研 究 速 報

Computational Efficiency on Optimal Shape Design Methods
—Sensitivity Analysis and Simulated Biological Growth Method—
最適形状設計技法に対する計算効率性比較-感度解析及び生体成長に基づく構造形状最適化の手法

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1. INTRODUCTION

By means of simulating the adaptation mechanism of natural tissues[1], one newly proposed optimal shape method was highly noticed, called "simulated biological growth method". This new method is impressive for its simplicity and gradientless process. Meanwhile, for very similar optimization problems, some other researchers could also find out the optimal solutions by their proposed methods. By observing the results acquired by these different methods, it seems the results solved by the newly proposed method show better efficiency but no direct comparison verified this observation. Therefore, in this paper, the authors would investigate the efficiency of this simulated biological growth method by comparing with an sensitivity-analysis optimization method also developed by the authors, both of which are based on the same optimality criterion for reducing the stress concentration.

In the following context, section 2 and 3 are given for the brief description of simulated biological growth method and the features of the sensitivity-analysis optimization method respectively. Section 4 offers the numerical examples as comparison. Through the comparisons, the efficiency of simulated biological growth method can be confirmed due to its excellent performance on saving computational cost, as concluded in section 5.

2. PROCEDURE OF SIMULATED BIOLOGICAL GROWTH METHOD

The biological structures[3] self-optimize their shapes by growth with respect to the natural load applied. In this case, the word "optimum" means that a state of constant stress at the surface of the biological components should be observed for all the natural loading case applied. As a result, the optimality criterion considered here is to change the shape of the design profiles to achieve the state of constant stress distribution. This objective can be alternatively defined as minimizing ratio of maximum stress to the object stress along the design profile, as shown in eq. 1. (where Γ_K : the design profile, s : the design point along the design profile, $\sigma_{eq}(s)$: equivalent stress of each design point and σ_{obj} : object stress. By the same definition of objective function for different optimal methods, the effect of optimization for each method can be clearly clarified.

$$\text{Min } \Omega = \left\{ I \,\middle|\, I = \frac{\text{Max}(\sigma_{eq}(s))}{\sigma_{obj}},\ s \subset \Gamma_K \right\} \tag{1}$$

As mentioned, the simulated biological growth method was originated from the simulation on biological structures which can adapt themselves to external loads for reducing stress peak with growth or atrophy. The basic procedure for a simulated biological growth method consists of two stages : the first one is FEM static analysis for obtaining the stress distribution over the design domain, which can be one part or the whole domain of the design structure according to the need of the designer. Then by introducing the growth law using fictitious strain which was proposed quite divergently by different research groups[2),3)], an incremental growth analysis based on this growth law can be carried out to generate the incremental displacements for updating the design structure. After the design structure is updated, the aforementioned process is repeated again until the convergence of eq.1 can be recognized.

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3. OPTIMALITY CRITERION APPROACH-SENSITIVITY-ANALYSIS METHOD

In order to evaluate the simulated biological growth method, one optimality-criterion approach using sensitivity analysis was also proposed by the authors. In the following, the features of this method are explained. To reduce the stress concentration of the design structure, the nodes on the design profile are selected as the evaluation points used for the calculation on the objective function as shown in eq.1. The stresses for each evaluation point can be represented by the equivalent stress (here, the von Mises stress is used); that is, the equivalent stress is the functional of the stress tensor and the stress tensor is the function of nodal coordinates, as expressed in eq.2.

$$\sigma_{eq} = \frac{\sqrt{2}}{2}(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\sigma_{xy}^2)^{1/2}$$
$$= f(\sigma_x, \sigma_y, \sigma_{xy}) = f(\sigma_x(x, y), \sigma_y(x, y), \sigma_{xy}(x, y)) \quad (2)$$

