.14}$$

where $\mathbf{c}$ is the vector of c_i, $\mathbf{V}$ is the matrix of V_{ij}, Y is the consumer's income, $\mathbf{a}$ is the vector of a_i, and $\mathbf{p}$ is the price vector of goods. We obtain the consumer's optimum $\mathbf{a}^*$ and $\mathbf{c}^*$ from the maximization problem above.

Next consider the problem that gives the same answer as the problem above. Here, the consumer chooses quantities of goods to achieve the optimum $\mathbf{c}^*$.

$$\min_{\mathbf{a}} \; \mathbf{p}\mathbf{a} \tag{5.15}$$

$$s.t. \;\; \mathbf{V}\mathbf{a} = \mathbf{c}^*. \tag{5.16}$$

The dual problem can be written as

$$\max_{\rho} \ \rho \mathbf{c}^* \tag{5.17}$$

$$s.t. \ \rho \mathbf{V} = \mathbf{p}. \tag{5.18}$$

where ρ can be regarded as the shadow price vector of the characteristics. This constraint is the hedonic function itself.

In empirical analysis, measurable characteristics are regressed on the price of the good. Then, we can obtain the shadow price of characteristics, ρ. If there are measurable improvements of characteristics in the good, we can calculate what is the contribution of the improvement to the price increase. For example, the measurable characteristics of personal computers are Central Processing Unit (CPU), memory, display, and so on. Hedonic approach regresses these characteristics on the price of personal computers. If there are marginal improvements of CPU, the coefficient of the regression calculates the contributions of the improvements to price. As a result, we can measure quality-adjusted price. This is the basic idea of the hedonic approach.

5.6.2 Our Approach

Our approach estimates the estimation equation (5.5)

$$\overline{\Delta p_i} = \mu \overline{\Delta x_i} + (1 - \mu)\overline{\Delta z_i} + \epsilon_i. \tag{5.19}$$

The instrumental variables, slopes of quality Engel curves, is estimated by

$$\log x_{ijt} = \alpha_{it} + \theta_i \log Y_j \tag{5.20}$$

$$\log x_{ijt} = \alpha_{it} + \theta_{it} \log Y_j \tag{5.21}$$

$$\Delta\theta_{it} = \frac{\theta_{it} - \theta_{it-1}}{\theta_{it-1}}. \tag{5.22}$$

What is the difference between the hedonic approach and our approach? What is the advantages and disadvantages of our approach? First of all, our approach can apply to the goods in which **V** and **a** are invisible or unmeasurable. For example, we can not apply the hedonic approach to service goods including clothes and haircut. This is because quality of clothes and haircut is unmeasurable: quality of clothes is affected not only by the amount of wool, cotton, and rayon but also by design. However, design is unmeasurable. In this sense, the applications of hedonic approach is limited mainly to durable goods like personal computers and automobiles.

However, our approach can apply to these types of service goods. What is needed for our approach is the data of quality-adjusted price, market price and slopes of quality Engel curves. Especially for estimating quality Engel curves, the only requirement for applying our method is that the price and quantity components of the expenditure data be separable for statistical calculations. Our approach can not only measure durable goods, but also service goods.

There is also disadvantages in our approach. One of them is that our approach can not deal with each good separately. Our approach measures how the CPI inflation rate control quality change rate of sample goods on average. We obtain one coefficient μ in the estimation. In contrast, hedonic approach can apply to each good separately. the Hedonic approach answers to the question about what is the quality upgrading and pure price change on each good. It should be noticed that this property of our approach can be advantageous. This is because in our approach we do not need to treat each good, and not need to run regression on each good. Our method can simultaneously consider a wide variety of goods. The overall performance of quality-adjusted prices can be examined through a single estimation.

In summary, our approach can apply wider variety of goods than the preceding hedonic approach. Additionally, our approach is easier to calculate overall performance of CPI because we only need to run a single regression. However, our approach can not tell the quality-adjusted price of each good.

5.7 Quality Engel Curves and Product Cycles

Why do unit prices and the slopes of quality Engel curves display such a positively high correlation? In this section, we introduce one theoretical background of this fact and interpret it.

Our explanation to this question is in line with Aoki and Yoshikawa (2002) and Matsuyama (2002). Both studies theoretically describe the diffusion of new goods to the economy. Aoki and Yoshikawa (2002) models the economy in which life cycle of demand takes S-shaped: when a new good emerges in the market, the demand increases almost exponentially. However, the growth of the demand becomes slow down and approaches the ceiling in the end.

They claim that the household heterogeneity leads to this life cycle of demand. Our explanation also focuses on the relationship between household heterogeneity and product cycles.

We assume that there are three stages in product cycles: First, consider the stable market where all households purchase the good of similar quality and price. Then, the estimated quality Engel curve θ_{it} is flat. Second, when a new and high-priced good comes into the economy, only wealthy households purchase the new and high-priced good, and the other households still purchase the old type. Then, the estimated quality Engel curve

θ_{it} gets steeper. Third, competitions and productivity improvements lower the price of the new good, and the non-wealthyer households also become to purchase the good. Then, estimated quality Engel curves get flatter again.

