A Simple Expression for the Additional Sky Radiance Produced by Polarization Effects

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Abstract

In sky radiance calculations for clear and turbid atmospheres, the neglect of polarization effects produces systematic errors up to about 10 %. This paper describes a technique for reducing such errors by adding a correction term to the result of scalar transfer code. A semi-empirical expression for the correction term was constructed based on the successive order theory and several parameterizations. Our expression reduces the error to about 0.6 % in the case of homogeneous, optically thin or moderately thick atmospheres.

1. Introduction

Various methods have been developed for sky radiance calculation and some of them have been generalized to include the polarization effects (Chandrasekhar, 1960; Herman and Brouwning, 1966; Kattawar and Plass, 1968; Tanaka, 1971a; Garcia and Siewert, 1986). Scalar versions (we refer the term *scalar* to the formulation neglecting polarization effects) of these methods are widely used by various investigators since they are comparatively simple to formulate and can be easily coded for computers. Their solutions, however, involve systematic errors up to about 10 % due to the neglect of polarization effects (Chandrasekhar, 1960; Tanaka, 1971b). Vector versions, in which we refer to the formulation taking account of polarization effects, on the other hand, give exact solutions for all Stokes parameters I, Q, U and V, but are comparatively more complicated and naturally require considerable computer time. In many of the practical problems, the scalar approximation turns out to be fairly accurate, so that one would not want to go through all the labor required for fully-fledged computer codings. For this reason, tables, library computer codes, and approximate formulations have been used extensively in the field of remote-sensing (see, *e.g.*, Box and Deepak, 1981).

In this study, we divide the true sky radiance into two parts, *i.e.*, the radiance calculated using the scalar approximation and that produced by polarization effects, as follows:

$$I = I_{scalar} + \Delta I.$$

We shall hereafter refer to the second term as *additional sky radiance.* If, therefore, some simple expression for ΔI is available, we can correct the results from scalar transfer codes at the expense of only a small amount of additional computer time. To the best of our knowledge, there has been no study in this direction. We use the successive order theory to yield a correction term. Unfortunately, however, the efficiency of this method is not satisfactory in its original form because of a slow rate of convergence (Sec. 3.1). We will therefore modify this technique in order to include more efficiently the contributions from the higher orders of scattering and present a simple and yet practical expression (Secs 3.2 and 3.3), and finally examine its accuracy (Sec. 4). A complete formulation for the correction term is summarized in Appendix B for convenience of application.

2. Characteristics of the additional sky radiance

First of all, we investigate several important characteristics of the additional sky radiance. Consider a plane-parallel homogeneous atmosphere of optical thickness τ_0 illuminated by solar radiation propagating toward (μ_0, ϕ_0) with its Stokes vector $(F_0, 0, 0, 0)$. The additional sky radiance is ex-

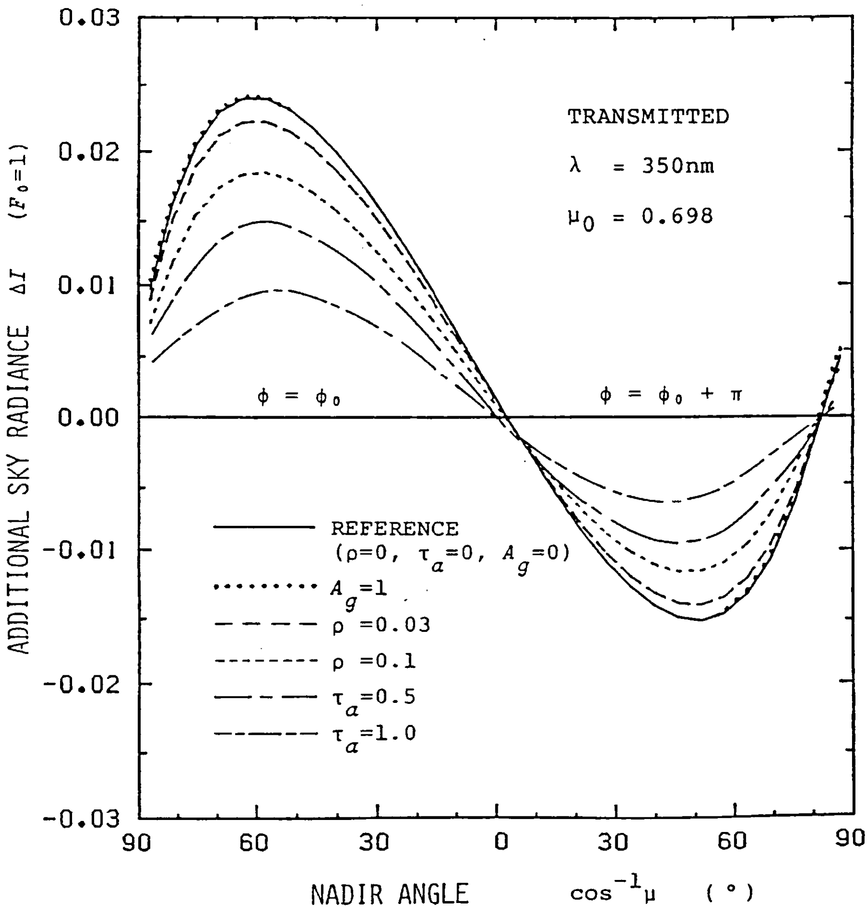


Fig. 1. The additional sky radiance in the principal plane for transmission and for various atmospheric conditions. The assumed values of the parameters are ω_m=1, $\tilde{m}$=1.5 − 0i, J=4 ,and p_0=1013 mb.

pressed as

$$\begin{aligned}\Delta I(\mu, \mu_0, \phi-\phi_0; \tau, \tau_0) \\ = I(\mu, \mu_0, \phi-\phi_0; \tau, \tau_0) \\ -I_{scalar}(\mu, \mu_0, \phi-\phi_0; \tau, \tau_0). \qquad (1)\end{aligned}$$

In the equation, I is the exact sky radiance (vector intensity), I_{scalar} is that calculated by a scalar approximation (scalar intensity), and τ is the optical depth of the observer's position measured from the top of the atmosphere. The optical thickness of the molecular atmosphere, τ_m, is assumed to be given by the following expression:

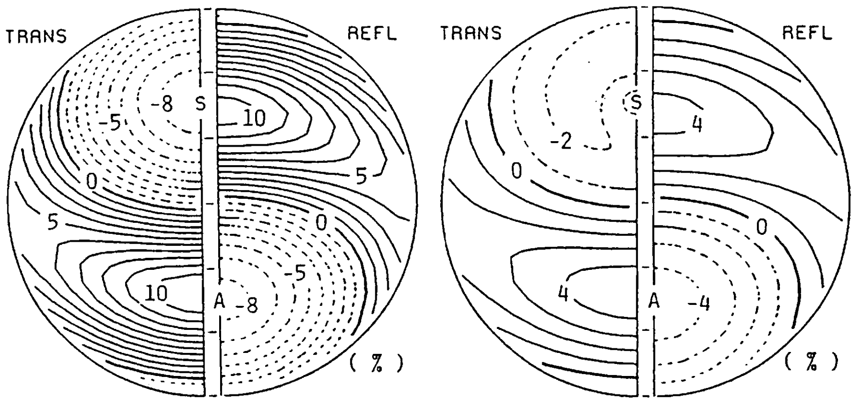
$$\tau_m = \frac{p_0}{1013} \cdot \frac{6+3\rho}{6-7\rho} \times 0.0825\lambda^{-(3.916+0.074\lambda+0.050/\lambda)}, \qquad (2)$$

where p_0 is the ground pressure, ρ is the depolarization factor of molecular scattering, and λ is the wavelength. This relation was originally proposed by Frölich and Shaw (1980) for the case of $\rho = 0.0095$. In the calculations for turbid atmospheres, the size distribution of aerosols is assumed to obey the following power law:

$$n(r) = \begin{cases} 0 \\ \quad \text{for } r < 0.01\mu\text{m and } r > 30\mu\text{m}, \\ C(0.1)^{-J} \\ \quad \text{for } 0.01\mu\text{m} \le r \le 0.1\mu\text{m}, \\ Cr^{-J} \\ \quad \text{for } 0.1\mu\text{m} \le r \le 30\mu\text{m}, \end{cases} \qquad (3)$$

where $n(r)dr$ is the particle number with radii between r and $r + dr$, and C is a constant.

