研 究 速 報

A minimization theory applied to minimization of drag of a two dimensional strut with cavity flow

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1. Introduction

It is considered that a two-dimensional strut with a blunt base moves in unbounded fluid with a constant speed, where a cavity is generated behind the blunt base. In this paper, a minimization theory is proposed and applied to a problem for finding optimal body shape of a strut which minimizes its drag due to cavitation. The minimization theory is based on the Hilbert space theory[1]. From physical point of view, a small variation of the shape gives rise to a small change of drag of the strut; the small change of drag can be expressed by the inner product in a Hilbert space, furthermore, the way of expression of the small change is unique. In this study, its representation is realized by introducing orthogonal functions, so that theoretical results for the reduced drag of the strut and its shape can be driven.

2. Mathematical formulation

A two-dimensional strut with a blunt base, or trailing edge is considered to move steadily in unbounded fluid with a speed U as shown in Fig. 1. Under the assumption that fluid flow is irrotational and fluid is incompressible and inviscid, a velocity potential can be defined to describe the fluid motion. Therefore, Lapace's equation must be satisfied in the fluid, the perturbation velocity and pressure should vanish at infinity, and the velocity at the trailing edge of the strut is finite like Kutta condition for the wing theory. In addition, a solution space is made to be a linear vector space by an appropriate linearization of the boundary condition. Here the coordinates have been nondimensionalized in terms of the chord length of the strut, with the leading edge at x=1, and the nondimensional strut thickness is $2y_0(x)$. Downstream of the trailing edge, x=0, a cavitation of length L exists.

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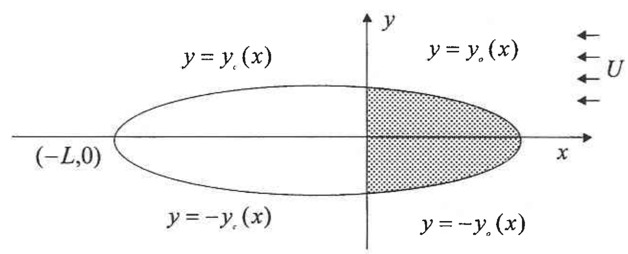

Fig.1 Cavity Flow past a Strut

Considering the flow is symmetrical about y=0, it is logical to describe the flow with the help of a source distribution. From the linearized potential theory, the drag can be written as follow[2];

$$D = \frac{\rho U^2}{\pi L}\left(\int_0^1 \frac{dy_0(x)}{dx}\frac{(L+2x)}{[x(L+x)]^{1/2}}dx\right)^2 \quad \cdots\cdots (1)$$

Because the cavity must be closed, the integration of the source strength, over the strut and the cavity, is equal to zero. This gives a functional relation;

$$0 = \frac{\pi}{4}\sigma L + \int_0^1 \sqrt{\frac{L+t}{t}}\frac{dy_0(t)}{dt}dt \quad \cdots\cdots (2)$$

Here the cavitation number σ is defined by

$$\sigma = \frac{p_0 - p_v}{(1/2)\rho U^2} \quad \cdots\cdots (3)$$

where p_0 is the ambient pressure of the flow at a large distance from the strut, and p_v is the vapor pressure of the fluid. Therefore, the drag is a functional of the shape of the strut $y_0(x)$;

$$D = D(y_0) \quad \cdots (4)$$

Let us assume $h=h(x)$ is a variation of the shape $y_0(x)$ with some regularity, which has a small norm;

$$\| h \| = \left(\int_0^1 |h(x)|^2 dx \right)^{1/2} = O(\varepsilon) \quad \cdots (5)$$

where $\varepsilon << 1$ indicates a positive small parameter. A constraint will be imposed, that is, the small variation $h=h(x)$ vanishes at the trailing edge (x=0) and the leading edge (x=1).

$$h(0) = h(1) = 0 \quad \cdots (6)$$

Then a perturbed drag can be described as following;

$$D(y_0 + h) - D(y_0) = A(h) + E(h) \quad \cdots (7)$$

Here, the operator A is a bounded linear functional and $E(h)$ is a nonlinear functional which will be ignored, provided $\varepsilon << 1$. From the Riesz representation theorem[3], there exits a unique function $g(x) \in X$ where X is called the Lebesgue space[4];

$$X = \left\{ f(x) \middle| \int_0^1 f^2(x) dx < \infty \right\} \quad \cdots (8)$$

$$A(h) = \langle h, g \rangle = \int_0^1 h(x) g(x) dx \quad \cdots (9)$$

Therefore, it is possible to write eq. (7) as following;

$$R(y_0 + h) \approx R(y_0) + \langle h, g \rangle \quad \cdots (10)$$

where g is unique and to be determined. $g(x)$ can be expressed with the help of a complete orthonormal set $\{ e_i(x) \}$;

$$g(x) = \sum_{i=1}^{\infty} g_i e_i(x) \quad \cdots (11)$$

Considering the constraint, the following complete orthonomal set is introduced in this study;

$$e_i(x) = \sqrt{2} \sin i\pi x, \quad 0 \le x \le 1, \quad i = 1, 2, 3, \ldots \quad \cdots (12)$$

3. Theoretical results

From the Schwarz inequality,

$$|\langle h, g \rangle| \le \| h \|_X \cdot \| g \|_X = \sqrt{\int_0^1 h^2(x) dx} \cdot \sqrt{\int_0^1 g^2(x) dx} \quad \cdots (13)$$

which means that the change of the drag $<h, g>$ is always bounded by the norm products $\| h \|_X \cdot \| g \|_X$. By the way, the minimization of the drag is attained when $h(x)$ is spanned by $g(x)$, i.e., when $h_{min} = -\varepsilon g$[5]. Considering the continuity of the inner product, substitution of eq. (11) to eq. (10) yields

$$D(y_0 + h) = D(y_0) + \lim_{n \to \infty} < h, \sum_{i=1}^{n} g_i e_i > \quad \cdots (14)$$

Variations are assumed as follow;

$$h^k(x) = \sum_{i=1}^{\infty} < h^k, e_i > e_i, \quad k = 1, 2, 3, \ldots \quad \cdots (15)$$

From eq. (10), the following linear matrix equation is obtained, which can be solved to yield g_i;

$$D_k = \sum_{i=1}^{\infty} A_{k,i} g_i \quad \cdots (16)$$

where

$$D_k = D(y_0 + h^k) - D(y_0)$$

$$A_{k,i} = \langle h^k, e_i \rangle = \int_0^1 h^k(x) e_i(x) dx$$

Using eq. (16), it is possible to find the function $g(x)$. Therefore, the desired form of the strut which minimize drag is

$$y_{new}(x) = y_0(x) + h_{min}(x) \quad \cdots (17)$$

where

$$h_{min}(x) = -\varepsilon g(x) \quad \cdots (18)$$

The corresponding reduction drag is

$$\langle h, g \rangle = \langle h_{min}, g \rangle = \langle -\varepsilon \cdot g, g \rangle = -\varepsilon \| g \|^2 \quad \cdots (19)$$

4. Conclusions

In this paper, a minimization theory based on the Hilbert space theory is proposed. As an application of it, cavity flow past a strut is considered and it is theoretically shown that cavity drag of the strut can be reduced and its optimal shape can be found by the proposed theory.

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