研　究　速　報

Application of Biological-Growth Strain Method on Optimal Shape Design of Mechanical Components

生体成長ひずみ法に基づく構造形状最適化への適用

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1. INTRODUCTION

By the notch stress theory developed by Baud[1] and Schack[2], etc., the shape optimization problem of minimizing stress concentration in the continuous structures (also termed as stress optimal design) can be focused on the adjacent area where stress concentration occurs in stead of dealing with the whole design structure. On the other hand, from the comparative viewpoint on the mechanism of minimizing stress concentration, it is found that the adaptation process of biological tissues by shape variation to their environment is very effective in reducing stress concentration in the biological structures. The high similarity between the biological adaptation process and the stress optimal design in the continuous structures motivated us to present one new optimal shape design method based called "biological-growth strain method (BGS)" by simulating biological adaptation process.

In the following, the characteristics of BGS will be given in section 2; then, the numerical verification of the proposed method and application on various fields of structural design are illustrated in section 3. Finally, the concluding remarks is included in section 4.

2. CHARACTERISTICS OF BIOLOGICAL-GROWTH STRAIN METHOD

(1) Similarity between stress optimal design and biological adaptation process

Through years of researches, "uniform stress distribution design (or equi-strength design)"—the hypothesis post-

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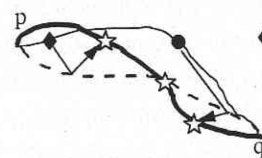


Fig. 1 Illustration for curvature variation on the design profile based Baud's postulation

demonstration[3]. Based on this validated Baud's postulated by Baud showed its practical validation by analytical demonstration[3]. Based on this validated Baud's postulation, the stress optimal design can be achieved by varying the outer-surface shape (i.e. curvature of the design points along the design profile) of the design structure. By iteratively repeating the above process, the outer-surface shape can be considered to be optimized once if the condition Eq. (1) is satisfied. The iterative optimization process can be illustrated by Fig. 1. On the other hand, by looking at the load carriers in animals or plants like bones or brunches, it is undoubtedly convinced that the load carriers develop their optimum by well adapting themselves to their environment at a certain loading conditions[4]. With the further inspect on those biological adaptation process, several interesting aspects can be summarized in the following points: (1) the state of constant stress can be found on the surface of those well-adapted biological structures and (2) the adaptation process of these biological tissues is carried out by growth or atrophy of their live tissues near the stress-concentrated surface. By comparing these observed facts on the biological adaptation process with stress optimal design process, it is not difficult to find out the high similarity between them on the optimality condition and the optimization process. Here, it is worth mentioning that the gradients for objective function and the constraint conditions is usually necessary for the conventional stress optimal design while the biological optimization process needs no gradient information. As a result, it was motivated to propose one gradientless stress optimal design method by simulating the biological adaptation process to update the design variables instead of using the gradient optimization techniques.

In fact, the derivation of BGS is similar to "initial stress method", i.e. the shape updating of the design structure is an iterative process which consists of two stages: (i) generation of fictitious strain by some built-in rules (e.g. nonlinear constitutive relationship or the biological growth rules) and (ii) production of fictitious external loads by the given fictitious strain. In stage (i), due to the definition of the fictious strain derived from the simulation on biological adaptation, it is termed as the biological-growth strain. In the followings, first of all, the concept and the definition of the biological-growth strain is given; then, the algorithm for the production of the fictious external loads termed as "biological-growth strain analysis" is derived.

(2) Brief Description on Biological-Growth Strain Method

In order to optimally design the shape of the structure, one optimal shape design method proposed by the authors[5] was utilized to design the structures. Compared to other conventional structural optimization technique, the feature of this proposed method was its ignorance on gradient calculation, which could be regarded as the dominant factor on increasing the computational cost of structural optimization. The design objective of this method is to minimize the stress concentration within the design structure, which can be expressed as

$$\mathrm{Min}\left\{ \sum_{(x_i,\, y_i),\ i=1..n} (\bar{\sigma}_i - \sigma_{ref})^2 \right\} \tag{1}$$

Here, the design variables are the coordinates (x_i, y_i) of the design points selected along the design profile Γ, where i = 1..n. The reference stress is $\bar{\sigma}_{ref}$ and the equivalent stress along Γ is $\bar{\sigma}_i$. The algorithm of this method was proposed by simulating the biological adaptation to their loading environment like trees or bones; i.e. they change their shapes by the growth or trophy of the living tissue near the high stressed area. Based on this concept, one parameter called biological growth strain was defined as follow

$$\{\varepsilon_k^{BP}\}_j = \begin{Bmatrix} \varepsilon_1^{BP} & 0 \\ 0 & \varepsilon_2^{BP} \end{Bmatrix}_j = \begin{cases} \text{for dectile material: } \dfrac{\sigma_j - \sigma_{mean}}{\sigma_{mean}} \nabla h, & \text{if element } j \in \Gamma^* \\ \text{for brittle material: } \dfrac{\sigma_j - \sigma_{mean}}{\sigma_{mean}} \left|\dfrac{\sigma_k}{f_s}\right| \nabla h, & \text{if element } j \in \Gamma^* \\ \{0\}, & \text{if element } j \notin \Gamma^* \end{cases} \tag{2}$$