Figure 1 shows the values of ΔI in the principal plane for various conditions of molecular anisotropy, ground reflection, and aerosol optical thickness. The notations ω_m, τ_a, and A_g represent the single-scattering albedo of molecules, optical thicknesses of aerosols, and Lambertian surface albedo, respectively. The complex refractive index of aerosols, $\tilde{m}$, is assumed to be 1.5−0i. In the case of the molecular atmosphere, we cannot ignore the depolarization effect produced by the molecular anisotropy (Behethi and Fraser, 1980) since it affects both the extinction coefficient and the degree of polarization. The value of ΔI is reduced by about 10 % as the depolarization factor increases from 0 to 0.03. The presence of ground reflection, on the other hand, has little effect on the value of ΔI. In turbid atmospheres,



λ =350nm,
τ_m=0.60, (ω_m=1, ρ=0),
τ_a=0,
A_g=0, p_0=1013mb, μ_0=0.698.

(a)

λ =350nm,
τ_m=0.63, (ω_m=1, ρ=0.03),
τ_a=0.50, ($\tilde{m}$=1.5−0.01i, J=4),
A_g=0.1, p_0=1013mb, μ_0=0.698.

(b)

Fig. 2. The angular distribution of the relative errors in the sky radiance calculated by the scalar approximation for a molecular atmosphere (a) and a turbid atmosphere (b). Left halves are for transmitted radiances and 'S' denotes the solar point; right parts are for reflected radiances and 'A' denotes the antisolar point. Nadir/zenith angle is linearly plotted in degrees, and the outer arcs indicate the horizon. Meanings of the parameters are given in the text.

the presence of aerosols gives rise to a noticeable decrease in ΔI, although the overall pattern of angular distribution is almost the same as those for molecular atmospheres.

Figure 2 illustrates the angular distributions of the error in scalar intensity, $-\Delta I/I$, for both molecular and turbid atmospheres. The error distributions exhibit a symmetric feature for transmitted (TRANS) and reflected (REFL) radiations; a negative error appears around the solar (or antisolar) point, and a positive error along the line 90° distant from the solar (or antisolar) point.

To investigate the azimuth dependence of ΔI, we expand the additional sky radiance in a Fourier series as

$$\Delta I(\mu,\mu_0,\phi-\phi_0;\tau,\tau_0) = \sum_{m=0}^{M} \Delta I^m(\mu,\mu_0;\tau,\tau_0)\cos m(\phi-\phi_0). \quad (4)$$

The highest order of expansion M is equal to 2 for Rayleigh atmospheres. In turbid atmospheres, the value of M depends on the degree of polarization caused by the aerosols. Fortunately, for ordinary earth's atmospheres, the representative size parameters of aerosols are large enough that the degree of resulting polarization is small compared with the case of molecular scattering. The values of ΔI^m for $m \geq 3$ have been found to be smaller by about 2 orders of magnitude than those for $m \leq 2$, so that we can safely adopt $M = 2$.

3. Semi-empirical expression for the additional sky radiance

3.1 An expression for ΔI based on the successive order theory

In view of the origin of the additional sky radiance, an expression for ΔI can be derived based on the theory of the successive order of scattering (Hovenier, 1971), according to which ΔI^m can be expanded in the scattering order n:

$$\begin{aligned}\Delta I^m(\mu,\mu_0;\tau,\tau_0) &= \sum_{n=1}^{N} I^m{}_{(n)}(\mu,\mu_0;\tau,\tau_0) \\ &\quad - \sum_{n=1}^{N} I_{scalar}{}^m{}_{(n)}(\mu,\mu_0;\tau,\tau_0) \\ &= \sum_{n=2}^{N} \Delta I^m{}_{(n)}(\mu,\mu_0;\tau,\tau_0). \quad (5)\end{aligned}$$

In the equation above, the identity $I^m{}_{(1)} = I_{scalar}{}^m{}_{(1)}$ for unpolarized incident radiations has been used. The expansion coefficients $\Delta I^m{}_{(n)}$ are expressed as

$$\begin{aligned}\Delta I^m{}_{(n)}(\mu_n,\mu_0;\tau,\tau_0) &= \frac{1}{1+\delta_{0m}}\left(\frac{\omega}{2}\right)^n \int_{-1}^{1} d\mu_{n-1}\cdots\int_{-1}^{1} d\mu_1 \\ &\quad \times \left\{ \left[\prod_{j=n,-1}^{1} \mathbf{P}^m(\mu_j,\mu_{j-1}) \right]_{11} \right.\end{aligned}$$

Table 1. Maximum Error and Root-Mean-Square Errors (in Parentheses) in Transmitted Radiance for a Molecular Atmosphere Corrected by Eqs. (5) and (6) with Assuming $N=2$.

λ (nm)	τ_m	Maximum Error (RMS Error) (%) Scalar approx.	corrected by Eqs. (5),(6) with $N=2$
600	0.07	3.4(1.8)	−0.5(0.2)
500	0.14	6.0(3.0)	1.4(0.7)
400	0.34	10.1(5.1)	4.2(1.9)
350	0.60	12.3(6.3)	−6.8(3.1)
300	1.16	−14.9(6.8)	−11.6(4.3)

Note: The following conditions are assumed: $\omega_m=1$, $\rho=0$, $A_g=0$, and $p_0=1013$ mb.

$$- \prod_{j=n,-1}^{1} [\mathbf{P}^m(\mu_j,\mu_{j-1})]_{11} \Bigg\} \times e_{(n)}(\mu_n,\mu_{n-1},\cdots,\mu_0;\tau,\tau_0)F_0, \quad (6)$$

where ω is the single-scattering albedo, $[\ ]_{kl}$ denotes the matrix element of the k-th row and l-th column, and $e_{(n)}$ is the geometrical factor for n-th-order scattering (of which formulations for $n=1$ and 2 were given by Hovenier, 1971). The matrix $\mathbf{P}^m$ is the m-th Fourier coefficient of the phase matrix $\mathbf{P}$ such that

$$\mathbf{P}(\mu,\mu_0,\phi-\phi_0) = \mathbf{P}^0(\mu,\mu_0) + 2\sum_{m=1}^{M} \mathrm{Re}\left\{\mathbf{D}^{-1}\mathbf{P}^m(\mu,\mu_0)\mathbf{D}e^{-im(\phi-\phi_0)}\right\}, \quad (7)$$

where $\mathbf{D} = \mathrm{diag.}(1,1,i,i,)$. The simplest expression of Eq. (5) is obtained by assuming the maximum order of scattering N to be 2. The residual errors in radiances corrected by this method are listed in Table 1. The correction with $N=2$ provides a fairly good result at $\lambda = 600$ nm, but the errors increase as the wavelength decreases and reach a maximum of 6.8 % at $\lambda = 350$ nm. The errors are not negligible even for $N = 3$ and are estimated to be about 4 % maximum at 350 nm. The error would decrease if a larger scattering order is assumed, but the formulation of the geometrical factor would simultaneously becomes too complicated to write down, and the computer time would increase significantly. Obviously, therefore, Eqs. (5) and (6) are not appropriate for a handy and yet economical correction procedure.

