研 究 解 説

The Second Order Hydrodynamic Actions On A Flexible Body (Part I)

弾性浮体に働く二次の流体力学的作用（その１）

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The second order hydrodynamic forces due to rigid body rotations and variations of instantaneous wetted surface area may provide non-linear loads to the structural responses of a ship traveling in rough seas, and may also induce low frequency oscillation or springing of a multi-module very large floating structure moored in waves. As a preliminary study of the second order hydroelastic behavior of marine structures, the contributions of the first order wave potentials and responses to the second order hydrodynamic actions for a flexible body are formulated, and the corresponding expressions. The velocity potential, The hydrodynamic pressure and The hydrodynamic forces are presented in this paper.

1. Introduction

The second order hydrodynamic forces induced by the rigid body rotations and the variation of the instantaneous wetted surface area have been a great concern in the estimation of responses of a floating platform, a mooring system, or a VLFS in random waves. When a ship travels in rough seas, the large motions and the hydrodynamic forces acting on the instantaneous wetted surface also result in the non-linear behaviors of the structural loads and responses. This may be observed in many ways, including the phenomenon of greater magnitudes of midship sagging moments than those of hogging moments of a ship traveling in waves, for example. To meet the requirements of safety, reliability, and performance of ships and other marine structures, the naval architects have to make continuous effort in seeking the possible ways to predict the non-linear loads more reasonably, rationally, as well as practically for application.

The second order wave forces have been widely investigated for motion and drift force predictions of stationary floating marine structures. In most cases the structure is approximated as a rigid body responding to first order and second order wave forces. When the structure has forward speed as the case of a ship, or when it encounters current for the case of a stationary floating system, the modeling and solution of second order problems get more complicated.

For a ship traveling in irregular waves, the non-linear effect induced by rotations of the hull has not yet been thoroughly investigated. Many publications concern about the instantaneous wetted surface effect. In this connection the strip theories have shown to be easily handled and mostly used[1]. The non-linear three-dimensional hydrodynamic actions on a ship have also been the subject of research, though not much (see, for example, ITTC'96[2]). Lin and Yue[3] presented a three-dimensional method of solving the large-amplitude motions of a rigid ship. In this method the time-domain Green Function is employed, the free surface condition is linear, while the hydrodynamic integral equations are taken over the instantaneous wetted surface and its water line.

If the structural responses (distortions, stresses, bending moments, etc.) to the non-linear second order hydrodynamic actions are to be examined together with the rigid body motions, the solution of corresponding fluid-structure interactions is even more complicated. It should be reminded that the inclusion of the flexible body distortions in the analysis does not give notable influence to the resultant hydrodynamic forces and moments required for the rigid body motion predictions. The key advantage of

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doing so is to obtain more rational predictions of local and global structural (not only the rigid body) loads and responses.

During the past decade great progress has been made in the development and application of the linear three-dimensional hydroelasticity theories[4)-7)]. How to account for the non-linear hydrodynamic forces in the three-dimensional fluid-structure interactions remains a field to be further investigated. This paper discusses the contributions of the first order velocity potentials and responses to the second order hydrodynamic actions on a flexible body. No attempt is made at the present stage to consider the non-linear effect due to slamming impact and wet deck loading. Only allowed for in the analysis are the second order hydrodynamic actions induced by the rigid body rotations and the variation of the instantaneous wetted surface area. The structure is still assumed linear, and the existing linear three-dimensional hydroelasticity approach is used to obtain the first order velocity potentials and responses of the structure[8)]. The expressions of the generalized second order hydrodynamic forces are formulated, and the non-linear equations of motion both in frequency domain and in time domain are presented. The methods introduced may be used to develop a numerical procedure for prediction of non-linear loads and responses of a ship traveling in rough seas, or a multi-module VLFS in waves, where each of the modules may have rigid body rotations. The corresponding numerical study is now being continued at Institute of Industrial Science of Tokyo University, and China Ship Scientific Research Center.