Here j: jth design element within design domain Γ^*; σ_j: equivalent stress within jth design element; σ_{mean}: mean of equivalent stress of all the design elements; σ_k: kth principal stress, k=1..2 ; f_s: uniaxial compressive or tensile strength; ∇h: constant for search step. With the introduction of this parameter, the shape of the design structure can be changed by means of updating vectors of nodes coordinates (i.e. the shrinking or swelling of the design elements), which is formulated as

$$[K]\{u\} = \{\Delta g\} \tag{3}$$

Here [K]: global stiffness matrix and {u}: nodal coordinate updating vectors. The equivalent nodal force vector $\{\Delta g\}$ is

$$\{\Delta g\} = \int_{\Omega_e} [B]^T [D]\{\varepsilon^B\} \partial\, \Omega_e \tag{4}$$

That is, the nodal coordinate updating vectors {u} can be attained by solving the global governing equation superimposed by Eq. (3) without figuring out the gradient of the objective function or constraint conditions. Therefore, the

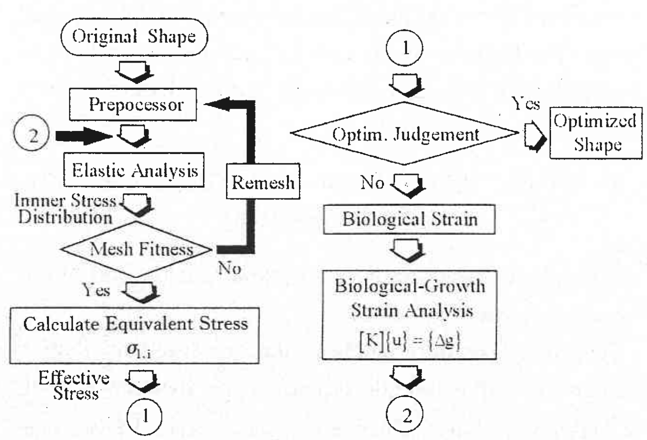


Fig. 2 Computational flow of BGS

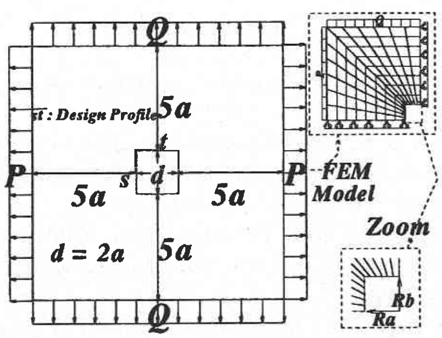
new shape of the structure is obtained by adding the nodal coordinate updating vectors to the old coordinates of nodes. For brevity, the computational flow of BGS is illustrated in Fig. 2. The termination of this iterative computation can be recognized when Eq. (1) is satisfied.

3. NUMERICAL VERIFICATION ON CORRECTNESS OF BGS

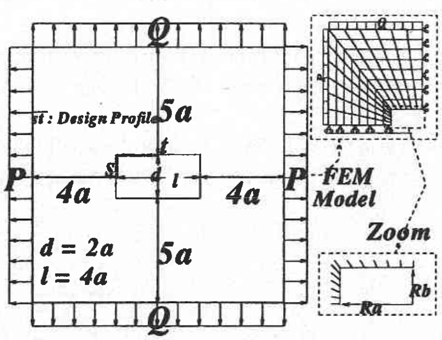
After explaining characteristics of BGS, the availability of this method is necessary to be verified before applying this method to general structures. To verify the availability of this method, the design problem to find out the optimal shape of the hole in a plate under biaxial stress was considered. According to the elasticity theory[6], the tangential stress for the elliptical hole in an infinite plate under biaxial stress is given as

$$\sigma_\theta = \sigma_1 \frac{(1+k)^2 \sin^2\theta - k^2}{\sin^2\theta + k^2\cos^2\theta} + \sigma_2 \frac{(1+k)^2\cos^2\theta - 1}{\sin^2\theta + k^2\cos^2\theta} \quad (5)$$

Here, k: axis ratio (=b/a; b: vertical axis and a: transversal axis) and θ: directional angle. With the introduction of the assumption $\sigma_2/\sigma_1 = k$, Eq. (5) can be rewritten as follow