3.2 An approximate formula of ΔI for Rayleigh atmospheres

We would like to modify Eqs. (5) and (6) in order to include the contributions of higher order scatterings. First, we expand the phase matrix $\mathbf{P}^m$ as in Siewert (1981):

$$\mathbf{P}^m(\mu,\mu_0) = \sum_{l=m}^{L} \mathbf{A}_l{}^m(\mu)\mathbf{B}_l\mathbf{A}_l{}^m(\mu_0). \quad (8a)$$

The matrices $\mathbf{A}_l{}^m$ and $\mathbf{B}_l$ are of the forms:

$$\mathbf{A}_l{}^m(\mu) = \begin{vmatrix} P_l{}^m(\mu) & 0 & 0 & 0 \\ 0 & R_l{}^m(\mu) & -T_l{}^m(\mu) & 0 \\ 0 & -T_l{}^m(\mu) & R_l{}^m(\mu) & 0 \\ 0 & 0 & 0 & P_l{}^m(\mu) \end{vmatrix}, \quad (8b)$$

$$\mathbf{B}_l = \begin{vmatrix} \beta_l & \gamma_l & 0 & 0 \\ \gamma_l & \alpha_l & 0 & 0 \\ 0 & 0 & \zeta_l & -\varepsilon_l \\ 0 & 0 & \varepsilon_1 & \delta_l \end{vmatrix}, \quad (8c)$$

where

$$P_l{}^m(\mu) = i^m P^l_{m,o}(\mu),$$
$$R_l{}^m(\mu) = -\frac{i^m}{2}\left\{P^l_{m,2}(\mu) + P^l_{m,-2}(\mu)\right\},$$
$$T_l{}^m(\mu) = -\frac{i^m}{2}\left\{P^l_{m,2}(\mu) - P^l_{m,-2}(\mu)\right\},$$

with $P^l_{m,n}(\mu)$ being the generalized spherical function (Gel'fand and Sapiro, 1956; Hovenier and van der Mee, 1983). In these expressions, we use somewhat different definitions for $P_l{}^m$, $R_l{}^m$, $T_l{}^m$ and $\mathbf{B}_l$ from those employed by Siewert (1981) in order to simplify the formulations that follow. In the case of a Rayleigh atmosphere, the coefficient matrix, $\mathbf{B}^R$, can be expressed by the following 3×3 matrices (Vestrucci and Siewert, 1984),

$$\mathbf{B}^R{}_0 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad \mathbf{B}^R{}_1 = \mathbf{O},$$

$$\mathbf{B}^R{}_2 = \begin{vmatrix} \frac{1}{2} & -\sqrt{\frac{3}{2}} & 0 \\ -\sqrt{\frac{3}{2}} & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \quad (9a,b,c)$$

and

$$\mathbf{B}^R{}_l = \mathbf{O} \quad \text{for} \quad l \geq 3. \quad (9d)$$

By using Eqs. (8) and (9), Eqs. (5) and (6) are modified as

$$\begin{aligned}\Delta I^m(\mu,\mu_0;\tau,\tau_0) &= \frac{1}{1+\delta_{0m}}\Sigma^m_{ll_0} P_l{}^m(\mu) P_{l_0}{}^m(\mu_0) \\ &\times \sum_{n=2}^{N}\frac{1}{2^n}\int_{-1}^{1} d\mu_{n-1}\cdots\int_{-1}^{1} d\mu_1 \\ &\times X^m_{ll_0(n)}(\mu_{n-1},\cdots,\mu_1;\omega,\mathbf{B}^R) \\ &\times e_{(n)}(\mu,\mu_{n-1},\cdots,\mu_0;\tau,\tau_0)F_0, \end{aligned} \tag{10a}$$

where

$$\Sigma^m_{ll_0} = \begin{cases} \sum_{l=0,2}^{2}\sum_{l_0=0,2}^{2} & \text{for } m=0, \\ \sum_{l=2}^{2}\sum_{l_0=2}^{2} & \text{for } m=1 \text{ and } 2, \end{cases} \tag{10b}$$

and

$$\begin{aligned} X^m_{ll_0(n)}&(\mu_{n-1},\cdots,\mu_1;\omega,\mathbf{B}) \\ &= \omega^n \Bigg\{ \Bigg[\mathbf{B}_l \mathbf{A}_l{}^m(\mu_{n-1}) \prod_{j=n-1,-1}^{2} \mathbf{P}^m(\mu_j,\mu_{j-1})\cdot \mathbf{A}_{l_0}{}^m(\mu_1)\mathbf{B}_{l_0}\Bigg]_{11} \\ &\quad -\beta_l P_l{}^m(\mu_{n-1}) \prod_{j=n-1,-1}^{2} [\mathbf{P}^m(\mu_j,\mu_{j-1})]_{11}\cdot P_{l_0}{}^m(\mu_1)\beta_{l_0}\Bigg\}. \end{aligned} \tag{10c}$$

Here we make a rather rough approximation as

$$\begin{aligned} \frac{1}{1+\delta_{0m}}&\sum_{n=2}^{\infty}\frac{1}{2^n}\int_{-1}^{1} d\mu_{n-1}\cdots\int_{-1}^{1} d\mu_1 \\ &\times X^m_{ll_0(n)}(\mu_{n-1},\cdots,\mu_1;1,\mathbf{B}^R) \\ &\times e_{(n)}(\mu,\mu_{n-1},\cdots,\mu_0;\tau,\tau_0) \\ &\simeq \chi E(\mu,\eta,\mu_0;\tau,\tau_0), \end{aligned} \tag{11}$$

where χ and η are adjustable parameters and E is a newly defined geometrical factor given by

$$\begin{aligned} E(\mu,\eta,\mu_0;\tau,\tau_0) &= |\eta|\{e_{(2)}(\mu,\eta,\mu_0;\tau,\tau_0) \\ &+ e_{(2)}(\mu,-\eta,\mu_0;\tau,\tau_0)\}. \end{aligned} \tag{12}$$

The precise expression for E is given in Appendix B. The meaning of this approximation is that the sum of the multiple-scattered light transferred through the polarization components Q, U or V at least one time can be represented by the fictitious second-order scattering of the first order scattered light with radiance χ and direction η(or $-\eta$). We have examined this assumption with numerical calculations, and have found that χ and η can be determined almost uniquely for each set of m, l, l_0 and τ_0.