2. Coordinate systems

Three coordinate systems will be used to define the fluid actions and the structural responses as shown in Fig. 1, where $Ax_0y_0z_0$ is a space-fixed frame of reference; Oxyz is an equilibrium frame of axes moving with forward seed U

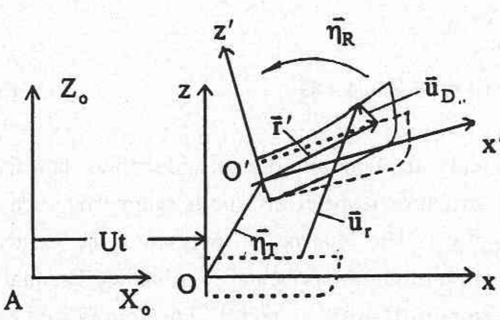

Fig. 1 Coordinate systems

and remaining parallel to $Ax_0y_0z_0$; O'x'y'z' is a body-fixed axes system. O'x'y'z' coincides with Oxyz in the absence of any disturbance. The O'z' axis passes through the gravity center G and points upwards. The plane lies on the undisturbed water surface.

3. Structural dynamics

3.1 Definition of the Displacement Components

The structure is discretized by finite element method as a system of m degrees of freedom. The rigid body transitions, rotations and the flexible body distortions of the structure are denoted by the vectors $\bar{\eta}_T$, $\bar{\eta}_R$ and $\bar{u}_D$ respectively. $\bar{\eta}_T$ is described with respect to the equilibrium frame of axes, while $\bar{\eta}_R$ and $\bar{u}_D$ are both described with respect to the body fixed axes system. They are

$$\bar{\eta}_T = \sum_{r=1}^{3} \bar{u}_r = (u_1, u_2, u_3), \tag{3.1a}$$

$$\bar{\eta}_R = \sum_{r=4}^{6} \bar{\theta}_r = (\theta_4, \theta_5, \theta_6) = \frac{1}{2}\sum_{r=4}^{6} \nabla \times \bar{u}_r, \tag{3.1b}$$

$$\bar{u}_D = \sum_{r=7}^{m} \bar{u}_r = (u_r, v_r, w_r), \tag{3.1c}$$

where $\bar{u}_r$ ($r \geqq 7$) denotes the distortion of the structure in its r-th principal mode, and

$$\bar{\theta}_r = \frac{1}{2}\nabla \times \bar{u}_r .$$

The displacement due to the 6 rigid body modes are respectively

$$\bar{u}_1 = \{u_1, 0, 0\},\ \bar{u}_2 = \{0, u_2, 0\},\ \bar{u}_3 = \{0, 0, u_3\},$$
$$\bar{u}_4 = \{0, -z'\theta_4, y'\theta_4\},\quad \bar{u}_5 = \{z'\theta_5, 0, -x'\theta_5\},$$
$$\bar{u}_6 = \{-y'\theta_6, x'\theta_6, 0\}. \tag{3.2}$$

Evidently,

$$\sum_{r=4}^{6} \bar{u}_r = \bar{\eta}_R \times \bar{r}'$$

In linear theory the total displacement at any point of the structure may be expressed as

$$\bar{u}^{(1)} = \{u^{(1)}, v^{(1)}, w^{(1)}\} = \sum_{r=1}^{m} \bar{u}_r$$
$$= \bar{\eta}_T + \bar{\eta}_R \times \bar{r}' + \bar{u}_D . \tag{3.3}$$

In a general case an order parameter ε is introduced that $|\bar{\eta}_T/L| = O(\varepsilon) = |\bar{\eta}_R/L|$, where L is a characteristic length of the structure. The order of distortions may be represented as $|\bar{u}_D/L| = O(\varepsilon^{\nu})$. For a conventional ship ν may be about 1.5~2.0. However for some kind of VLFS, for example a shallow draft barge-type very large floating airport, taking *L*as the draft, ν may be 1.0 or less.

If the effect of the rigid body rotations are to be examined to the second order, the total displacement at any point of the structure described in the axes system Oxyz may be written as

$$\bar{u} = [\bar{\eta}_T + \tilde{T}(\bar{r}' + \bar{u}_D)] - \bar{r}'. \qquad (3.4)$$

