研 究 速 報
研 究 速 報

An Optimization theory and its Application to CFD based Design for Marine Propeller behind Ship

Taek S. JANG* and Takeshi KINOSHITA*

1. Introduction

An optimization theory is proposed based on the Hilbert space theory[1], which is applied to design of a propeller in non-uniform wake. To accelerate the optimization, Euler beam theory is introduced to generate a small change of variation for the propeller. From the propeller designer's point of view, it is difficult to consider a complex non-uniform wake behind a hull. In this study, an optimization theory is proposed to maximize its efficiency in the non-uniform wake with the help of functional approach. Furthermore, the beam theory is introduced to generate a variation, which is intuitively and physically appropriate in this study. Because of the non-uniform wake behind the hull, propeller blades at each radius r as shown in Fig.1 experiences different angle of attack with respect to the incoming fluid of the non-uniform wake. For a systematic analysis, let $\alpha(r)$ be an angle between the linear line intersecting L.E and T.E of a blade at each radius r and the y-z plane. A small variation of $\alpha(r)$ would change the propulsive efficiency of the ship. In this study, it is shown that although there may be infinitely many variations of $\alpha(r)$, the variation of $\alpha(r)$ which gives an optimal propulsive efficiency is to be found by using the Hilbert space theory. According to this study, there is no reason to exclude a cavitation flow generated by the propeller. To proceed the present theory, CFD simulation will be used, i.e. the governing equations for the fluid flow are the Navier-Stokes equation and the equation of continuity. The present study gives theoretical results for improved propeller efficiency and corresponding optimal shape of the propeller based on CFD simulation.

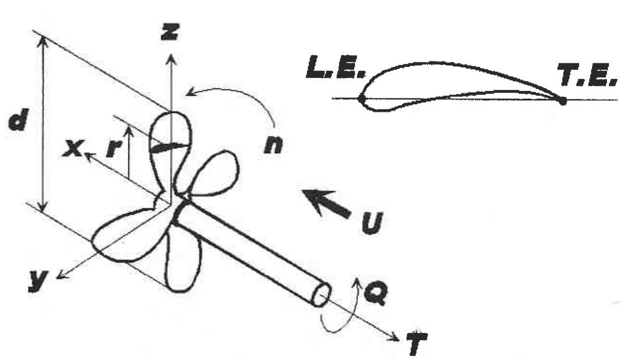

Fig.1 Definition sketch of a propeller

2. Mathematical formulation

Forward speed of a ship U, a diameter of a propeller d and revolution per time n being fixed constant, then the advance ratio $J=U/nd$ is constant, so that a thrust T and torque Q of the propeller are a function of the geometry of the propeller. Physically, the geometry is faithfully described with help of the slope $\alpha(r)$. In mathematical view point, the thrust T and the torque Q are a functional of $\alpha(r)$;

$$T = T(\alpha) \quad \cdots\cdots (1)$$

$$Q = Q(\alpha) \quad \cdots\cdots (2)$$

By the way, the propulsive efficiency of the ship[2] is

$$\eta = \frac{DU}{2\pi nQ} = \frac{(1-t)U}{2\pi n} \cdot \frac{T}{Q} \quad \cdots\cdots (3)$$

where D is the drag force of the ship and t is its thrust-deduction coefficient. From eq. (3), it is known that the propulsion efficiency increases in proportion to the rise in the ratio T/Q. According to eq. (1) and eq. (2), the ratio T/Q is also a functional of the slope $\alpha(r)$;

* Institute of Industrial Science, University of Tokyo

$$\frac{T}{Q}=\frac{T(\alpha)}{Q(\alpha)}=\tau(\alpha) \quad \cdots\cdots (4)$$

where $\tau(\alpha)$ is defined to be the ratio T/Q. Let $h=h(r)$ is a small variation of the sectional slope $\alpha(r)$ such that

$$\|h\|=\left(\int_{hub}^{d/2}|h(r)|^2 dr\right)^{1/2}=O(\varepsilon) \quad \cdots\cdots (5)$$

and

$h=h(r)$ has a regularity

where $\varepsilon<<1$ indicates a positive small parameter. Then a perturbation of the ratio $\tau(\alpha)$ can be described as following;

$$\tau(\alpha+h)-\tau(\alpha)=A(h)+E(h) \quad \cdots\cdots (6)$$

Here, the operator A is a bounded linear functional and $E(h)$ is a nonlinear functional which will be ignored, provided $\varepsilon<<1$. From the Riesz representation theorem[3], the bounded linear functional A has the functional form as following;

$$A(h)=<h,g>=\int_{hub}^{d/2} h(r)g(r)dr \quad \cdots\cdots (7)$$

Here, a function $g(r)$ is unique and belongs to the Lebesgue space X such that

$$X=\left\{f(r)\left|\int_{hub}^{d/2} f^2(r)dr<\infty\right.\right\} \quad \cdots\cdots (8)$$