After rewriting χ and η to $\chi^m_{ll_0}(\tau_0)$ and $\eta^m_{ll_0}(\tau_0)$, respectively, we obtain an approximate expression for ΔI for Rayleigh atmospheres:

$$\begin{aligned} \Delta I^m(\mu,\mu_0;\tau,\tau_0) &\simeq \Sigma^m_{ll_0} P_l{}^m(\mu) P_{l_0}{}^m(\mu_0) \\ &\times \chi^m_{ll_0}(\tau_0) E(\mu,\eta^m_{ll_0}(\tau_0),\mu_0;\tau,\tau_0)F_0. \end{aligned} \tag{13}$$

Equation (13) is simple enough for practical use. The values of $\chi^m_{ll_0}(\tau_0)$ and $\eta^m_{ll_0}(\tau_0)$ have been determined to fit to the values of ΔI for $0 < \tau_0 \le 2$, as listed in Appendix B. The maximum optical thickness of $\tau_0 = 2$ is large enough for the ordinary earth's atmosphere. The contribution of the surface albedo is neglected since it hardly affects the additional sky radiance at least in the case of the Lambertian surfaces.

3.3 Approximate formula of ΔI for realistic earth's atmospheres

In the case of an actual atmosphere, the value of ΔI is affected considerably by molecular depolarization and aerosol loading, as shown in Fig. 1. To deal with such effects, we consider an atmosphere characterized by the optical thickness of $\tilde{\tau}_0$, the single-scattering albedo of $\tilde{\omega}$, and a Rayleigh-like phase matrix $\tilde{\mathbf{B}}$ given by

$$\begin{aligned} \tilde{\mathbf{B}}_1 &= \mathbf{O}, \\ \tilde{\mathbf{B}}_2 &= a \begin{vmatrix} \frac{1}{2} & -b\sqrt{\frac{3}{2}} & 0 \\ -b\sqrt{\frac{3}{2}} & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \\ \tilde{\mathbf{B}}_l &= \mathbf{O} \quad \text{for} \quad l \ge 3, \end{aligned} \tag{14a,b,c}$$

where a and b are adjustable parameters and $\varepsilon_l = 0$ for $l \ge 2$. Note that $\mathbf{B}_0$ is always equal to diag.(1,0,0) in 3×3 form. Most of the atmospheres can be approximated by $\tilde{\tau}_0$, $\tilde{\omega}$, a and b, as shown in the following cases.

Molecular atmospheres.

When the anisotropic polarizability of molecules is taken into account, the phase matrix is expressed by (Hansen and Hovenier, 1974; Vestrucci and Siewert, 1984)

$$\begin{aligned} \mathbf{B}_1 = \mathbf{O}, \quad \mathbf{B}_2 &= \frac{1-\rho}{1+\rho/2}\mathbf{B}^R{}_2, \\ \mathbf{B}_l = \mathbf{O} \quad &\text{for} \quad l \ge 3. \end{aligned} \tag{15}$$

By comparing Eq. (15) with Eq. (14), we have

$$\tilde{\tau}_0 = \tau_m, \quad \tilde{\omega} = \omega_m, \quad a = \frac{1-\rho}{1+\rho/2}, \quad b = 1. \tag{16a,b,c,d}$$

Aerosol atmospheres.

Here we must treat the phase matrix in its full 4×4 form. But it is expected that the neglect of ε_l causes a quite small error because, when the incident radiation is unpolarized, only the photons scattered at least four times will affect the radiance by ε_l. Hansen (1974) has shown that the error due to the assumption of $\varepsilon_l=0$ is less than 0.0001 % even for an optically thick atmosphere. We therefore omit ε_l from the phase matrix in the subsequent discussion.

In order to eliminate the contribution of higher orders of $l \geq 2$, we use the truncation method discussed in Appendix A. If the truncation fraction f is assumed to be $(2\beta_2-1)/9$, the expansion coefficients of the truncated phase matrix, $\mathbf{B}^t$, are approximated as

$$\mathbf{B}^t{}_1 = \frac{1}{1-f}\mathrm{diag.}(\beta_1 - 3f, 0, 0) \approx \mathbf{O}, \tag{17a}$$

$$\mathbf{B}^t{}_2 = \frac{1}{1-f}\begin{vmatrix} \frac{1}{2}(1-f) & \gamma_2 & \\ \gamma_2 & \alpha_2 - 5f & 0 \\ 0 & 0 & \varsigma_2 - 5f \end{vmatrix} \approx \begin{vmatrix} \frac{1}{2} & \frac{\gamma_2}{1-f} & 0 \\ \frac{\gamma_2}{1-f} & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix}, \tag{17b}$$

and for $l \geq 3$,

$$\mathbf{B}^t{}_l = \frac{1}{1-f}\begin{vmatrix} \beta_l - (2l+1)f & \gamma_l & 0 \\ \gamma_l & \alpha_l - (2l+1)f & 0 \\ 0 & 0 & \xi_l - (2l+1)f \end{vmatrix} \approx \mathbf{O}. \tag{17c}$$

By using these approximations, the parameters are obtained as

$$\tilde{\tau}_0 = (1 - f\omega_a)\tau_a, \quad \tilde{\omega} = \frac{(1-f)\omega_a}{1 - f\omega_a}, \tag{18a,b}$$

$$a = 1, \quad b = \sqrt{\frac{2}{3}}\frac{\gamma_2}{1-f}, \quad \text{and } \tilde{\tau} = \frac{\tilde{\tau}_0}{\tau_0}\tau, \tag{18c,d,e}$$

where $\tilde{\tau}$ is the modified optical depth of the observer's position. The parameters for a homogeneous turbid atmosphere involving both molecules and aerosols are easily obtained by the combination of Eq. (16) and Eq. (18), as listed in Appendix B.

We now derive the expression for Eq. (10a) to include $\tilde{\tau}_0$, $\tilde{\omega}$ a and b. The order of magnitude of each of the quantities in Eq. (10a) is estimated as

$$X^m_{ll_0(n)}(\mu_{n-1}, \cdots, \mu_1; \tilde{\omega}\tilde{\mathbf{B}}) \sim O(\omega^n a^{n-n_{ll_0}} b^{2n_b}), \tag{19a}$$

$$e_{(e)}(\mu, \mu_{n-1}, \cdots, \mu_0; \tilde{\tau}, \tilde{\tau}_0) \sim O(\tilde{\tau}_0^n), \tag{19b}$$

and hence,

$$\begin{aligned} &\Delta I_{(n)}(\mu, \mu_0; \tau, \tau_0) \\ &\quad\simeq \Delta I_{(n)}(\mu, \mu_0; \tilde{\tau}, \tilde{\tau}_0)\Big|_{\omega=\tilde{\omega}, \mathbf{B}=\tilde{\mathbf{B}}} \\ &\quad\sim O(\tilde{\omega}^n a^n b^{2n_b} \tilde{\tau}_0{}^n). \end{aligned} \tag{19c}$$

