研　究　速　報

AN UPRIGHT SINGLE BEAM EQUIVALENT TO GROUPED PILES

群抗と等価な直立単純梁について

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1. INTRODUCTION

Piles, grouped beneath a superstructure, interact with the surrounding soil during an earthquake, and the dynamic pile-soil-pile interaction often affects the motion of the superstructure to some considerable extent. Straight-forward evaluation of the pile-soil-pile interaction, however, is cumbersome especially in dealing with tens or hundreds piles groped together, and no small number of attempts for simpler approaches have been made so far. These attempts include the Ring-Pile method (Takemiya, 1986) and Closely-Spaced-Plates model (Ohira, *et al.* 1985). In these methods, respectively, piles with the soil caught among them are re-grouped into several concentric cylinders (piles arranged in concentric circles) and soil-pile-striped upright plates, allowing close evaluation of interaction effects to be made with less time and trouble. This paper presents further simplified approach in which a group of piles is viewed as a single equivalent upright beam.

2. SUPERPOSITION METHOD FOR EVALUATION OF GROUP EFFECT

Prior to introducing the equivalent upright beam, straight-forward evaluation of pile-soil-pile interaction is necessary to provide rigorous solutions. Based on the numerical scheme presented by Tajimi and Shimomura (Thin-Layered Method, 1976) that allows soil-embedded foundation interaction effects to be rigorously evaluated, a numerical program *"TLEM"*(Ver. 1.1) has been developed for soil-pile group interaction analyses (Konagai, 1998). The piles are assumed to be upright Timoshenko or Beronoulli-Euler beams embedded in a horizontally layered soil deposit with infinite extent. The evaluation of pile-soil-pile interaction effects in this program is based on the superposition method that was originally proposed by Poulos (1968, 1971). In this approximation, only two piles are considered in the formulation of global flexibility matrix, and other piles' effects on these two piles are totally ignored (**Fig. 1**). Kanya and Kausel (1982) have shown that the superposition scheme gives reasonable results not only for static loads but for dynamic loads as well.

A dynamic interaction factor $I_{u,F}$ for a pair of piles (in which a unit harmonic load is applied to the first pile head (active pile) and the displacements are evaluated for the second one (passive pile)) is defined as follows:

$$I_{u,F} = \frac{\textit{Dynamic displacement of passive pile}}{\textit{Static displacement of active pile}} \quad \cdots\cdots\cdots(1)$$

When rocking motions or moments are concerned, displacements and forces are denoted by $r_0\phi$ and M/r_0, respectively, where r_0 is the radius of pile, ϕ is the angle of rotation of the passive pile and M is the moment applied to the head of the active pile.

Figs. 2a-2c show the variations of interaction factors with respect to non-dimensional frequency $a_0(=\omega d/v_s$ with $d = 2r_0)$. The parameters used in the analyses are listed in **Tables 1** and **2**. The results are compared in these figures with the solutions by Kanya and Kausel (1982). The curves calculated with *"TLEM"*