Therefore, it is possible to write eq. (6) as following;

$$\tau(\alpha+h)\approx\tau(\alpha)+<h,g> \quad \cdots\cdots (9)$$

where $g(r)$ is to be determined. A function $g(r)$ can be expressed with the help of a complete orthogonal set with unit norm, i.e. a complete orthonormal set $\{e_i(r)\}$;

$$g(x)=\sum_{i=1}^{\infty} g_i e_i(r) \quad \cdots\cdots (10)$$

and

$$\|e_i(r)\|=1 \quad i=1,2,\ldots$$

Considering the continuity of the inner product

$$\tau(\alpha+h)\approx\tau(\alpha)+\lim_{n\to\infty}<h,\sum_{i=1}^{n} g_i e_i> \quad \cdots\cdots (11)$$

The variation is

$$h(x)=\sum_{i=1}^{\infty}\varepsilon h_i e_i(x) \quad \cdots\cdots (12)$$

From the Parseval formula[4], the norm of h is $O(\varepsilon)$. The coefficients εh_i can be found as

$$\varepsilon h_i=<h,e_i>=\int_{hub}^{d/2} h(r)e_i(r)dr \quad \cdots\cdots (13)$$

therefore

$$h(r)=\sum_{i=1}^{\infty}<h,e_i>e_i(r) \quad \cdots\cdots (14)$$

If a variation

$$h^k(r)=\sum_{i=1}^{\infty}<h^k,e_i>e_i, \quad k=1,2,3,\ldots \quad \cdots\cdots (15)$$

is substituted into eq. (9), then

$$\begin{aligned}\tau(\alpha+h^k)&=\tau(\alpha)+<h^k,g>\\&=\tau(\alpha)+\sum_{i=1}^{\infty}<h^k,e_i><e_i,g>\\&=\tau(\alpha)+\sum_{i=1}^{\infty}<h^k,e_i>g_i, \quad k=1,2,3,\ldots\end{aligned} \quad \cdots\cdots (16)$$

Therefore, the following linear matrix equation is obtained from eq. (15), which can be solved to yield g_i;

$$\tau_k=\sum_{i=1}^{\infty} A_{k,i}g_i \quad \cdots\cdots (17)$$

where

$$\tau_k=\tau(\alpha+h^k)-\tau(\alpha)$$

$$A_{k,i}=<h^k,e_i>=\int_{hub}^{d/2} h^k(r)e_i(r)dr$$

By the way, from the beam theory[5], a deflection due to a concentrated force P is governed by the differential equation;

$$\frac{d^2h^k}{dr^2}=-\frac{M_k}{EI}, \quad r_{hub}<r<d/2 \quad \cdots\cdots (18)$$

where E is modulus of elasticity, I is moment of inertia and M_k is a moment generated by the force P at $r=r_k$, $r_{hub}<r_k<d/2$.

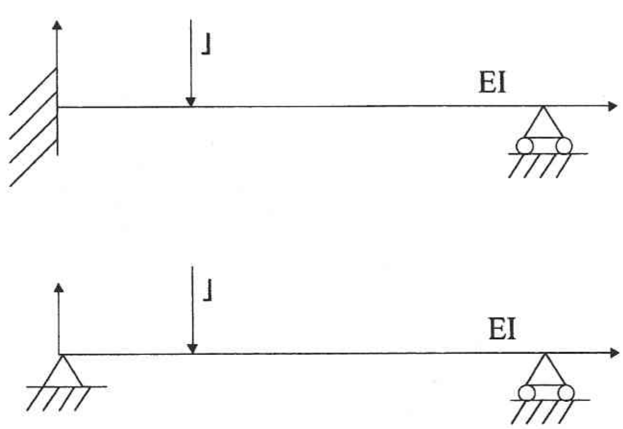

Fig. 2　Definition sketch of the Euler Beam

Using the set of the functions h^k, it is possible to construct

$$g(x)=\sum_{i=1}^{\infty} g_i h^k(r) \quad \cdots\cdots (19)$$

From eq.(9), the following matrix equation is obtained;

$$\tau_k=\sum_{i=1}^{\infty} B_{k,i} g_i \quad \cdots\cdots (20)$$

where

$$\tau^k=\tau(\alpha+h^k)-\tau(\alpha)$$
$$B_{k,i}=<h^k,h^i>=\int_{hub}^{d/2} h^k(r)h^i(r)dr$$