In these equations, n_{ll_0} is given by $n_{00}=2$, $n_{02}=1$ and $n_{22}=0$, and n_b is an unknown real number. The factor 2 appearing before n_b indicates that b^2 is the fundamental value for the estimate of ΔI. By using Eq. (19c), we assume

$$\frac{\Delta I^m{}_{(n+1)}(\mu, \mu_0; \tau, \tau_0)}{\Delta I^m{}_{(n)}(\mu, \mu_0; \tau, \tau_0)} \approx \text{const.} \equiv a\omega\Delta_a{}^m(\tau_0) \quad \text{for } b = 1, \tag{20a}$$

$$\frac{\Delta I^m{}_{(n+2n_b)}(\mu, \mu_0; \tau, \tau_0)}{\Delta I^m{}_{(n)}(\mu, \mu_0; \tau, \tau_0)} \approx \text{const.} \equiv b^2\Delta_b{}^m(\tau_0) \quad \text{for } a = \omega = 1. \tag{20b}$$

In Eq. (20b), a hypothetical scattering order $n+2n_b$ is used. These relations are similar to those presented by Lenoble (1954) and Dave (1964), indicating that the ratio of two successive values of $I_{(n)}$ in each direction is almost constant in Rayleigh atmospheres. By summing up the geometrical progressions of $\Delta I^m{}_{(n)}$ for $n \geq 2$ with Eqs. (20a) and (20b) and by using Eq. (13), we obtain the desired approximate expression for the additional sky radiance for general atmospheres:

$$\begin{aligned} \Delta I^m(\mu, \mu_0; \tau, \tau_0) &\simeq (\tilde{\omega}ab)^2 \\ &\times \frac{1 - \Delta_a{}^m(\tilde{\tau}_0)}{1 - \tilde{\omega}a\Delta_a{}^m(\tilde{\tau}_0)} \cdot \frac{1 - \Delta_b{}^m(\tilde{\tau}_0)}{1 - b^2\Delta_b{}^m(\tilde{\tau}_0)} \\ &\times \Sigma^m_{ll_0}(\tilde{\omega})^{n_{ll_0}} P_l{}^m(\mu) P_{l_0}{}^m(\mu_0) \chi^m_{ll_0}(\tilde{\tau}_0) \\ &\times E(\mu, \eta^m_{ll_0}(\tilde{\tau}_0), \mu_0; \tilde{\tau}, \tilde{\tau}_0) F_0. \end{aligned} \tag{21}$$

Here we added the term $(\omega)^n ll_0$ to complete the ω-dependence of ΔI. The values of $\Delta_a{}^m(\tau_0)$ and $\Delta_b{}^m(\tau_0)$ have been estimated with numerical calculations as listed in Appendix B.

4. Numerical results

To validate the efficiency of our expression for ΔI, we have calculated the maximum and rms residual errors for various conditions. We used the doubling-adding method with 30 discrete streams for the calculation of sky radiance. The results of our transfer code closely coincided with those of Coulson *et al.* (1960) for Rayleigh atmospheres, except that the differences were more than 0.1 % for the grazing angles $0 < |\mu| \leq 0.2$ or $0 < \mu_0 \leq 0.2$. The radiances for aerosol atmospheres were compared with the results of Garcia and Siewert (1986) and Fraser (1988). The former authors compiled the intensities only for the small solar elevation of $\mu_0 = 0.20$, and their values agreed with ours to within 0.3 %

Table 2. Maximum and Root-Mean-Square Errors in Transmitted and Reflected Radiances for Molecular Atmospheres Calculated with Scalar Transfer Code (I_{scalar}) and Corrected by Eq. (21)($I_{scalar} + \Delta I$).

λ	τ_m	ω_m	ρ	A_g	Maximum Error (RMS Error) (%)			
					Transmitted		Reflected	
(nm)					I_{scalar}	$I_{scalar} + \Delta I$	I_{scalar}	$I_{scalar} + \Delta I$
600	0.07	1.	0.	0.	3.4(1.8)	0.1(0.0)	3.4(1.8)	0.1(0.0)
500	0.14	1.	0.	0.	6.0(3.0)	−0.1(0.0)	5.9(3.1)	0.1(0.0)
400	0.34	1.	0.	0.	10.1(5.1)	0.1(0.0)	9.8(5.0)	−0.0(0.0)
350	0.60	1.	0.	0.	12.3(6.3)	0.3(0.0)	11.5(6.0)	0.1(0.0)
300	1.16	1.	0.	0.	−14.9(6.8)	−0.5(0.1)	11.6(6.2)	0.8(0.2)
350	0.60	0.9	0.	0.	11.1(5.8)	0.5(0.2)	10.4(5.5)	−0.3(0.1)
		0.5			6.3(3.5)	0.5(0.2)	5.8(3.2)	−0.4(0.2)
350	0.63	1.	0.03	0.	10.7(5.6)	0.2(0.0)	10.0(5.3)	0.1(0.0)
	0.71		0.1		−8.6(4.3)	0.3(0.0)	7.2(4.0)	−0.2(0.0)
350	0.60	1.	0.	0.2	11.1(5.3)	0.2(0.0)	9.0(4.5)	0.1(0.0)
				1.0	−7.2(2.8)	0.2(0.0)	−6.1(1.9)	0.1(0.0)

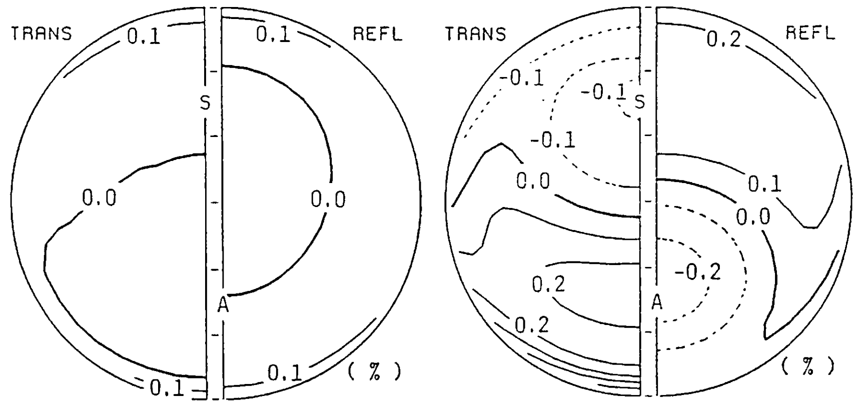
Note: Ground pressure $p_0 = 1013$ mb is assumed.

for $0.20 \leq |\mu| \leq 1$. The errors reduced to less than 0.1 % when we used the 90-streams transfer code. This was also the case for the polarization components Q, U and V. The results of Fraser (1988) were calculated for the aerosol atmosphere with $\tau_a = 1$, $J = 4$, $\mu_0 = 0.695$ and 0.242 by using the transfer code developed by Dave. A discrepancy of 0.3 % is also found in this case. This discrepancy is considered to be due partly to the spline interpolation of our results on the coarse grid of the zenith angles for the comparison, and due partly to the inaccuracy of the matrix method with 30-streams. These errors are not always negligible when the absolute value is discussed. The main problem in this study is, however, the relative value of the difference between the scalar and vector intensities, *i.e.* $\Delta I/I$. A comparison of the value of $\Delta I/I$ is made with the results of Fraser (1988) for Rayleigh atmospheres, and an overall agreement was found within 0.1 % except for the near-grazing angles. The self-consistency in our results calculated by 30- and 90-streams code was also 0.1 % for both Rayleigh and aerosol atmospheres.

To ensure the precision, the thirteen quadrature points corresponding to $0.251 \leq |\mu|(\text{and } \mu_0) \leq 0.997$ were adopted for the data points in the following discussion. The mean and rms errors were calculated from the radiance values in 1118 directions with different combinations of μ, μ_0 and $\phi - \phi_0$.