Here $\tilde{T}$ is the transformation matrix between the coordinate systems O'x'y'z' and Oxyz. It has the form

$$\tilde{T} = \tilde{I} + \tilde{R} + \tilde{H}, \qquad (3.5)$$

where

$$\tilde{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} 0 & -\theta_6 & \theta_5 \\ \theta_6 & 0 & -\vartheta_4 \\ -\theta_5 & \theta_4 & 0 \end{bmatrix}, \qquad (3.6a)$$

$$\tilde{H} = \frac{1}{2}\begin{bmatrix} -(\theta_5^2 + \theta_6^2) & 0 & 0 \\ 2\theta_4\theta_5 & -(\theta_4^2 + \theta_6^2) & 0 \\ 2\theta_4\theta_6 & 2\theta_5\theta_6 & -(\theta_4^2 + \theta_5^2) \end{bmatrix}. \qquad (3.6b)$$

The formula (3.4) may therefore be written to the second order in the form

$$\bar{u} = \bar{u}^{(1)} + \tilde{H}\bar{r}' + \tilde{R}\bar{u}_D. \qquad (3.7)$$

The second term at the right hand side represents the second order influence of the rigid body rotations to the local displacement. The third term describes the first order influence of the rigid body rotations to the representation of distortions in the equilibrium frame of axes. The three terms of (3.7) are of the orders O(ε), O(ε^2) and O(ε^3) respectively for a conventional ship; however may be of O(ε), O(ε^2) and O(ε^2) respectively for a VLFS.

In a similar way, the relationship of normal vectors of the body's wetted surface, defined in the two coordinate systems O'x'y'z' and Oxyz, relating to the steady-state and disturbed conditions, may be written as

$$\tilde{N}(x, y, z, t) = \tilde{T}\bar{n}(x', y', z', t) = \bar{n} + \tilde{R}\bar{n} + \tilde{H}\bar{n}. \qquad (3.8)$$

3.2 Principal modes and principal coordinates

Under the assumption of linear structure a set of principal modes of the dry structure, $\bar{u}_r^o(x', y', z') = \{u_r^o, v_r^o, w_r^o\}$ (r = 1, 2, ..., m), may be obtained. The first six modes are the rigid body modes, defined as

$$\bar{u}_1^0 = \{1, 0, 0\}, \quad \bar{u}_2^0 = \{0, 1, 0\}, \quad \bar{u}_3^0 = \{0, 0, 1\},$$
$$\bar{u}_4^0 = \{0, -z', y'\}, \bar{u}_5^0 = \{z', 0, -x'\}, \bar{u}_6^0 = \{-y', x', 0\} \quad (3.9)$$

These dry modes may be used as an aggregation of orthogonal functions to represent the structural responses in the form

$$\bar{u} = \sum_{r=1}^{m} \bar{u}_r(x', y', z') = \sum_{r=1}^{m} \bar{u}_r^o(x', y', z') p_r(t) \qquad (3.10)$$

where $P_r(t)$ (r = 1, 2, ..., m) are the principal coordinates. This expression may be used for either the first order responses, or the second order responses. For the first order responses (3.10) gives (3.3), with

$$\bar{u}_r = \bar{u}_r^o p_r^{(1)}(t). \qquad (3.11)$$

When representing the second order responses, (3.10) is non-linear to the fluid excitations, and gives

$$\mathit{\bar{u}} = \sum_{r=1}^{m} \bar{u}_r(x', y', z') = \sum_{r=1}^{m} \bar{u}_r^o(x', y', z') p_r^{(2)}(t).$$

In this case (3.7) becomes

$$\bar{u} = \mathit{\bar{u}} + \tilde{H}\bar{r}' + \tilde{R}\bar{u}_D$$

According to (3.2) and (3.9), the first six principal coordinates are respectively

$$p_1(t) = u_1, \quad p_2(t) = u_2, \quad p_3(t) = u_3,$$
$$p_4(t) = \theta_4, \quad p_5(t) = \theta_5, \quad p_6(t) = \theta_6. \qquad (3.12)$$

3.3 Generalized equations of motion

If the expression (3.10) is introduced, it is found that the matrix equation of motion of the structure, may be written in the form

$$\mathbf{a}\ddot{\mathbf{p}} + \mathbf{b}\dot{\mathbf{p}} + \mathbf{c}\mathbf{p} = \mathbf{Z} + \boldsymbol{\Delta} + \mathbf{G} \qquad (3.13)$$

where a and c are generalized mass and stiffness matrices of the dry structure respectively, both symmetric such that $c_{rr} = \omega_r^2 a_{rr}$. The matrix b represents the structural damping in terms of the principal coordinates. The matrix p is the vector $\{p_1(t), p_2(t), ..., p_m(t)\}$. The generalized forces and Z, Δ, and G will be examined individually.