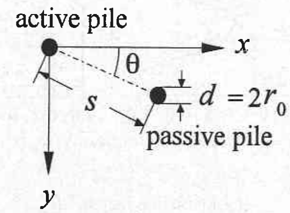

Fig. 1 Active and Passive piles

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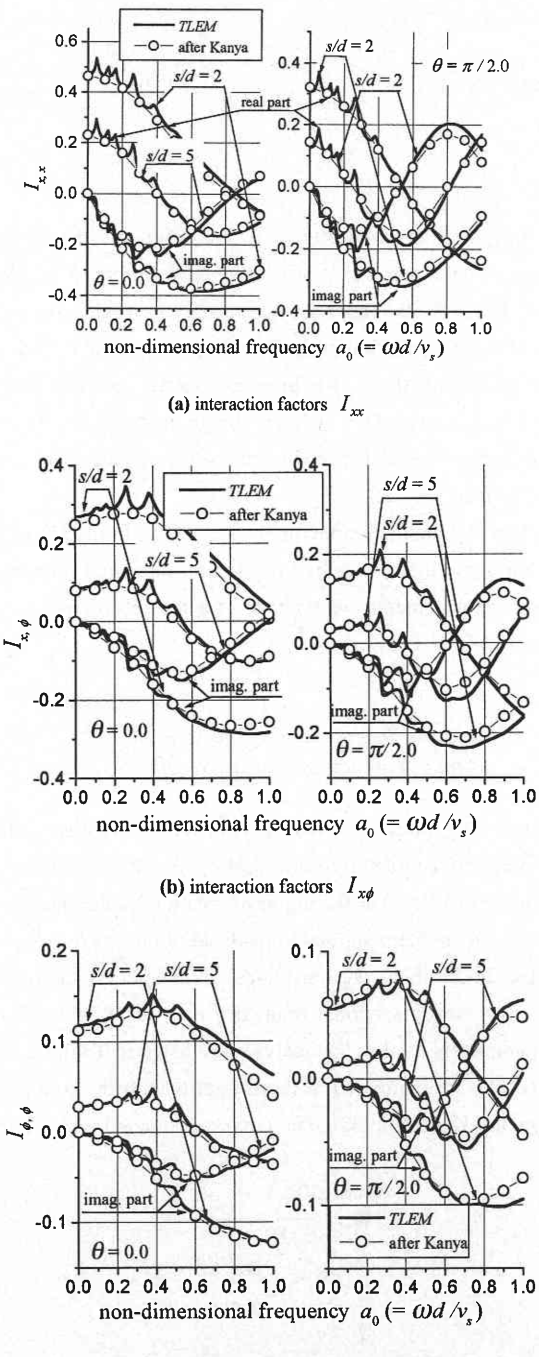
show small spikes at the resonance frequencies of the surface soil deposit, because the bedrock underlying the surface layer is assumed to be completely rigid in *"TLEM"* analyses. Except for these upward spikes, the interaction factors are in good agreement with the solutions by Kanya and Kausel (1982).



(a) interaction factors I_{xx}

(b) interaction factors $I_{x\phi}$

(c) interaction factors $I_{\phi\phi}$

Fig. 2 Variations of interaction factors

Given the interaction factors for all the possible pairs in a grouped piles, the global flexibility matrix of the pile group is assembled. **Fig. 3** shows the variation of 2 × 2 pile-cap stiffness in x-direction that is compared with that obtained by Kanya and Kausel (1982). The good agreement between them validates *"TLEM"*.

3. EQUIVALENT SINGLE BEAM

The soil and n_p piles system is divided into horizontal slices as shown in **Fig. 4**. The following assumptions are tentatively adopted herein to derive the stiffness matrix of the equivalent single beam:

(1) Pile elements within a horizontal soil slice are deformed all at once keeping their intervals constant, and the soil caught among piles moves in a body with the piles. The cross-section of the equivalent single upright beam, thus, comprises both the firmly joined piles and the soil.

(2) Frictional effects due to bending of piles (external moments on each pile) are ignored.

(3) Top ends of the piles are fixed to a rigid cap.

(4) All upper or lower ends of the sliced pile elements arranged on the cut-end of a soil slice remain on one