In Table 2 are shown the maximum and the rms errors in the scalar and the corrected intensities for molecular atmospheres. The correction with Eq. (21) is almost perfect for Rayleigh atmospheres (upper part of the table). Even if we consider the absorption and depolarization (lower part of the table), the maximum and the rms errors are reduced to less than 0.6 % and 0.2 %, respectively. In many applications for practical problems, these errors must be insignificant because of uncertainties inherently caused by a horizontal inhomogeneity of the atmosphere, and by observational errors. The angular distributions of % error are shown in Fig. 3a. By comparing with Fig. 2a, it is found that the correction is quite satisfactory. In Table 3 are listed the errors for the downward and upward radiances at various pressure levels in Rayleigh atmospheres. It is also found that our expression is valid for the internal field.

Results for homogeneous turbid atmospheres with several aerosol polydispersions are summarized in the upper and middle parts of Table 4 and Fig. 3b. The maximum and the rms errors were reduced to less than 0.6 % and 0.3 %, respectively, in most cases. The largest error of 0.9 % was found for the case of strongly absorbing aerosols with $\tilde{m} = 1.5 - 0.05i$. Such large errors, however, occur for limited angles; small values of μ for small values of μ_0 in transmitted radiances near $\phi - \phi_0 = 180°$, and large values of $|\mu|$ for large values of μ_0 in reflected radiances. The variety of the phase matrix is well represented with the two parameters, β_2 and γ_2. Since γ_2 is the only unknown quantity within a scalar transfer code, we need no additional Mie calculation for the correction if the parameter γ_2 is assumed to be zero. The errors calculated with this assumption are about twice as large as those in the previous calculation, as listed in the lower part of Table 4. This result indicates that an introduction



CORRECTED
λ =350nm,
τ_m=0.60, (ω_m=1, ρ=0),
τ_a=0,
A_g=0, p_0=1013mb, μ_0=0.698.

(a)

CORRECTED
λ =350nm,
τ_m=0.63, (ω_m=1, ρ=0.03),
τ_a=0.50, ($\tilde{m}$=1.5-0.01i, J=4),
A_g=0.1, p_0=1013mb, μ_0=0.698.

(b)

Fig. 3. Same as Fig. 2 but corrected by Eq. (21).

Table 3. Maximum and Root-Mean-square Errors in Internal Fields at Several Pressure Levels in a Molecular Atmosphere.

λ	τ_m	τ/τ_m	Maximum Error (RMS Error) (%)			
			Downward		Upward	
(nm)			I_{scalar}	$I_{scalar}+\Delta I$	I_{scalar}	$I_{scalar}+\Delta I$
500	0.14	0.2	4.6(2.4)	−0.2(0.1)	−3.3(1.4)	0.0(0.0)
		0.5	5.2(2.6)	−0.1(0.0)	−3.2(1.1)	0.0(0.0)
		0.8	5.6(2.7)	−0.1(0.0)	−2.5(0.6)	0.1(0.0)
350	0.60	0.2	9.6(4.9)	−0.3(0.2)	10.9(5.3)	0.2(0.1)
		0.5	10.9(5.5)	−0.2(0.0)	−11.2(4.9)	0.1(0.0)
		0.8	11.5(5.8)	0.2(0.1)	−10.9(3.4)	0.4(0.1)

Note: The following conditions are assumed: ω_m=1, ρ=0, p_0=1013 mb, and A_g=0.1.

of γ_2 is important for the improved correction.

The computer time required for the correction, on the other hand, depends on the number of data points but is usually negligible. If the corrections are performed on the scalar intensities in 150 directions, for example, the required CPU-time is 1/200 of that of scalar transfer code which is quite small compared with the increment of CPU-time for vector calculation (about 5 times that of scalar transfer code).

Now let us turn to the applicability of our method to inhomogeneous atmospheres. We have tested a two-layered atmosphere consisted of a homogeneous turbid layer and an overlaying pure molecular layer. Equation (21) is used to correct the scalar reflection and transmission matrices of each layer before applying the adding procedure. The results are listed in Table 5. Since this method can account for the polarization effects only roughly, the errors are larger by about one order of magnitude than those of homogeneous atmospheres (corresponding to the rows of 0-1013mb in the table). Nevertheless a considerable improvement is achieved, especially for the case of low-level haze layers, say at 900 – 1013 mb pressure levels, as are frequently observed in the actual atmosphere.

5. Summary

We have analyzed the angular distribution of the errors involved in sky radiances caused by the use of the scalar approximation, and proposed a semi-empirical expression for reducing the errors. The validity and efficiency of our expression are found to be quite satisfactory for homogeneous, optically thin

Table 4. Maximum and Root-Mean-Square Errors in Transmitted and Reflected Radiances for Turbid Atmospheress.

λ (nm)	τ_m	τ_a	$\tilde{m}$	J	Maximum Error (RMS Error) (%) Transmitted I_{scalar}	Transmitted $I_{scalar}+\Delta I$	Reflected I_{scalar}	Reflected $I_{scalar}+\Delta I$
600	0.07	0.5	$1.5-0i$	4	1.0(0.4)	−0.1(0.0)	−1.3(0.6)	−0.1(0.0)
500	0.14	0.5	$1.5-0i$	4	1.8(0.7)	−0.1(0.1)	−2.1(1.0)	−0.2(0.1)
400	0.36	0.5	$1.5-0i$	4	3.7(1.6)	0.3(0.1)	3.7(2.0)	−0.3(0.1)
350	0.63	0.5	$1.5-0i$	4	5.1(2.4)	0.6(0.1)	5.2(2.9)	−0.3(0.1)
300	1.22	0.5	$1.5-0i$	4	−7.1(3.2)	0.6(0.2)	6.2(3.5)	0.5(0.2)
350	0.63	0.1	$1.5-0i$	4	9.1(4.4)	0.3(0.1)	8.7(4.7)	−0.2(0.1)
		1.0			−2.7(1.3)	0.6(0.1)	3.0(1.7)	−0.3(0.1)
350	0.63	0.5	$1.5-0.01i$	4	5.4(2.6)	0.7(0.2)	5.4(3.1)	−0.5(0.2)
			$1.5-0.05i$		6.3(3.0)	0.9(0.2)	−6.2(3.5)	−0.9(0.3)
350	0.63	0.5	$1.5-0i$	3	5.6(2.6)	0.7(0.1)	5.7(3.1)	−0.3(0.1)
				4	5.1(2.4)	0.6(0.1)	5.2(2.9)	−0.3(0.1)
				5	5.4(2.5)	0.5(0.2)	5.5(3.1)	−0.4(0.1)
350	0.63	0.5	$1.5-0i$	3		1.0(0.2)		0.4(0.1)
				4		0.6(0.1)		−0.3(0.1)
				5		1.0(0.5)		−1.0(0.5)

Note: The following conditions are assumed: ω_m=1, ρ=0.03, p_0=1013 mb, and A_g=0. For the last three rows, γ_2=0 is assumed in the calculation of ΔI

Table 5. Muximum and Root-Mean-Square Errors in Transmitted and Reflected Radiances for Two-Layered Inhomogeneous Atmospheres.