3.4 Generalized forces

The generalized fluid force Z corresponding to p may contain the linear and non-linear hydrodynamic actions. In linear theory its r-th component is

$$Z_r(t) = -\iint_{\bar{S}} \bar{n}\cdot\bar{u}_r^o p\, dS \tag{3.14}$$

where p denotes the pressure acting on the wetted surface $\bar{S}$, $\bar{n}$ is the outward unit normal vector into the fluid. If the second order hydrodynamic forces are included it is expressible as

$$Z_r(t) = -\iint_{S(t)} \bar{N}\cdot\bar{u}_r^o p\, dS \tag{3.15}$$

where the integration is taken over the instantaneous wetted surface S(t), and $\bar{N}$ denotes its unit normal vector, defined by (3.8).

The matrix Δ representing generalized concentrated forces at the principal coordinates may include the actions of linear or non-linear mooring forces $\bar{T}_j(t)$ (j = 1, 2, ..., m_o), acting at.(X'_j, y'_j, z'_j). In this case $\bar{T}_j(t)$ is a function of the principal coordinates. If only the mooring forces are considered, the generalized concentrated force at P_r is

$$\Delta_r(t) = \sum_{j=1}^{m_o} \bar{u}_r^o(x'_j, y'_j, z'_j)\cdot\bar{T}_j(t) \tag{3.16}$$

Provided that the hydrodynamic drag and damping forces acting on the mooring lines are neglected, the motions of the attachment points of the mooring lines are small, and the linear assumption is employed, $\bar{T}_j(t)$ will be proportional to the principal coordinates. In this simple case Δ_r (r = 1, 2, ..., m) are always expressible as

$$\Delta_r(t) = \sum_{k=1}^{m} (Cm_{rk}) p_k(t) \tag{3.17}$$

where Cm_rk is a restoring coefficient representing the mooring line effect. If the motions of the attachment points are not small, $\bar{T}_j(t)$ (j = 1, 2, ..., mo) and Δ_r (r = 1, 2, ..., m) will be non-linearly dependent on the principal coordinates. For chain mooring lines they may be calculated by the catenary theory in time domain[9].

The generalized force G represents that of distributed gravity, with the r-th component in the form

$$G_r = -\rho\iiint_{\Omega} \rho_b g w'_r d\Omega \tag{3.18}$$

4. Hydrodynamic actions

4.1 Velocity potential

4.1.1 Decomposition of velocity potentials

The fluid is assumed inviscid, incompressible, and the flow is irrotational. If the body travels with a constant forward speed U in x direction, the velocity potential may be decomposed in the form[10]

$$\Phi(x_o, y_o, z_o, t) = U\bar{\phi}(x,y,z) + \phi(x,y,z,t) \tag{4.1}$$

with the unsteady component expressed as

$$\phi(x,y,z,t) = \phi_o(x,y,z,t) + \phi_D(x,y,z,t) + \sum_{k=1}^{m} \phi_k(x,y,z,t) \tag{4.2}$$

$\bar{\phi}$ denotes the velocity potential for the steady motion of the body in calm water. The velocity of the steady flow relative to the moving equilibrium frame of reference is

$$\bar{W} = U\nabla(\bar{\phi} - x). \tag{4.3}$$

ϕ_0, ϕ_D and ϕ_k denote the incident, diffracted and radiation wave potentials respectively. k = 1, 2, ..., m correspond to all the dry modes of the structure, including the 6 rigid body modes.

With the wave steepness as the perturbation parameter α, the diffracted and radiation potentials ϕ_D and ϕ_k can be expressed as a power series of α:

$$\phi = \alpha\phi^{(1)} + \alpha^2\phi^{(2)} + \cdots \tag{4.4}$$

The first order potentials can be calculated by the linear hydroelasticity theories[4~8]. The solution of the second order potentials $\phi^{2)}$ is complicated for engineering applications. In the present paper only the second order wave forces, closely related to the contribution of the first order potentials and responses are examined using the pressure integration method. Therefore hereafter the superscript of[1)] will be omitted.

