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Theoretical Analysis of Erbium-Doped Fiber Laser with a Ring Cavity

リング共振器を用いたエルビウムドープ・ファイバレーザの理論解析

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The operation of erbium-doped fiber lasers with ring cavities is discussed by solving the rate equations and propagation equations. A small signal model is proposed for analyzing the lasing threshold. There exists an analytic solution in the case of small signal. It is shown that the threshold input pump power is relational to the cavity length and coupler efficiency. The optimum cavity length and coupling efficiency exist for the minimum threshold. The case of big signal is analyzed as well by solving the equations numerically. The output spectrum is given, which shows that the spectral width of the output is approximately 2 nm. The effects of the cavity length, coupler efficiency, and input pump power on the output characteristics are discussed.

1. INTRODUCTION

The idea of using rare-earth ion doped glasses in fiber form as lasing media is attracting more and more attention since the recent work of Poole *et al.* [1),2)], where single mode fused silica fibers were doped with a variety of rare-earth ions, and laser action was shown to occur on several different transitions. One of the great advantages of a longitudinally pumped fiber laser over its bulk counterpart is the possibility of producing an extremely small spot size (and hence a high pump intensity) and yet maintaining this small pump size over a distance long compared with its Rayleigh length.

This permits very low threshold operation of a four-level laser oscillator, although as was pointed by Digonnet and Gaeta[3)], the slop efficiency of such a laser is not significantly different from that of the same laser in bulk form. If sufficient pump power is available to pump bulk oscillator to well above threshold, there would seem to be no reason, from the point of view of efficiency, for constructing such a laser in a fiber form. There is however a considerable advantage to be gained in operating a three-level laser in fiber form, since the pump thresholds for such systems are generally significantly higher than comparable four-level lasers.

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Of the rare-earth ion doped fiber lasers that have been demonstrated to date, it is the transition in Er^{3+} at 1.5 μm that is of particular interest in telecommunication, as this wavelength coincides with the low-loss window of fused silica fiber. Furthermore, since an Er^{3+} doped fiber has an absorption band at 800 nm, the possibility would seem to exist for a variety of active fiber devices pumped by compact, efficient GaAlAs laser diodes.

In this paper, a theoretical analysis on the operation of a ring fiber laser under various cavity conditions in the forward propagation direction is presented. This paper is organized as follows: first the formalization of the fundamental model is described; in order to analyze the threshold of fiber laser, a small-signal approximation is used; further, a self-consistent method is established for analyzing the output characteristics of the laser; finally, the conclusions are summarized.

2. FORMALIZATION OF THE MODEL

It is well known that the characteristics of erbium-doped fiber can be described by using propagation equations and rate equations. Here we use the form presented by E. Desurvire *et al.*[4)].

The propagation equation governing the evolution with longitudinal fiber coordinate z of the pump power $P_p(z)$ and the signal power $P_s(z, \nu_i)$ (corresponding the ASE power propagating in the direction as the pump power) write:

$$\frac{dP_p(Z,)}{dZ} = -\gamma_p(Z)P_p(Z) \quad (1)$$

$$\frac{dP_s(Z,\nu_i)}{dZ} = G_z(Z,\nu_i)[P_s(Z,\nu_i)+P_0] - G_a(Z,\nu_i)P_s(Z,\nu_i) \quad (2)$$

Where P_0 represents an equivalent input noise power, which is given by $P_0 = h\nu_i\Delta\nu$. P_s represents the ASE power contained in a frequency slot of arbitrary width $\Delta\nu$.

Due to the complex expressions of the absorption and emission coefficient γ_p, Ga, and Ge involved in Eqs. (1) and (2), numerical solving of the above system is a lengthy task in the most general case. However, it is possible to consider the case of a uniform pump distribution across the fiber core, corresponding to a highly multimode pump regime, as well as the case of a step-like Er^{3+} concentration distribution, for which the coefficients can be expressed in a closed form. Here a uniform pump distribution with cylindrical symmetry is considered:

$$\psi_p(x,y) \equiv \psi_p(r) = \begin{cases} S = 1/\pi a^2, & for\ r \leq a \\ 0, & for\ r \geq a, \end{cases} \quad (3)$$

where a is the fiber core radius. Like wise, a step-like Er^{3+} concentration profile is assumed, in which the fiber core of radius a is uniformly doped:

$$\rho(x,y) \equiv \rho(r) = \begin{cases} \rho_0 & for\ r \leq a \\ 0, & for\ r \geq a. \end{cases} \quad (4)$$

In addition, it is assumed that the signal mode profile dependence is only radial, which is the case of the actual TEM_{00} mode fiber. For simplicity, a Gaussian approximation is selected for the TEM_{00} mode, i.e.:

$$\psi_s(r) = \frac{\exp(-r^2/w_f^2)}{\pi w_s^2}, \quad (5)$$

where ω_s is the power spot size of the signal mode.