λ (nm)	τ_m	τ_a	Upper-Lower Pressure Levels of Haze Layer (mb)−(mb)	Maximum Error (RMS Error) (%) Transmitted I_{scalar}	Transmitted $I_{scalar}+\Delta I$	Reflected I_{scalar}	Reflected $I_{scalar}+\Delta I$
500	0.14	0.5	0 − 1013	1.8(0.7)	−0.1(0.1)	−2.1(1.0)	−0.2(0.1)
			200 − 1013	1.9(0.7)	0.6(0.2)	−2.2(1.1)	−0.7(0.3)
			500 − 1013	2.1(0.7)	0.9(0.3)	2.5(1.3)	−1.0(0.5)
			900 − 1013	2.6(0.8)	0.8(0.3)	3.4(1.7)	−0.8(0.4)
350	0.63	0.5	0 − 1013	5.1(2.4)	0.6(0.1)	5.2(2.9)	−0.3(0.1)
			200 − 1013	5.5(2.4)	2.1(0.7)	6.5(3.4)	2.6(1.3)
			500 − 1013	6.6(2.6)	−2.9(1.1)	8.4(4.2)	2.6(1.4)
			900 − 1013	7.6(2.8)	−1.4(0.5)	10.2(5.0)	−0.9(0.5)

Note: The following conditions are assumed: ω_m, $\rho = 0.03$, $\tilde{m} = 1.5 - 0i$, $J = 4$, $p_0 = 1013$ mb, and $A_g = 0$.

and moderately thick, but not strongly absorbing atmospheres. The errors are reduced to less than 0.6 % in most cases, except for strongly absorbing atmospheres. The present method should also reduce the amount of labor required for reprogramming a scalar transfer code in order to include the polarization effects. Rapid and yet rather accurate calculation of sky radiance can be carried out if the correction term proposed in this work is added to a fast scalar transfer code (Stamnes, 1986; Nakajima and Tanaka, 1986). Such a fast transfer code is required, for example, to retrieve some optical properties of aerosols from sky radiance measurements.

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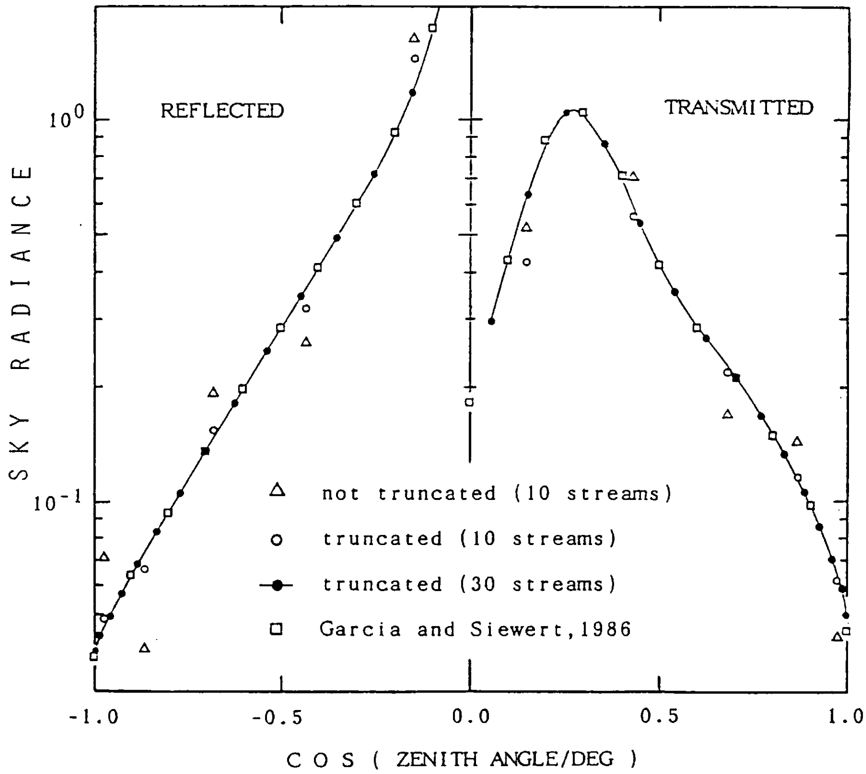

Fig. A. Sky radiance in the solar meridian for L=60 model of Garcia and Siewert (1986) calculated with 10-streams code for a not-truncated phase matrix (triangle); 10-streams for the truncated case (open circle); 30-streams for the truncated case (full circle). Squares are taken from Garcia and Siewert(1986).

carried out by ACOS-1000 and SX-1 of Computer Center, Tohoku University.

Appendix

A. Truncation Technique Including Polarization Effects.

Wiscombe (1977) has developed the δ-M method which truncates the forward peak of the phase function $P(\xi)$ as

$$P(\xi) \simeq 2f\delta(1-\xi) + (1-f)P^t(\xi), \tag{A1}$$

where $\xi = \cos\Theta$ (Θ is the scattering angle) and f is the truncation fraction. We extend this technique to apply to the phase matrix with strongly asymmetric diagonal elements.

When the phase matrix is expressed for the modified Stokes vector I_l, I_r, Q and V, the forward parts of the diagonal elements can be represented by one value since the amplitude functions, S_1 and S_2, coincide with each other at $\Theta = 0$. Then the truncation formula for the phase matrix is given by

$$\mathbf{P}(\xi) \simeq 2f\delta(1-\xi)\mathbf{E} + (1-f)\mathbf{P}^t(\xi), \tag{A2}$$

where $\mathbf{E}$ is the unit matrix. This equation is still valid if we use the Stokes vector (I, Q, U and V) or the CP-representation (I_2, I_0, I_{-0} and I_{-2}). From the orthogonal completeness of the generalized spherical function (Gel'fand and Sapiro, 1956):

$$\int_{-1}^{1} P^l_{m,n}(\xi)P^{l'}_{m,n}(\xi)d\xi = \frac{2}{2l+1}\delta_{ll'},$$

the expansion coefficients for the forward peak, $2f\delta(1-\xi)\mathbf{E}$, are easily estimated by using the CP-representation as

$$2f\delta(1-\xi) = \sum_{l=|m|}^{\infty}(2l+1)fP^l_{m,m}(\xi), \tag{A3}$$

for $m = 2, 0, -0$ and -2. After translating this into the Stokes vector, we have

$$\mathbf{B}_l = \begin{cases} (2l+1)f\,\mathrm{diag.}(1,0,0,1) & \text{for } l = 0 \text{ and } 1, \\ (2l+1)f\mathbf{E} & \text{for } l \geq 2. \end{cases}$$

The truncated part of the phase matrix, $\mathbf{P}^t(\xi)$, is then expressed by

$$\mathbf{B}^t{}_l = \frac{1}{1-f}\begin{vmatrix} \beta_l-(2l+1)f & \gamma_l & 0 & 0 \\ \gamma_l & \alpha_l-(2l+1)f^* & 0 & 0 \\ 0 & 0 & \xi_l-(2l+1)f^* & -\varepsilon_l \\ 0 & 0 & \varepsilon_l & \delta_l-(2l+1)f \end{vmatrix}, \tag{A5}$$

where $f^*=0$ for $l=0$ and 1, and $f^*=f$ for $l \geq 2$. If we assume the truncation fraction to be β_L, each element in $\mathbf{B}^t{}_l$ almost vanishes for $l \geq L$, and can be approximated by $\mathbf{O}$.