Through this product diffusion process, high unit price and high θ_{it} is observed at the second stage. New and high-priced goods are purchased by wealthy households. This can be one explanation about why unit price and θ_{it} has such a high correlation. Similarly to Aoki and Yoshikawa (2002), our explanation implies that household heterogeneity has a major impact on product cycles.

Matsuyama (2002) also describes how new goods are spread to various income classes. It puts emphasis on the goods' diffusion process from wealthy to non-wealthy consumers. As far as households' heterogeneity comes from income dispersion, we can regard Matsuyama (2002) as a special case of Aoki and Yoshikawa (2002).

Here, we take an example of televisions market and describe the product cycles in the real economy. In the first half of 2000's, cathode ray tube (CRT) televisions were replaced by Liquid Crystal Display (LCD) televisions. LCD televisions came into the market around 2001, and spread to the whole market around 2006 in Japan.

The figure 5.3 describes the quality Engel curves for televisions before 2001. Before LCD televisions come into the market, the slopes of quality Engel curves were relatively flat. This fact that both wealthy and non-wealthy consumers buy televisions of similar quality implies that there are not high quality televisions that wealthy consumers want to purchase in the market. There may be unmet needs of wealthy consumers for high quality television.

When LCD televisions enter the market around 2001, quality Engel curves get steeper, which is described in the figure 5.4. This is because LCD televisions were expensive at that time, and hence, only wealthy consumers can afford them. While non-wealthy consumers still purchase normal CRT televisions, wealthy consumers switch to LCD televisions. However, after the introduction of LCD televisions to the market, the slopes of quality Engel curves gradually got flatter. This occurred around 2006, which is described in the figure 5.5.

There are two reasons: first, the price of LCD televisions were decreasing by the the process innovations, or competition among firms. Second, non-wealthy consumers became to buy such cheaper LCD televisions. In short, in addition to wealthy consumers, even non-wealthy consumers began to purchase LCD televisions that is cheaper than before.