With such assumptions one can yield the following equations for calculating the parameters in Eqs. (1) and (2):

$$G_{ae}(z,\nu_i) = \frac{\rho_0}{2}\sigma_{a,e}(\nu_i)\left\{\pm\frac{P_p(z)/P_p^{th}-1}{P_p(z)/P^{th}+1\ r(z)}\ln\frac{1+r(z)}{1+\eta r(z)}+1-\eta\right\}, \quad (6)$$

$$\gamma_p(z) = \frac{\rho_0\sigma_p}{1+P_p(z)/P_p^{th}}$$

$$\left\{1+\frac{1}{2}\left(\frac{w_s}{a}\right)^2\left(P_p(z)/P_p^{th}-1\right)ln\left[\frac{1+r(z)}{1+\eta}r(z)\right]\right\}, \quad (7)$$

with

$$r(z) = \frac{1}{1+P_p(z)/P_p^{th}}\frac{P_p^*}{P_{sat}}, \quad (8)$$

$$P_p^{th} = \pi a^2\frac{h\nu_p}{\sigma_p\tau}, \quad (9)$$

$$P_{sat} = \pi 2\pi^2 n_i^2 h\nu_s\Delta\nu_s/\lambda_s^2, \quad (10)$$

and $\eta = \exp(-a^2/w_s^2)$. The factor P_s^* in Eq. (8) corresponds the ASE power falling in the homogeneous linewidth centered at $\nu_i = \nu_s$. The factor P_{th} corresponds to the pump power threshold for Er^{3+} population inversion[5] and P_{sat} to a homogeneous gain saturation power[6].

3. SMALL SIGNAL MODEL

The Equations (1) and 82) cannot be solved analytically, in general. However for the special case of small signal, analytical solutions can be obtained. Considering the small signal regime, i.e., $P_s^* \ll P_{sat}$, or $r(z) \ll 1$, it can be found that the coefficients reduce to following equations by expending the logarithm functions in Eqs. (6) and (7):

$$\gamma_p(z) = \frac{\rho_0\sigma}{1+P_p(z)/P_p^{th}}, \quad (11)$$

$$G_e(z,\nu_i) = \rho_0\sigma_e(\nu_i)(1-\eta)\frac{P_p(z)/P_p^{th}}{P_p(z)/P_p^{th}+}, \quad (12)$$

$$G_a(z,\nu_i) = \rho_0\,\sigma_a(\nu_i)\,(1-\eta/(P_p(z)/P_p^{th}+1)). \quad (13)$$

The absorption coefficient γ_p found in Eq. (11) corresponds to that of a pump light propagating through the absorbed regime. Eqs. (12) and (13) show that the ASE gain and the absorption coefficients are proportional to the signal active core overlap factor $(1-\eta)$, which is an important parameter for the gain optimization.

With such assumptions, Eq. (1) has its solution about $P_p(z)$ as following:

$$P_p(z) + P_p^{th}\ln\frac{P_p(z)}{P_p(0)} = P_p(0) - \rho_0\sigma_p P_p^{th} z, \quad (14)$$

where $P_p(0)$ is the pump power at the start of the cavity. At the position near the start, i.e., $z \to 0$ or $P_p(z) \to P_p(0)$, the solution becomes to the linear form as

$$P_p(z) = P_p(0) - \rho_0\sigma_p P_p^{th} z. \quad \left(P_p^{th} \ll P_p(0)\right) \quad (15)$$

In the other hand, at the position near the end of the cavity, i.e., $P_p(z) \to 0$ the solution becomes to an exponential function of z:

$$P_p(z) = P_p(0) \exp\left(\frac{P_p(0) - \rho_0 \sigma_p P_p^{th} z}{P_p^{th}}\right) \tag{16}$$

Furthermore, Eq. (2) can be written as

$$\frac{dP_s(z, v_i)}{dZ} = G(Z, v_i)P_s(Z, v_i) - G_e(Z, v)P_0 , \tag{17}$$

where G represents the net stimulated gain coefficient, writing $G = G_e - G_a$. The gain coefficient G is independent on the signal P_s, hence Eq. (17) has a solution in integral form as

$$P_s(z, v_i) = \exp\left(\int_0^z G(z, v_i)dz\right) \times\left\{\int_0^z G_0G_e(z,v_i)\exp\left(-\int_0^z G(z,v_i)dz\right) dz + P_s(0,v_i)\right\} . \tag{18}$$