Table 1 Parameters for piles

E_pI_p (tf m^2)	ρ_p (t/m^3)	r_0 (m)	length (m)
2.4×10^5	2.0	0.5	15

Table 2 Parameters for surface soil deposit

ρ_p (t/m^3)	v_s (m/s)	ν	Thickness (m)
1.75	100	0.40	20

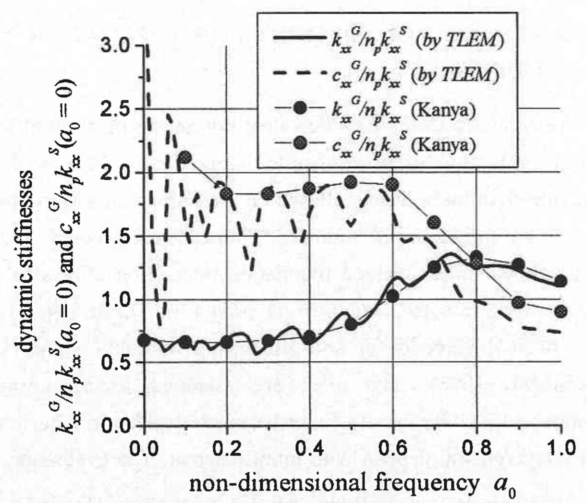


Fig. 3 Variations of stiffness for sway motion of 2 × 2 pile group

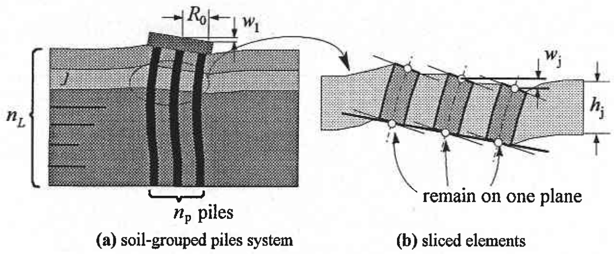


Fig. 4 Assumptions for evaluation of equivalent single beam

plane (Note this assumption does not necessarily mean that each pile's cross-section remains in parallel with that plane. See **Fig. 4(b)**).

With assumptions (1), (2) and (3), lateral external forces $\{\mathbf{p}_x\}$ are described in terms of lateral displacements $\{\mathbf{u}_x\}$ and anti-symmetric vertical motion of the cap ω_1 as:

$$\{p_x\} = [\mathbf{L}]\,[\mathbf{D}]^{-1}\left\{[\mathbf{L}]\{\mathrm{u}_x\} + \left\{\frac{w_1}{R_0} \,\vdots\, 0 \cdots 0\right\}^T\right\} \cdots\cdots\cdots\cdots (2)$$

where, R_0 is the radius of the equivalent single beam, and

$$[\mathbf{L}] = \begin{bmatrix} -\frac{1}{h_1} & \frac{1}{h_1} & 0 & 0 & \cdots & 0 \\ \frac{1}{h_1} & -\frac{1}{h_1}-\frac{1}{h_2} & \frac{1}{h_2} & 0 & & \vdots \\ 0 & \frac{1}{h_2} & -\frac{1}{h_2}-\frac{1}{h_3} & -\frac{1}{h_3} & & \vdots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & -\frac{1}{h_{n_L-1}} \\ 0 & \cdots & \cdots & 0 & \frac{1}{h_{n_L-1}} & -\frac{1}{h_{n_L-1}}-\frac{1}{h_{n_L-1}} \end{bmatrix} \cdots (3\,\mathrm{a})$$

$$[\mathbf{D}] = \frac{1}{6}\begin{bmatrix} 2\frac{1}{E_1I_1} & \frac{h_1}{E_1I_1} & 0 & 0 & \cdots & 0 \\ \frac{1}{E_1I_1} & 2\left(\frac{h_1}{E_1I_1}-\frac{h_2}{E_2I_2}\right) & \frac{h_2}{E_2I_2} & 0 & & \vdots \\ 0 & \frac{h_2}{E_2I_2} & 2\left(\frac{h_2}{E_2I_2}+\frac{h_3}{E_3I_3}\right) & \frac{h_3}{E_3I_3} & & \vdots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \frac{h_{n_p-1}}{E_{n_p-1}I_{n_p-1}} \\ 0 & \cdots & \cdots & 0 & \frac{h_{n_L-1}}{E_{n_p-1}I_{n_p-1}} & 2\left(\frac{h_{n_p-1}}{E_{n_p-1}I_{n_p-1}}+\frac{h_{n_p}}{E_{n_p}I_{n_p}}\right) \end{bmatrix} \cdots (3\,\mathrm{b})$$

with $E_jI_j = \mathrm{n} \times E_pI_p$ (E_pI_p = bending stiffness of a single pile).

Moment at the top ends of rigidly capped piles due to the lateral displacements $\{\mathbf{u}_x\}$ is expressed as:

$$M_1 = \{1st\ row\ of\ matrix\ [\mathbf{D}]^{-1}[\mathbf{L}]\}\,\{\mathbf{u}_x\}^T + D^{-1}_{1,1}\cdot\frac{w_1}{R_0} \cdots\cdots (3\,\mathrm{c})$$

where, $D^{-1}_{1,1}$= upper-left corner component of the matrix $[\mathbf{D}]^{-1}$.