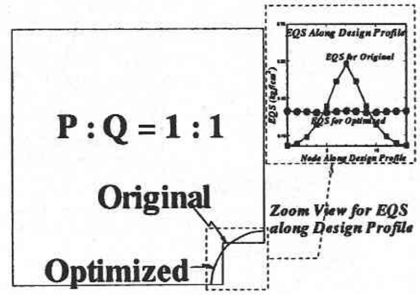


(a) square hole

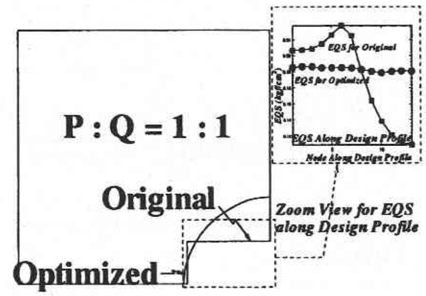


(b) rectangle hole

Fig. 3 Sketch of FEM model for the hole in an infinite plate under biaxial stress with different initial shapes



(a) case A1



(b) case B1

Fig. 4 Optimal shapes and EQS designed by BGS (P/Q = 1)

Table 1 Material property

Fc*	Ft*	E*	ν
1000	1000	7.00E+04	0.3

* : unit (kgf/cm²)

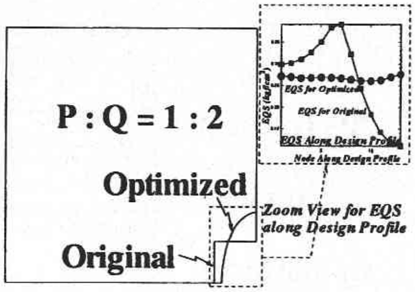
Table 2 Loading condition and case label

P : Q	A	B
1 : 1	A1	B1
1 : 2	A2	B2

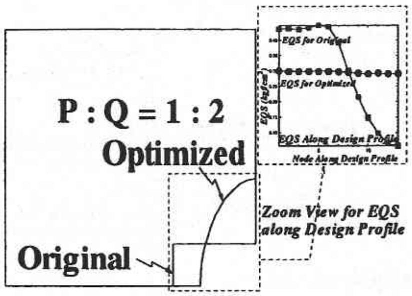
$$\sigma_\theta = \sigma_1 \frac{(1+k)^2\sin^2\theta + k(1+k)^2\cos^2\theta - k^2 - k}{\sin^2\theta + k^2\cos^2\theta} \quad (6)$$

$$= (1+k)\sigma_1$$

Thus, provided $\sigma_2/\sigma_1 = b/a = k$, the tangential stress will be constant along the edge. According to Baud's postulation, the shape with uniform strength distribution is considered as optimum. Furthermore, to verify the availability of BGS , different initial shapes of holes including (a) square and (b) rectangle were used as numerical examples as shown in Fig.3. Due to symmetry, only one quarter of these structural models were analyzed. The material property and loading condition with case label were given in Table.1 and 2 respectively. Due to ductile material used in these cases, von Mises stress was selected as the equivalent stress(abbr. as EQS). By applying BGS, the optimized shape for different initial shapes under the equal



a) case A2



(b) case B2

Fig. 5 Optimal shapes and EQS designed by BGS (P/Q = 0.5)

Table 3 Comparison on Ra and Rb (P/Q = 1)

P : Q = 1 : 1				
		Ra	Rb	Rb/Ra
A	Ori	5.00	5.00	1.00
	Opm	6.38	6.38	1.00
B	Ori	10.00	5.00	0.50
	Opm	10.40	10.29	0.99
Analytical				1.00

Table 4 Comparison on Ra and Rb (P/Q = 0.5)

P : Q = 1 : 2				
		Ra	Rb	Rb/Ra
A	Ori	5.00	5.00	1.00
	Opm	4.17	8.54	2.05
B	Ori	10.00	5.00	0.50
	Opm	6.66	12.67	1.90
Analytical				2.00

biaxial stress ratio (i.e. P/Q = 1) and EQS along the design profile before and after optimization was given in Fig.4. In addition, the axis ratio of the design profile for the results of BGS and analytical solution was shown in Table 3. For the unequal biaxial stress ratio (i.e. P/Q = 0.5), the optimized shapes and EQS distribution was shown in Fig.5 while Table 4 offered the comparison between the results of BGS and the analytical solution.

By reviewing the definition given in Eq. (1), the reason causing the variation of the optimized shape for different initial shape can be understood because the reference stress is defined as the mean stress along the design profile rather than a fixed value. However, the axis ratio in Table 3 and 4 for different initial shape under P/Q = 1 and 0.5 showed the good agreement between analytical result and the numerical result given by BGS. In addition, the equivalent stress along the design profile could be quite uniform for all the cases shown in Fig.4 and 5. Thus, the correctness of BGS could be verified.

4. CONCLUDING REMARKS

The following conclusions can be extracted from this study:

(1) By observing the algorithm of BGS, the gradientless optimization technique by simulating the biological adaptation made this method very simple without special coding procedure, unlike the conventional approach for the need to code their special algorithm.

(2) The availability of BGS was proved from the viewpoint of correctness and applicability by finding the conformity between the numerical result obtained by BGS and the analytical result.

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