Corresponding to (3.10) the first order radiation potentials may be represented as

$$\phi_k(x,y,z,t) = \varphi_k(x,y,z) p_k(t) \tag{4.5}$$

4.1.2 The governing equations of the first order potentials for a flexible body

The unsteady velocity potential for a flexible structure satisfies the following governing equations

$$\begin{cases} \nabla^2\phi = 0 & P\in V \\ (\frac{\partial}{\partial t}-U\frac{\partial}{\partial x})^2\phi+g\frac{\partial}{\partial z}\phi=0 & z=0 \\ \frac{\partial}{\partial n}\phi=0 & \text{(on sea bed)} \\ \frac{\partial}{\partial n}\phi=\bar{n}\cdot[\dot{\bar{u}}^{(1)}+\bar{\theta}^{(1)}\times\bar{W}-(\bar{u}^{(1)}\cdot\nabla)\bar{W}] & P\in\bar{S} \\ \text{Radiation Condition.} & R=\sqrt{x^2+y^2}\to\infty \end{cases} \tag{4.6}$$

where $P = (x, y, z)$ u, V is the fluid domain.

With the substitution of (3.3) and (3.11), the wetted surface boundary condition of the radiation potentials may be represented in the form

$$\begin{cases} \frac{\partial}{\partial n}\phi_k = a_k(P)\dot{p}_k(t)+b_k(P)p_k(t), \\ a_k=\bar{n}\cdot\bar{u}_k^o, \\ b_k=\bar{n}\cdot[\bar{\theta}_k^o\times\bar{W}-(\bar{u}_k^o\cdot\nabla)\bar{W}], \end{cases} \tag{4.7}$$

In frequency domain the separation of the time and the space variations allows the radiation potentials, corresponding to each of the flexible body modes, to be solved by employing a suitable boundary integral or boundary element method. The details may be found in references[4,5,7,8], where the three-dimensional pulsating source Green function, or the pulsating and translating source Green function were used for floating, or traveling flexible marine vehicles.

In time domain, the radiation potentials may be transferred to three new functions in the following way

$$\phi_k(P;t)=\int_0^t\tilde{\phi}_k(P;t-\tau)\dot{p}_k(\tau)d\tau\ , \tag{4.8a}$$

with

$$\tilde{\phi}_k(P;t)=\psi_{1k}(P)\delta(t)+\psi_{2k}(P)H(t)+\chi_k(P;t). \tag{4.8b}$$

Here δ(H) and H(t) are defined as

$$\delta(t)=\begin{cases}\infty & t=0,\\ 0 & t\neq 0,\end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty}\delta(t)f(t)dt=f(0)\ ,$$

$$H(t)=\begin{cases}0 & t<0,\\ 1 & t\geq 0.\end{cases}$$

The functionsand ψ_{1k}, ψ_{2k} satisfy the following equations.

$$\begin{cases} \nabla\psi_{1k}=0 & P\in V, \\ \psi_{1k}=0 & z=0, \\ \frac{\partial}{\partial n}\psi_{1k}=a_k & P\in\bar{S}, \end{cases} \tag{4.9a}$$

$$\begin{cases} \nabla\psi_{2k}=0 & P\in V, \\ \psi_{2k}=0 & z=0, \\ \frac{\partial}{\partial n}\psi_{2k}=b_k & P\in\bar{S}, \end{cases} \tag{4.9b}$$

$$\begin{cases} \nabla\chi_k(P,t)=0 & P\in V, \\ [(\frac{\partial}{\partial t}-U\frac{\partial}{\partial x})^2+g\frac{\partial}{\partial z}](\chi_k+\psi_{2k})=0 & \\ & P\in\bar{S}, z=0, t\succ 0, \\ \frac{\partial}{\partial n}\chi_k(P,t)=0 & P\in\bar{S}, \\ \chi_k(P,0)=0 & z=0, \\ \frac{\partial}{\partial t}\chi_k\big|_{t=0}=-g\frac{\partial}{\partial z}\psi_{1k} & z=0. \end{cases} \tag{4.9c}$$

The numerical solutions of ψ_{1k}, ψ_{2k} and χ_k may be obtained by employing the time domain Green functions[11,12], and the boundary integral method[6].