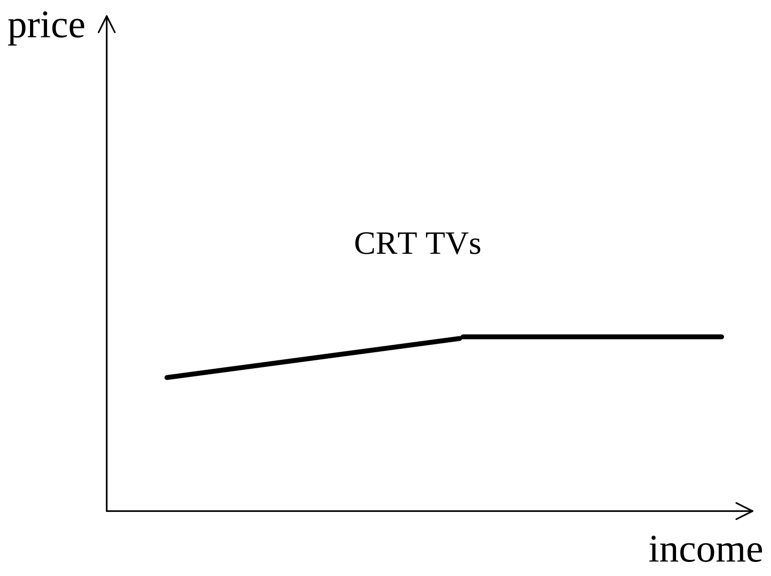

Figure 5.3: Quality Engel Curves of Televisions in 2001

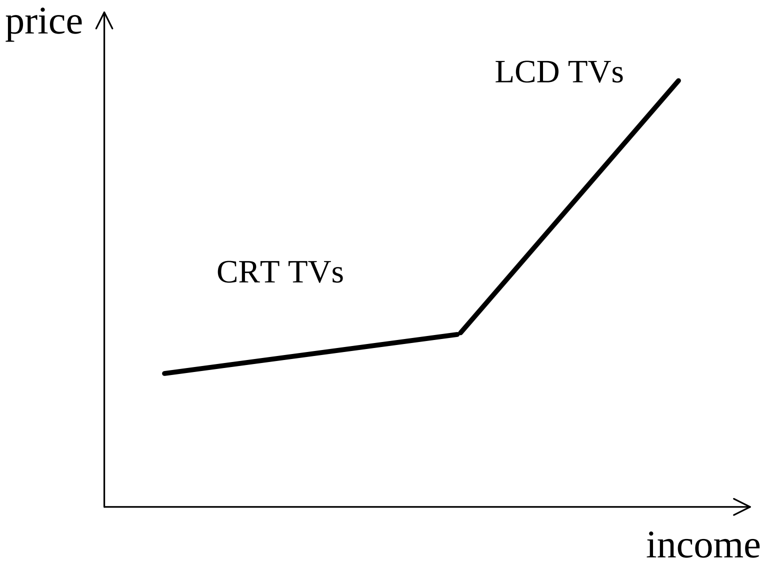

Figure 5.4: Quality Engel Curves of Televisions in 2003

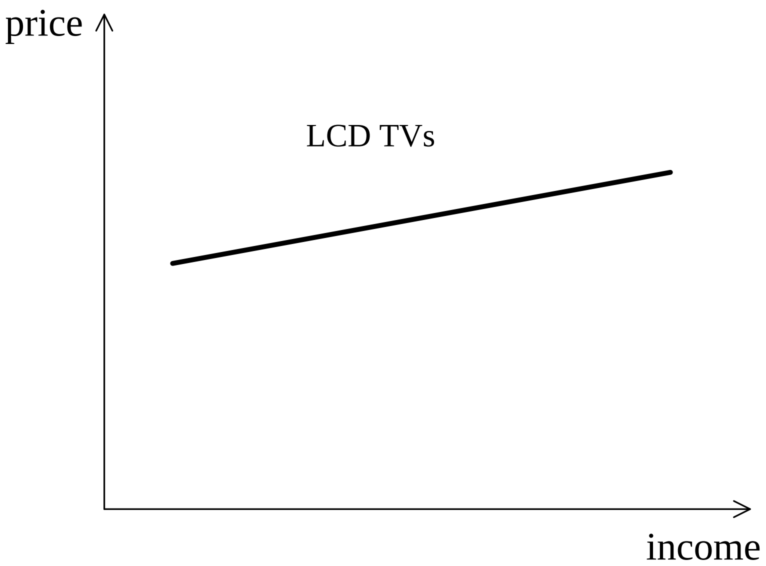

Figure 5.5: Quality Engel Curves of Televisions in 2006

Apart from the main point, investigating the dynamic movements of quality Engel curves, we observe three phases in product cycles:

1. wealthy consumers purchase newly-invented expensive goods.

2. The price of the new goods gradually decreased through process innovations or competitions.

3. non-wealthy consumers also begin to purchase these goods.

These observations imply some relationships between income distributions and product cycles. Recall that when LCD televisions came into the market, only wealthy consumers first purchased them. Their purchase encouraged the firms to do process innovations and to compete each other. As a result, the price decreased and LCD televisions spread to the whole economy.

If there were no consumers who first purchased the LCD televisions, what happens to the economy? One possible story is that there were not process innovations and competitions among firms, and hence, LCD televisions does not spread to the economy. In other words, the existence of wealthy consumers may encourage the product innovation.

On the other hand, the existence of middle and non-wealthy consumers may encourage the process innovations and competitions, because it means there is sufficient demand for the new goods and fierce competitions.

In this sense, our observations imply the relationships between income distribution and product cycles. We can discuss, first, Who is the engine of the product cycles and economic growth. Second, we can discuss whether income distribution affects product innovation and process innovation differently or not. These discussions are not developed in this study, and are left for the future research.

5.8 Conclusions

The unit price change is divided into two parts: quality change and pure price change. To distinguish between the two and measure the pure price change, we need some variables that is correlated with quality change and uncorrelated with pure price change. The slopes of quality Engel curves are legitimate candidates for such variables. Applying this basic idea to the GMM estimation that focuses on the accuracy of the CPI inflation, we estimate the extent to which the CPI controls for quality changes with the exception of food-related goods. According to our estimation, the Japanese CPI inflation rate captured only about 40% of the quality change rate in 2001-2004. Compared to related research, we can say that

(1) accuracy of the Japanese CPI in quality adjustment is at the similar level to the U.S. CPI, and

(2) our estimations are in line with the estimations by the hedonic approach in Japan.

In addition, we can say that our approach is suitable to the Japanese economy, because we can estimate the slopes of the quality Engel curves based on cross-sectional household data by making use of the noticeable features of the panel data in the Japanese FIES. Our method has two main advantages as compared to the hedonic approach, which is the preceding way to control for quality changes.

First, our method can simultaneously consider a wide variety of goods. In other words, the overall performance of quality-adjusted prices can be examined through a single estimation. On the other hand, the hedonic approach is only applicable to individual goods.

The second advantage of our approach is that it can be applied even to those goods whose 'quality' is unclear. One specific example of this is apparel goods, the quality of which depends heavily on taste and design. The quality components of each good need not be specified, as is required in the hedonic approach. Our approach can also measure the quality changes of service goods. The only requirement for applying our method is that the price and quantity components of the expenditure data be separable for statistical calculations.

It is worth mentioning that the high correlation between inflation rates and the slopes of quality Engel curves indicates some interesting features of the spread of new goods or product cycles. My conclusion is as follows: Initially, wealthy people are the first to purchase new and high-priced goods that enter the market. With the wealthy consuming the goods, through the competition and process innovations of firms, the prices of these goods gradually decrease. Finally, even the non-wealthy are able to afford them.

This story matches our empirical finding on quality Engel curves; this trend was generally observed in the market for LCD TVs in Japan from 2001 to 2006. As a future research topic, the relationship between income distribution and innovation should be examined.

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