We have performed several numerical calculations using this method by assuming the truncation order L to be twice the number of discrete streams. As shown in Fig. A, a considerable improvement can be achieved by such truncation. The principal cause for this improvement is thought to be the truncation of the β element, *i.e.* Eq. (A1). However, the conversion of the polarized light of forward scattering into the direct transmitted light cannot be attained with the scalar truncation method alone.

B. A Complete Formulation of the Expression for the Additional Sky Radiance

Here we summarize the formulae needed for the correction procedure. The scalar intensity, I_{scalar}, is corrected to the improved one I as

$$I(\mu,\mu_0,\phi-\phi_0;\tau,\tau_0) = I_{scalar}(\mu,\mu_0,\phi-\phi_0;\tau,\tau_0) + \sum_{m=0}^{2} \Delta I^m(\mu,\mu_0;\tau,\tau_0)\cos m(\phi-\phi_0).$$

The Fourier components ΔI^m are calculated by

$$\Delta I^m(\mu,\mu_0;\tau,\tau_0) \simeq (\tilde{\omega}ab)^2 \cdot \frac{1-\Delta_a{}^m(\tilde{\tau}_0)}{1-\tilde{\omega}a\Delta_a{}^m(\tilde{\tau}_0)} \cdot \frac{1-\Delta_b{}^m(\tilde{\tau}_0)}{1-b^2\Delta_b{}^m(\tilde{\tau}_0)} \times \Sigma^m_{ll_0}(\tilde{\omega})^{n_{ll_0}} P_l{}^m(\mu) P_{l_0}{}^m(\mu_0) \times \chi^m_{ll_0}(\tilde{\tau}_0) E(\mu,\eta^m_{ll_0}(\tilde{\tau}_0),\mu_0;\tilde{\tau},\tilde{\tau}_0) F_0,$$

where

$$\tilde{\tau}_0 = \tau_m + (1-f\omega_a)\tau_a, \quad \tilde{\omega} = \frac{\tau_{scat}}{\tilde{\tau}_0},$$

$$a = \frac{1}{\tau_{scat}}\left\{\frac{1-\rho}{1+\rho/2}\omega_m\tau_m + (1-f)\omega_a\tau_a\right\},$$

$$b = \frac{1}{a\tau_{scat}}\left\{\frac{1-\rho}{1+\rho/2}\omega_m\tau_m - \sqrt{\frac{2}{3}}\gamma_2\omega_a\tau_a\right\},$$

$$\tilde{\tau} = \frac{\tilde{\tau}_0}{\tau_0}\tau, \quad \tau_{scat} = \omega_m\tau_m + (1-f)\omega_a\tau_a,$$

$$f = (2\beta_2 - 1)/9;$$

$$\Delta_a{}^0(\tau_0) = 0.6\sqrt{\tau_0}, \quad \Delta_b{}^0(\tau_0) = 0.1\tau_0,$$
$$\Delta_a{}^1(\tau_0) = 0.5\sqrt{\tau}, \quad \Delta_b{}^1(\tau_0) = 0.1\tau_0,$$
$$\Delta_a{}^2(\tau_0) = 0.4\sqrt{\tau_0}, \quad \Delta_b{}^2(\tau_0) = 0.1\tau_0;$$

$$\Sigma^0_{ll_0} = \sum_{l=0,2}^{2}\sum_{l_0=0,2}^{2}, \; \Sigma^1_{ll_0} = \Sigma^2_{ll_0} = \sum_{l=2}^{2}\sum_{l_0=2}^{2};$$

$$n_{00} = 2, \quad n_{20} = n_{02} = 1, \quad n_{22} = 0;$$

$$P_0{}^0(\mu) = 1, \quad P_2{}^0(\mu) = \frac{1}{2}(3\mu^2 - 1),$$

$$P_2{}^1(\mu) = \sqrt{\frac{3}{2}}\mu\sqrt{1-\mu^2},$$

$$P_2{}^2(\mu) = \sqrt{\frac{3}{8}}(1-\mu^2);$$

$$\chi^0_{00}(\tau_0) = 0.0085\tau_0, \qquad \eta^0_{00}(\tau_0) = 0.5,$$
$$\chi^0_{20}(\tau_0) = -0.0415\tau_0^{0.52}, \quad \eta^0_{20}(\tau_0) = 0.5,$$
$$\chi^0_{02}(\tau_0) = -0.0415\tau_0^{0.52}, \quad \eta^0_{20}(\tau_0) = 0.5,$$

$$\chi^0_{22}(\tau_0) = 0.25,$$
$$\eta^0_{22}(\tau_0) = 0.514 - 0.373\exp(-0.826\tau_0),$$
$$\chi^1_{22}(\tau_0) = 0.36,$$
$$\eta^1_{22}(\tau_0) = 0.752 - 0.581\exp(-0.584\tau_0),$$
$$\chi^2_{22}(\tau_0) = 0.195,$$
$$\eta^2_{22}(\tau_0) = 1.248 - 1.160\exp(-0.843\tau_0).$$

The formulae for E are given by (Hovenier, 1971)

$$E(\mu,\eta,\mu_0;\tau,\tau_0) = \frac{\mu_0\eta}{\mu_0-\eta}\{c(\mu,\mu_0;\tau,\tau_0) - c(\mu,\eta;\tau,\tau_0)\} + \frac{\mu_0\eta}{\mu_0+\eta}\left\{c(\mu,\mu_0;\tau,\tau_0) - c(\mu,-\eta;\tau,\tau_0) \times \exp\left(-\frac{\tau_0}{\mu_0}-\frac{\tau_0}{\eta}\right)\right\},$$

where

$$c(\mu,\mu_0;\tau,\tau_0) = \begin{cases} \dfrac{\mu_0}{\mu_0-\mu}\left\{\exp\left(-\dfrac{\tau}{\mu_0}\right) - \exp\left(-\dfrac{\tau}{\mu}\right)\right\} & \text{for } \mu > 0, \\ \dfrac{\mu_0}{\mu_0-\mu}\left\{\exp\left(-\dfrac{\tau}{\mu_0}\right) - \exp\left(-\dfrac{\tau}{\mu}-\dfrac{\tau_0}{\mu_0}+\dfrac{\tau_0}{\mu}\right)\right\} & \text{for } \mu < 0. \end{cases}$$

Greek letter constants, β_2 and γ_2, are calculated by (Herman and Lenoble,1968; Hovenier and van der Mee, 1983)

$$\beta_2 = \frac{5}{2}\int_{-1}^{1}[\mathbf{P}(\xi)]_{11}P_2{}^0(\xi)d\xi,$$

$$\gamma_2 = \frac{5}{2}\int_{-1}^{1}[\mathbf{P}(\xi)]_{12}P_2{}^2(\xi)d\xi,$$

where $\mathbf{P}(\xi)$ is the scattering matrix expressed for the Stokes vector.

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天空光強度における偏光の影響の補正法

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分子大気および混濁大気における天空光強度を計算する場合、偏光効果を無視した計算では最大 10 %程度の誤差が生じる。この誤差を補正するための本研究では、偏光効果から発生する天空光強度を逐次散乱法に基づいて定式化し、いくつかのパラメタリゼーションによって簡便な補正式にまとめた。光学的に厚くない均一大気の場合、我々の得た補正式は偏光の影響による誤差を 0.6 %程度まで減らすことができた。