4.2 Hydrodynamic pressures

According to Bernoulli equation and an expansion from the instantaneous wetted surface S(t) to its steady state mean position $\bar{S}$, the fluid pressure acting on the body's wetted surface may be expressed as

$$p|_{S(t)}=-\rho[1+\bar{u}\cdot\nabla]\{\tfrac{\partial}{\partial t}\phi+(\bar{W}\cdot\nabla)\phi +\tfrac{1}{2}(W^2-U^2)+\tfrac{1}{2}(\nabla\phi)^2++gz\}|_{\bar{S}} \tag{4.10}$$

By adopting (3.7), the pressure on the instantaneous wetted surface of a flexible body with arbitrary geometric shape may be represented to the second order in the form

$$p|_{S(t)} = p^{(1)}|_{\bar{S}} + p^{(2)}|_{\bar{S}} \tag{4.11}$$

where

$$p^{(1)}=-\rho\{\tfrac{\partial}{\partial t}\phi+(\bar{W}\cdot\nabla)\phi+[gz'+\tfrac{1}{2}(W^2-U^2)] +[gw+\tfrac{1}{2}(\bar{u}^{(1)}\cdot\nabla)W^2]\}|_{\bar{S}} \tag{4.12}$$

$$p^{(2)}=-\rho\{g(\tilde{H}\bar{r}'+\tilde{R}\bar{u}_D)\cdot\nabla z' +(\bar{u}^{(1)}\cdot\nabla)(\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)\phi +\tfrac{1}{2}(\nabla\phi)^2+\tfrac{1}{2}[(\tilde{H}\bar{r}'+\tilde{R}\bar{u}_D)\cdot\nabla]W^2\}|_{\bar{S}} \tag{4.13}$$

Obviously the first and second terms at the right hand side of (4.13) are equivalent to

$$-\rho g(\widetilde{H}\bar{r}'+\widetilde{R}\bar{u}_D)\cdot\nabla z' = -\rho g\sum_{l=1}^{m}(\theta_4 v_l-\theta_5 u_l) -\rho g[\theta_4\theta_6 x'+\theta_5\theta_6 y'-\tfrac{1}{2}(\theta_4^2+\theta_5^2)z'] \tag{4.14}$$

For a body which is either slender, or thin, or flat, the corresponding first order and second order pressure components are respectively as follows.

$$p^{(1)} = -\rho\{(\tfrac{\partial}{\partial t}-U\tfrac{\partial}{\partial x})\phi+g(z'+w)\}|_{\bar{S}} \tag{4.15}$$

$$p^{(2)} = -\rho\{g(\widetilde{H}\bar{r}'\cdot\nabla)z'+g(\widetilde{R}\bar{u}_D\cdot\nabla)z' +(\bar{u}^{(1)}\cdot\nabla)(\tfrac{\partial}{\partial t}-U\tfrac{\partial}{\partial x})\phi+\tfrac{1}{2}(\nabla\phi)^2\}|_{\bar{S}} \tag{4.16}$$

4.3 Hydrodynamic forces

Substituting (3.8) and (4.11~13) into (3.15) yields

$$Z_r(t) = Z_r^{(0)} + Z_r^{(1)}(t) + Z_r^{(2)}(t) \tag{4.17}$$

where $Z_r^{(0)}$, $Z_r^{(1)}$ (t) and $Z_r^{(2)}$ (t) are respectively the constant, first order and second order forces. They are as follows.

$$Z_r^{(0)} = \rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}_r^o[gz'+\tfrac{1}{2}(W^2-U^2)]dS \tag{4.18}$$

which represents the generalized steady-state buoyancy forces and resistant forces. The generalized first order forces are

$$Z_r^{(1)}(t) = \Xi_r^{(1)} + H_r^{(1)} + R_r^{(1)} + \Delta R_r \tag{4.19}$$

where

$$\Xi_r^{(1)} = \rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}(\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)[\phi_o(t)+\phi_D(t)]dS, \tag{4.20}$$

$$H_r^{(1)} = \sum_{k=1}^{m}\rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}_r^o(\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)\phi_k(t)dS, \tag{4.21}$$

$$R_r^{(1)} = \rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}_r^o[gw+\tfrac{1}{2}(\bar{u}^{(1)}\cdot\nabla)W^2]dS, \tag{4.22}$$

$$\Delta R_r = \rho\iint_{\bar{S}}(\widetilde{R}\bar{n})\cdot\bar{u}_r^o[gz'+\tfrac{1}{2}(W^2-U^2)]dS. \tag{4.23}$$