From the Schwartz inequality[6],

$$|<h,g>|\leq \|h\|\cdot\|g\| = \sqrt{\int_0^L h^2(r)dr}\cdot\sqrt{\int_0^L g^2(r)dr} \quad \cdots\cdots (21)$$

which means that the change of the ratio T/Q, i.e. $<h, g>$ is always bounded by the norm products $\|h\|\cdot\|g\|$. Here, the maximization of the change of the ratio T/Q is attained when $h(r)$ is spanned by $g(r)$.Using eq.(20), it is possible to find the function $g(r)$. Therefore, the desired sectional slope of the propeller of the ship which maximize the propulsive efficiency is

$$\alpha_{new}(r)=\alpha(r)+h_{\max}(r) \quad \cdots\cdots (22)$$

where $\alpha(r)$ is the original sectional slope of the propeller. The corresponding maximal change of the ratio T/Q is

$$<h,g>=<h_{\max},g>=<\varepsilon_1\cdot g,g>=\varepsilon_1\|g\|^2 \quad \cdots\cdots (23)$$

where ε_1 is a constant to be determined, which depends on the functional τ.

3. Theoretical results for the propulsive efficiency

Suppose that the modified propeller with $\alpha_{new}(r)$ in eq. (22) operates. Then the speed U, the thrust-deduction coefficient t and the ratio T/Q would slowly vary and these slowly varying quantities can be asymptotically expressed;

$$U'=U+o(\varepsilon) \quad \cdots\cdots (24)$$

$$t'=t+o(\varepsilon) \quad \cdots\cdots (25)$$

$$T'/Q'=T/Q+\varepsilon\|g\|^2+o(\varepsilon) \quad \cdots\cdots (26)$$

where U', t', and T'/Q' are slowly varied speed, thrust-deduction coefficient and ratio of thrust and torque, respectively. Therefore, a new propulsive efficiency η_{new} is

$$\eta_{new}=\frac{DU'}{2\pi nQ'}=\frac{(1-t')U'}{2\pi n}\cdot\frac{T'}{Q'} =\frac{(1-t)U}{2\pi n}\cdot\frac{T}{Q}+\varepsilon_1\|g\|^2\frac{(1-t)U}{2\pi n}+o(\varepsilon) \quad \cdots\cdots (27)$$

Using eq.(3) and the definition of the norm, the above equation becomes

$$\eta_{new}=\eta+\varepsilon_1\|g\|^2\frac{(1-t)U}{2\pi n}+o(\varepsilon) =\eta+\frac{\varepsilon_1\cdot(1-t)U}{2\pi n}\cdot\int_{hub}^{d/2} g^2(r)dr+o(\varepsilon) \quad \cdots\cdots (28)$$

According to the Parseval formula, the above integral representation can be transformed into series;

$$\eta_{new}=\eta+\frac{\varepsilon_1\cdot(1-t)U}{2\pi n}\cdot\left\{\sum_{i=1}^{\infty}|g_i|^2\right\}+o(\varepsilon) \quad \cdots\cdots (29)$$

The gain of the propulsive efficiency is

$$\Delta\eta=\eta_{new}-\eta=\frac{\varepsilon_1\cdot(1-t)U}{2\pi n}\cdot\int_{hub}^{d/2} g^2(r)dr+o(\varepsilon) \quad \cdots\cdots (30)$$

$$=\frac{\varepsilon_1\cdot(1-t)U}{2\pi n}\cdot\left\{\sum_{i=1}^{\infty}|g_i|^2\right\}+o(\varepsilon) \quad \cdots\cdots (31)$$

4. Conclusions

In this paper, a minimization theory based on the Hilbert space theory is proposed and applied to design for propeller in a wake, which gives theoretical results for optimal efficiency and corresponding shape of the propeller based on CFD simulation. In addition, the gain efficiency is always non-negative, because the following term in eq. (30) is always non-negative;

$$\int_{hub}^{d/2} g^2(r)dr=\sum_{i=1}^{\infty}|g_i|^2\geq 0$$

Exact calculation of the gain of the propulsive efficiency can be obtained from the relation between self-propelled and open-water propelled tests using the modified propeller. From the analysis, it is concluded that if the above minimizing process repeats, then the optimization to increase the propulsive coefficient can be achieved and it is also known that the path of optimization is unique.

(Manuscript received, June 15, 2000)

References

1) P. Roman. Some *Mordern Mathematics For Physicists and Other Outsiders*, Vol. 2, Pergamon Press Inc. (1975).

2) J. N. Newman. *Marine Hydrodynamics,* the MIT Press. (1986).

3) Naylor, Arch W. and Sell, George R.(1982), *Linear Operator Theory in Engineering and Science,* Springer-Verlag, Berlin.

4) Oden, J.Tinsley (1979), *Applied Functional Analysis*, Prentice-Hall International, Inc., London.

5) J. M. Gere and S.P. Timoshenko, Mechanics of Materials, PWS (1990).

6) Reddy, J.N.(1987), *Applied Functional Analysis and Variational Methods in Engineering*, McGraw-Hill, New York.