By averaging the object stress on each evaluation point, average stress can be obtained. Then, by Taylor's first-order expansion, the average stress can be approximated as follows:

$$\sigma^* = f(\sigma_{ij}) + \frac{\partial f}{\partial \sigma_{ij}} \delta \sigma_{ij} + \exists(\varepsilon) \quad (3)$$

where $\exists(\varepsilon)$: the error functional

As it is shown, the sensitivity coefficients of stresses with respect to the design variable is essential. The sensitivity analysis in this paper adopted the semi-analytical method by a finite difference method[4). and we differentiated the components of stress tensor with respect to the design variables (i.e. x and y), as follows:

$$\delta\sigma_{ij} = \frac{\partial \sigma_{ij}}{\partial X_k} \delta_k \quad (4)$$

where σ_{ij}: component of stress tensor;
X_k: coordinate of design points
δ_k: perturebation of coordinates

by substituting eq. 4 into eq. 3, eq. 3 can be rearranged as

$$\sigma^* = \sigma(\sigma_x, \sigma_y, \sigma_{xy}) +$$

$$\frac{\left(\frac{\partial f}{\partial \sigma_x}\frac{\partial \sigma_x}{\partial x} + \frac{\partial f}{\partial \sigma_y}\frac{\partial \sigma_y}{\partial x} + \frac{\partial f}{\partial \sigma_{xy}}\frac{\partial \sigma_{xy}}{\partial x}\right)}{\alpha_{ij}} \delta x +$$

$$\delta x + \frac{\left(\frac{\partial f}{\partial \sigma_x}\frac{\partial \sigma_x}{\partial y} + \frac{\partial f}{\partial \sigma_y}\frac{\partial \sigma_y}{\partial y} + \frac{\partial f}{\partial \sigma_{xy}}\frac{\partial \sigma_{xy}}{\partial y}\right)}{\beta_{ij}} \delta y + \exists(\varepsilon) \quad (5)$$

where i, j: the node number of design point, i, j=1..N

Or it can be expressed in the form of matrix as in eq. 6 by rearranging eq. 5. As you can observe in eq. 6, the right side of equation can be approximated by multiplying one constant (ϕ) for diminishing the effect of error functional.

$$\alpha_{ij} S_{xj} + \beta_{ij} \delta_{yj} = \sigma^* - \sigma i - \exists(\varepsilon)$$
$$= \phi[\sigma^* - \sigma i] \quad (6)$$

where i, j : dummy index; $i, j = 1 \ldots N$

As shown in eq. 6, the order of coefficient matrix $[\alpha_{ij}, \beta_{ij}]$ is N * 2N. As a result, the solution of (dx, dy) cannot be solved by only N equations. Hence, some other information related to (dx, dy) needs to be introduced into eq. 6. For solving that, the updating vector for each design point is assumed to be in the direction of the bisector of the angle defined by the idea presented in Fig. 1[1). Then, the relationship for (dx, dy) can be established as follow:

$$\delta Y / \delta X = \frac{x_{i+1} - x_i^*}{y_i^* - y_{i+1}} \quad (7)$$

By substituting eq. 7 into eq. 6 and rearranging eq. 6, the order of coefficient matrix $[\alpha_{ij}, \beta_{ij}]$ becomes N * N; that is, after obtaining the derivatives of stress with respect to design variables and using the relationship given in eq. 7, the solution for the updating vector can be obtained. With these updating vectors, the shape of the design structure will be changed by adding these updating vectors to the coordinates of design points. This process will be repeated until the convergence of eq.1 can be verified. However, due to the possible existence of unsmoothed design profile after updation, some spline function will be used to smooth the design profile at each iterative step.