Assumption (4) implies that the overall anti-symmetric rocking motion of a pile group is controlled by axial motions of the piles. In other word, the external moments on the overall soil-pile system from its surrounding soil are eventually sustained by the piles that experience alternate push and pull in their axes. External moments due to the anti-symmetric vertical motions $\{\mathbf{w}\}$ are described as:

$$\left\{\frac{\mathbf{M}}{R_0}\right\} = [\mathbf{Q}]\,\{\mathbf{w}\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

with,

$$[\mathrm{Q}] = \begin{bmatrix} \frac{EI_1^G}{R_0^2h_1} & -\frac{EI_1^G}{R_0^2h_1} & 0 & 0 & \cdots & 0 \\ -\frac{EI_1^G}{R_0^2h_1} & \frac{EI_1^G}{R_0^2h_1}-\frac{EI_1^G}{R_0^2h_1} & -\frac{EI_2^G}{R_0^2h_2} & 0 & & \vdots \\ 0 & -\frac{EI_2^G}{R_0^2h_2} & \frac{EI_2^G}{R_0^2h_2}-\frac{EI_3^G}{R_0^2h_3} & -\frac{EI_3^G}{R_0^2h_3} & & \vdots \\ 0 & & & & & 0 \\ \vdots & & & & \ddots & -\frac{EI^G_{n_p-1}}{R_0^2h_{n_p-1}} \\ 0 & \cdots & \cdots & 0 & -\frac{EI^G_{n_p-1}}{R_0^2h_{n_p-1}} & -\frac{EI^G_{n_p-1}}{R_0^2h_{n_p-1}}+-\frac{EI^G_p}{R_0^2h_n} \end{bmatrix} \cdots\cdots (5)$$

where, EI^G is the bending stiffness of the equivalent single upright beam. This EI^G is evaluated following the same procedure as that used for the evaluation of bending stiffnesses of reinforced concrete beams.

Given equations (2)-(5),the global stiffness matrix of the equivalent single beam is finally expressed as:

$$\begin{Bmatrix} \mathbf{p}_x \\ \cdots \\ \frac{\mathbf{M}}{R_0} \end{Bmatrix} = \begin{bmatrix} [\mathbf{L}]\,[\mathbf{D}]^{-1}\,[\mathbf{L}] & \vdots & 1st\ column\ of\ [\mathbf{L}]^{-1}[\mathbf{D}]\ and\ zeros\ for\ other\ column \\ \cdots\cdots\cdots & & \cdots\cdots\cdots \\ 1st\ row\ of\ [\mathbf{D}]^{-1}[\mathbf{L}]\ and\ zeros\ for\ other\ rows & \vdots & D^{-1}_{1,1}\ and\ [\mathbf{Q}] \end{bmatrix} \begin{Bmatrix} \mathbf{u}_x \\ \cdots \\ \mathbf{w} \end{Bmatrix} \cdots\cdots (6)$$

"TLEM" has been upgraded for evaluation of the behaviors of an equivalent single beam (Ver. 1.2). **Fig. 5** show the variations

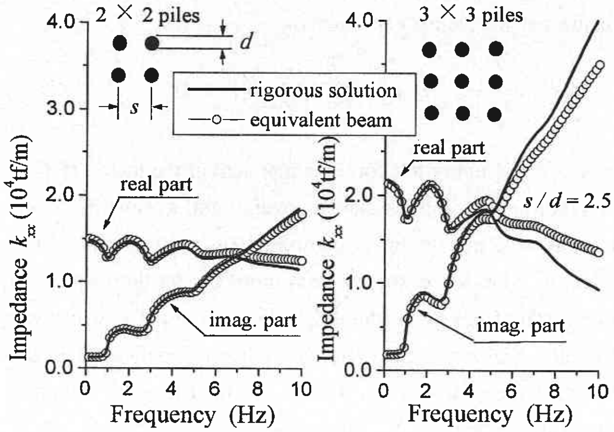


Fig. 5 Variations of stiffness parameters for sway motions of pile groups

Table 3 Parameters for steel piles

E_p (tf/m^2)	ρ_p (t/m^3)	r_0 (m)	thickness (m)	length (m)
2.1×10^7	7	0.3	0.0089	20

Table 4 Parameters for soil

ρ_p (t/m^3)	v_s (m/s)	ν
1.5	80	0.49

of pile cap stiffnesses for sway motions of 2 × 2 and 3 × 3 steel pile groups (**Table 3**) embedded in a homogeneous soil deposit (**Table 4**). The curves for the equivalent single beams agree well with rigorous solutions from *"TLEM"* (Ver. 1.1).