Considering the steady state, one has

$$P_s(l) \times r_s = P_s(0) , \tag{19}$$

signal power at the position $z = 0$ is obtained as the following equation:

$$P_s(0, v_i) = \frac{\int_0^l P_0G_e(z, v_i)\exp\left(-\int_0^z G(z, v_i)dz\right) dz}{\frac{1}{r_s} - \exp\left(\int_0^l G(z, v_i)dz\right)} \exp\left(\int_0^l G(z, v_i)dz\right) . \tag{20}$$

The output signal power spectrum is proportional to $P_s(0, v_i)$, and given by the next formula:

$$P_{out}(0, v_i) = \frac{1 - r}{r} P_s(0, v_i). \tag{21}$$

The total power is the sum for each i.

In this analysis, the notations are following to Desurvire and Simpson's paper 4), as follows: v i is the i-the frequency slot, v s is the center frequency slot of the spectrum, nf is the photon numbers of the laser mode and τ is the decaying constant of the laser resonator.

4. LASING THRESHOLD

The threshold of a erbium-doped fiber is generally defined as the power to pump the fiber to a population-inversed state, while in the case of a ring fiber laser, one must take the distribution of the pump power along the fiber cavity into account. This may make the population at the start of the cavity inversed while that at the end of the cavity not inversed. So the lasing threshold is defined here as the launched pump power which make the cavity gain equal to cavity loss (involved the coupling loss at the end of the cavity).

The threshold of *erbium-doped fiber* $P_p{}^{th}$ is calculated by Eq. (9).

The threshold lasing condition can be obtained from Eq. (20) letting the denominator equal zero:

$$\ln\left(\frac{l}{r_s}\right) = \int_0^l G(z, v_i)\, dz \tag{22}$$

For simplicity, it is assumed that $\sigma_a(v) = \sigma_e(v)$. Then the net stimulated gain coefficient $G = G_e - G_a$ can be written as

$$G(z, v_i) = \rho_0\sigma_e(v_i)\,(1 - \eta)\,\frac{P_p(z) / P_p^{th} - 1}{P_p(z) / P_p^{th} + 1} . \tag{23}$$

The gain coefficient dependency on the term of $(P_p(z) / P_p^{th} - 1)$ shows that a positive net gain G can exist as long as the population inversion is maintained along the fiber.

Then from Eqs. (1) (11) (23) one can derive

$$\int_0^l G(z, v_i)\, dz = \frac{\sigma_e}{\sigma_p}(1 - \eta)\left[2 \times \frac{P_p(0) - P_p(l)}{P_p^{th}} - \rho_0\sigma_p l\right]. \tag{24}$$

Lasing threshold is defined in the usual experimental way, as occurring when the previously broad-band spectrum develops one or more modes whose power increases rapidly with pump power, and whose spectral width is very much narrower than the broad-band emission spectrum. Since the lasing mode is likely to be at the center of the emission line where the gain is largest, it is clear that only the mode (i.e., frequency) at which the ASE gets the most efficient amplification should be taken into account.

From Eqs. (14) and (24), the lasing threshold condition can be derived:

$$P_p(0) - P_p(l) = \frac{1}{2} P_p^{th} \left(\rho_0\sigma_p l - \frac{\sigma_p}{\sigma_e}\,\frac{\ln\ (r_s)}{1\ -\eta}\right). \tag{25}$$

This equation gives the threshold lasing condition (threshold absorbed pump power) which depends on the fiber length, dopant concentration, pumping absorption cross section, fiber transverse profile and the coupling efficiency of the coupler.

The characteristics of the fiber coupler gives (see Fig. 1)

$$P_{pin} \times (1 - r_p) + P_p(l) \times r_p = P_p(0) \tag{26}$$

where r_p represents the coupling efficiency from cavity end to cavity start. Hence one can calculate the launched pump power corresponding to the lasing threshold condition. In the following discussion, this power P_{th} is defined as the threshold pump power of the ring fiber laser. P_{th} further can be written as following according to Eqs. (14), (24) and (26)

$$P_{th} = \frac{P_p^{th}\, y}{1 - r_p} \times \frac{1 - r_p e^x}{1 - e^x}. \tag{27}$$

where

$$x = \frac{1}{2}\left[\frac{\sigma_p \ln(\frac{1}{r_s})}{\sigma_e(1-\eta)} - \rho_0 \sigma_p l\right], \tag{28}$$

and

$$y = \frac{1}{2}\left[\frac{\sigma_p \ln(\frac{1}{r_s})}{\sigma_e(1-\eta)} - \rho_0 \sigma_p l\right], \tag{29}$$

If the cavity length l is significantly long, the term of e^x in Eq. (27) is negligible, hence the threshold launched power increases linearly as the cavity length growths. One can decrease the threshold power by selecting lower-dopant, and smaller-core size fiber, or using a higher coupling efficient coupler. But when the cavity length is short, the term of e^x in Eq. (27) cannot be neglected. Thus a higher threshold occurs in this regime.