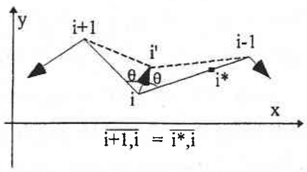

Fig. 1 Local Shifting of Node i to i′

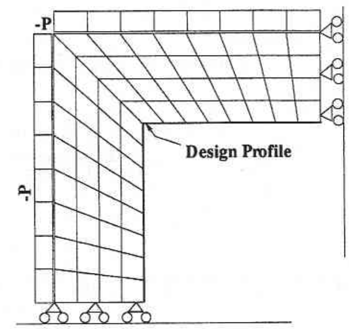

Fig. 2 FEM Model of Design Structure

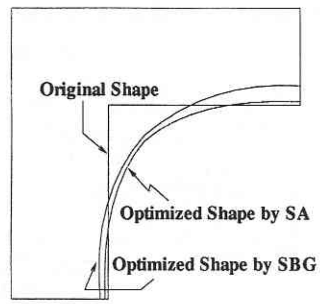

Fig. 3 Comparison on Shape of Structure

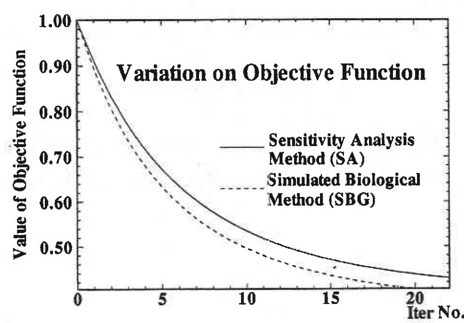

Fig. 4 Comparison on Objective Function

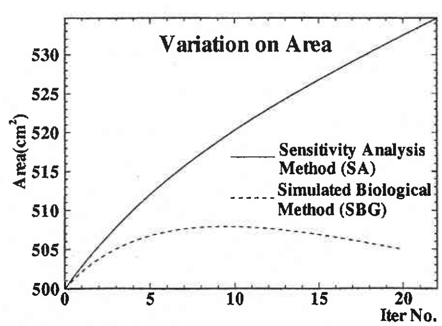

Fig. 5 Comparison on Area

4. ILLUSTRATION OF THE COMPARISON ON THE TWO APPROACHES

As the purpose of research mentioned previously, one numerical example for comparison is illustrated here. The example is a square plate under biaxial equi-tension with central diamond-shape notch, the FEM model of which is simplified in Fig. 2 due to its symmetry. The points on the notch (i.e. design profile) was chosen as design points. Fig. 3 indicates the optimized shapes obtained from the two methods with few difference. The convergence of objective function is shown in Fig. 4 with good acceptance for both methods. Fig. 5 gives the variation of structure area , in which the structure optimized by SA has similar area to the one optimized by SBG for the deviation of area is only 0.536%. And the strain energy within the design structure can be effectively reduced as shown in Fig. 6, which indicates the optimized structure can undergo higher external loading. And as emphasized in section 1, the effect of reducing stress concentration by the two methods can be clearly observed in Fig. 7. The stresses along the design profile after optimization can effectively tend to be uniform, which meets the requirement of the design problem. In fact, from Fig. 3 through 7, there really exists some variation between the two methods but the deviation can be regarded acceptable.

Indeed, through the above figures, the agreement of the optimized results by the two methods can be clearly observed. However, on the other hand, the computational cost spent by the two methods was quite different. Say, for the FEM model of the numerical case, in which there are 153 nodes and 128 elements, the programs were executed on SUN Sparc station 10. For SBG, one iterative step only took 1.2 sec while about 198 sec spent by SA; that is, only 98.4 sec was needed for SBG to find out the optimum (82 iterative loops) while about 17820 sec (about 4.95 hr) needed by SA. As a result, though the similar optimal solutions could be achieved by the two methods, the high

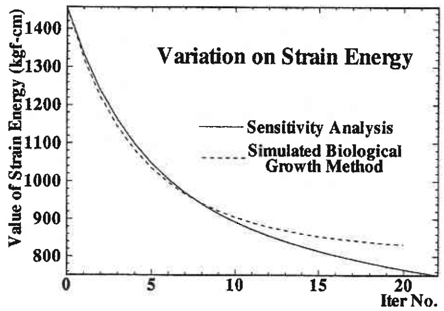

Fig. 6 Comparison on Strain Energy

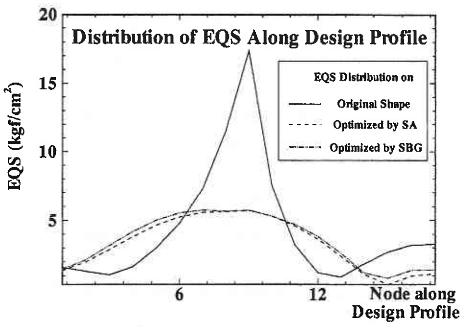

Fig. 7 Comparison on EQS

efficiency of the simulated biological growth method for saving a lot of computational cost was obviously confirmed.

5. CONCLUSIONS

By the observation on the two methods discussed in this research, the issues clarified in this study can be concluded as follows : (1) Due to the excellent saving on computational cost, the high efficiency of simulated biological growth method could be confirmed with the comparison on another optimality-criterion method. (2) With the high similarity of the optimal solution to the result by the simulated biological growth method, the availability of the method using sensitivity analysis suggested in the research was also verified. Nevertheless, the efficiency of this method was seriously deteriorated by the computation of sensitivity coefficients. And such disadvantage makes this method quite unattractive when compared with the simulated biological growth method.

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