An earthquake causes the free-field ground motion $\{\mathbf{u}^f\}$. The piles in this soil deposit, however, will not follow the free-field deformation pattern. This deviation of the displacements from the free-field soil displacements $\{\mathbf{u}^f\}$ is denoted by $\{\mathbf{u}^s\}$. Equation (6) is also used to evaluate effective foundation input motion $\{\mathbf{u}^f\}$ + $\{\mathbf{u}^s\}$. The effects of soil-embedded-foundation kinematic interaction are portrayed in the form of two kinematic displacement factors in sway and rocking motions

$$T_{e,sway} = \frac{(u^f + u^s)_{sway}}{(u^f)_{sway}}, \quad T_{e,rocking} = \frac{(u^f + u^s)_{rocking}}{(u^f)_{sway}} = \frac{u^s{}_{roking}}{u^f{}_{sway}} \quad \cdots (7\,\mathrm{a, b})$$

plotted as functions of frequency. In equation (7b), $u^s{}_{rocking}$ is defined as:

$$u^s{}_{rocking} = r_o \phi \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

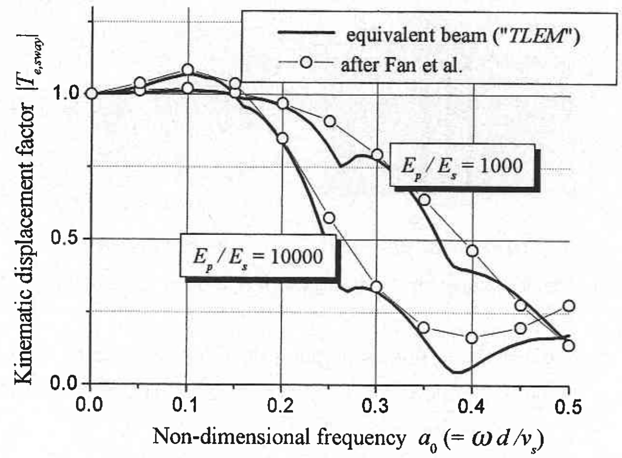


Fig. 6 Kinematic displacement factors of pile groups

and it is noticed that $u^f{}_{rocking}$ is essentially identical to zero.

Fig. 6 shows the kinematic displacement factors of a 2 × 2 PC pile group (*s*/*d* =, See **Tables 1** and **2**), and they are in good agreement with rigorous solutions by Fan *et al.* (1982).

4. CONCLUSIONS

Piles grouped beneath a superstructure can be viewed as a single equivalent upright beam when the piles are closely spaced. The stiffness matrix presented herein (equation (6)) yields close approximations of both dynamic pile-cap stiffness and kinematic displacement factors. This simplification will lead to expanding the feasibility of incorporating non-linear soil-pile group interaction effects in the analyses. Further detailed study on this point will be addressed in later publication.

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REFERENCES

1) Tajimi, H. and Y. Shimomura: Dynamic Analysis of Soil-Structure Interaction by the Thin Layered Element Method, Transactions of the Architechtual Institute of Japan, 243, 41-51, 1976.
2) Kanya, A. and E. Kausel: Dynamic Stiffness and Seismic Response of Pile Groups, NSF report, NSF/CEE-82023, 1982.
3) Konagai, K.: Guide to "TLEM", program manual No. 5, Konagai Lab., IIS, Univ. of Tokyo, June, 1998.
4) Fan, K., Gazetas, G., Kanya, A., Kausel, E. and Ahmad, S.: Kinematic Seismic Response of Single Piles and Pile Groups, Jour., Geotechnical Engineering, ASCE, 117 (12), 1860-1879, 1991.