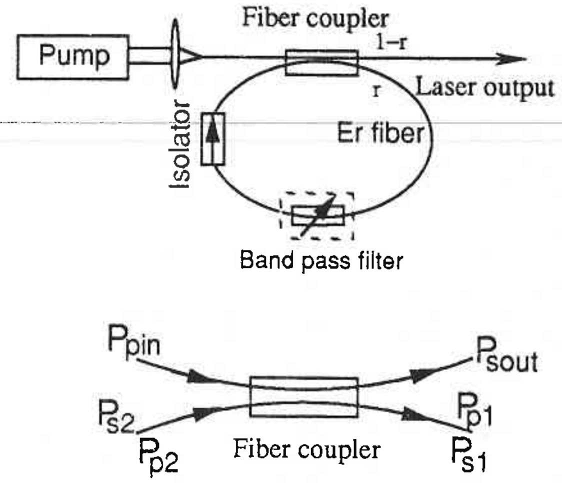
Fig. 2 plots the laser threshold power as a function of fiber length. The parameter used in the calculation is shown in Table 1. The general trend is for the threshold to increase with fiber length. This increase is required in order that the fiber may be bleached through. The launched power required to reach threshold rises again at short lengths since in this region the fiber only absorbs a small fraction of the launched power.

If the fiber length is too short, the laser will not operate since the absorbed pump power can not satisfy the threshold condition shown in Eq. (27), no matter what a high power is launched into the fiber. The shortest fiber length available to give a normal operating "theoretically" may be obtained by letting the dominator of Eq. (25) equal to zero, i.e., $e^x = 1$ or $x = 0$. Thus one can get the lasing-available condition as

$$l > \frac{\ln(\frac{1}{r_s})}{\rho_0 \sigma_e (1-\eta)}. \tag{30}$$

The word "theoretically" is used because the threshold near the shortest fiber length is too high hence the laser cannot operate actually, even the lasing-available condition is satisfied.

An optimum length of cavity fiber which is defined as the fiber length getting the lowest lasing threshold can determined by letting



$$P_{pin} \times (1 - r_p) + P_{p2} \times r_p = P_{p1}$$
$$P_{s2} \times r_s = P_{s1}$$
$$P_{sout} = P_{s2} \times (1 - r_s)$$

Fig. 1 Configuration of Er-doped ring fiber laser using a fiber coupler.

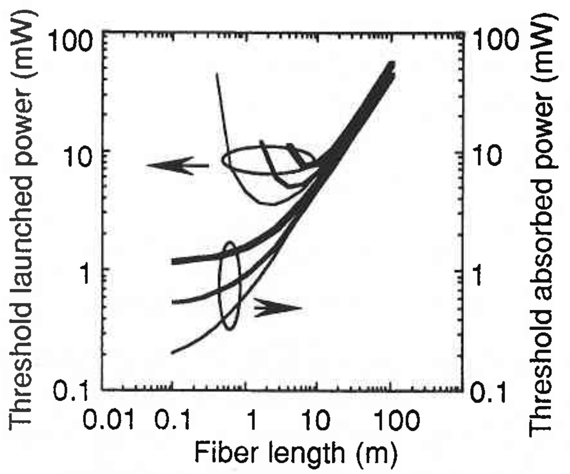


Fig. 2 Lasing threshold as a function of fiber length with varies coupling coefficients.
The coupling coefficients used in the calculation are 0.2, 0.5 and 0.8, respectively.

Table Parameters used in the analysis.

Item	Symbol	Value
Lasing wavelength	λ_s	1.53 μm
Wavelength of pump	λ_p	1.48 μm
Metastable-level lifetime	τ	1.0×10^{-2} s
Core radius of fiber	a	6 μm
Er^{3+} concentration in core	N_t	2.2×10^{24} /m^3
Input pump power	P_p	10 ~ 100 mW
Absorption cross section of pump	σ_p	2.4×10^{-25} /m^2
Coupling coefficient of pump	r_p	0.2
Coupling coefficient of signal	r_s	0.2 ~ 0.95
Cavity lengths	L	0 ~ 100 m

$$\frac{\partial P_{th}}{\partial l}=0\,. \qquad (31)$$

Fig. 3 shows the shortest lengths available to be used as the laser cavity and the optimum lengths for the lowest threshold with varies coupling efficiency of signal. It is clear that for a higher coupling (reflective) efficient, a shorter cavity length is necessary to get the minimum value of threshold pump power.