The last term ΔR_r represents the influence of the rigid body rotations to the first order forces, while the other terms are the same as the linear theory[4,5]. The generalized second order forces may be expressed as

$$Z_r^{(2)}(t) = F_r^{(2)}(t)+E_r^{(2)}(t)+D_r^{(2)}(t)+S_r^{(2)}(t)+\Delta Z_r^{(2)}(t) \tag{4.24}$$

where $F_r^{(2)}$, $E_r^{(2)}$, $D_r^{(2)}$ and $S_r^{(2)}$ are the contributions of the first order fluid pressures due to the movement of the wetted surface. These are

$$F_r^{(2)}(t) = \rho\iint_{\bar{S}}[(\widetilde{R}\bar{n})\cdot\bar{u}_r^o+(\bar{n}\cdot\bar{u}_r^o)(\bar{u}^{(1)}\cdot\nabla)] (\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)[\phi_o+\phi_D]dS +\rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}_r^o\,\tfrac{1}{2}[\nabla\phi_o+\nabla\phi_D]^2dS, \tag{4.25}$$

$$E_r^{(2)}(t) = \sum_{k=1}^{m}\rho\iint_{\bar{S}}\bar{n}\cdot\bar{u}_r^o\nabla[\phi_o(t)+\phi_D(t)]\cdot\nabla\phi_k(t)dS, \tag{4.26}$$

$$D_r^{(2)}(t) = \sum_{k=1}^{m}\rho\iint_{\bar{S}}[(\widetilde{R}\bar{n})\cdot\bar{u}_r^o+(\bar{n}\cdot\bar{u}_r^o)(\bar{u}^{(1)}\cdot\nabla)] (\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)\phi_k(t)dS + \sum_{k=1}^{m}\sum_{l=1}^{m}\tfrac{1}{2}\rho\iint_{\bar{S}}(\bar{n}\cdot\bar{u}_r^o)\nabla\phi_k(t)\cdot\nabla\phi_l(t)dS, \tag{4.27}$$

$$S_r^{(2)}(t) = \rho\iint_{\bar{S}}(\widetilde{R}\bar{n})\cdot\bar{u}_r^o[gw+\tfrac{1}{2}(\bar{u}^{(1)}\cdot\nabla)W^2]dS +\rho\iint_{\bar{S}}(\widetilde{H}\bar{n})\cdot\bar{u}_r^o[gz'+\tfrac{1}{2}(W^2-U^2)]dS +\rho\iint_{\bar{S}}(\bar{n}\cdot\bar{u}_r^o)(\widetilde{H}\bar{r}'+\widetilde{R}\bar{u}_D)(gz'+\tfrac{1}{2}W^2)dS. \tag{4.28}$$

The term $\Delta Z_r^{(0)}$ denotes the forces induced by the instantaneous variation of the wetted surface area $\Delta S = S(t) - \bar{S}$. This is

$$\Delta Z_r^{(2)}(t) = \rho\iint_{\Delta S}\bar{n}\cdot\bar{u}_r^o\{[(\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)\phi+\tfrac{1}{2}(W^2-U^2) +\tfrac{1}{2}(\bar{u}^{(1)}\cdot\nabla)W^2]+g(z'+w)\}dS. \tag{4.29}$$

The integration over ΔS may also be represented by an integral along the water line C_W and a line integral in vertical direction on the hull. It is also known that along C_W on $\bar{S}$, to leading order $0(\varepsilon)$, the following expression exists

$$\varsigma^* = -\tfrac{1}{g}[(\tfrac{\partial}{\partial t}+\bar{W}\cdot\nabla)\phi+\tfrac{1}{2}(W^2-U^2)+\tfrac{1}{2}(\bar{u}^{(1)}\cdot\nabla)W^2] \tag{4.30}$$

where ς^* is the local wave elevation. Thus (4.29) may further be expressed as

$$\Delta Z_r^{(2)}(t) = -\rho g\int_{C_W}\hat{Z}^2(\bar{n}\cdot\bar{u}_r^o)\frac{dl}{\sqrt{1-n_3^2}} \tag{4.31}$$

where

$$\hat{Z} = \hat{Z}(x',y',z',t) = \varsigma^* - w \tag{4.32}$$

denotes the relative vertical displacement of the structure and the local water surface.

The concluding remarks and the references are shown in the second paper.

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