Also the optimum reflective coefficient can be obtained, for a given cavity length, by letting

$$\frac{\partial P_{th}}{\partial r_s}0\,. \qquad (32)$$

The threshold is optimized at this reflection.

5. ANALYSIS OF LARGE SIGNAL

Because of the seed term in Eq. (2), the stationary value for the gain is actually slightly smaller than 1 at the lasing frequency. The small difference for having a complete compensation of the cavity losses is provided by the spontaneous emission.

When the pump power is high above the threshold the signal field in the cavity becomes very high in comparison with the homogeneous gain saturation power P_{sat}. In such cases, the small signal analysis will not be suitable for understanding the laser action, although it is appropriate for analyzing the threshold condition of the fiber lasers. Here a self-consistent method is developed to analyze the case of large signal.

5.1 Self-Consistent Method

To analyze the case of large signal, Eqs. (1), (2), (6), and (7) are used. A two-dimensional mesh along fiber length and spectral axis is used in the numerical analysis.

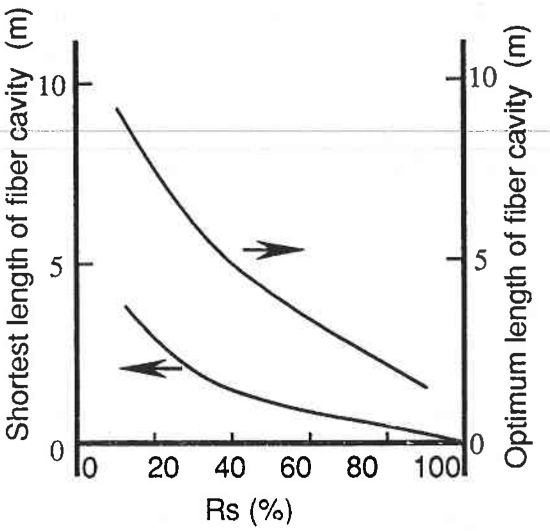


Fig. 3 The shortest lengths available to be used as the laser cavity and the optimum lengths for the lowest threshold with varies coupling efficiency of signal.

At first the initial value of the pump power and signal power at the start of the fiber are given as the product of the launched pump power and the coupling efficiency and spontaneous emission respectively. Then the gain efficient of the signal and the loss coefficient of the pump can be calculated from Eqs. (6) and (7). By using these parameters, the pump and signal power at the next step can be calculated from Eqs. (1) and (2). Thus the pump power and the signal power along the fiber can be obtained step by step.

If the signal power at the start of the cavity is not equal to the product of the signal power at the end of the cavity and the coupler efficiency, the calculation will repeat and the results from the former circulation is used as the initial value of the next circulation. The distribution of the pump power and signal power along the cavity can be obtained when the circulation get to the steady state.

5.2 Self-Consistent Method

By using the self-consistent method described in the previous section, the numerical solution of the case of large signal can be obtained. The parameters used in the following analysis are shown in Table 1. The fluorescence and absorption spectra $\sigma_{e,a}(\lambda)$ are shown in Fig. 5.

Fig. 5 shows the distribution of pump power and signal power along the fiber cavity. The launched pump power and the coupling efficiency used here are 100 mW and 0.5 respectively. While propagating along the cavity, the pump

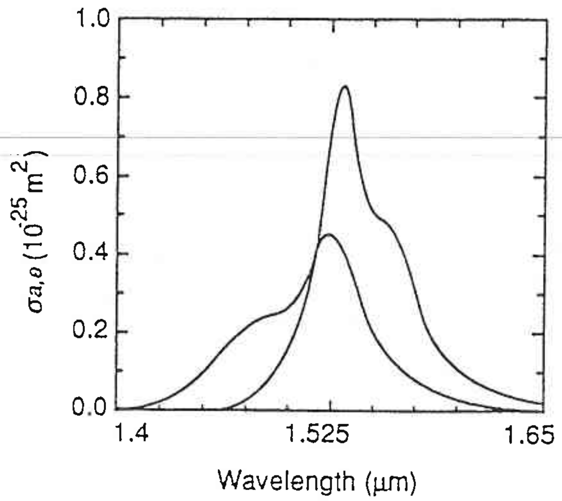


Fig. 4 Fluorescence and absorption cross sections around / = 1.53 μm.

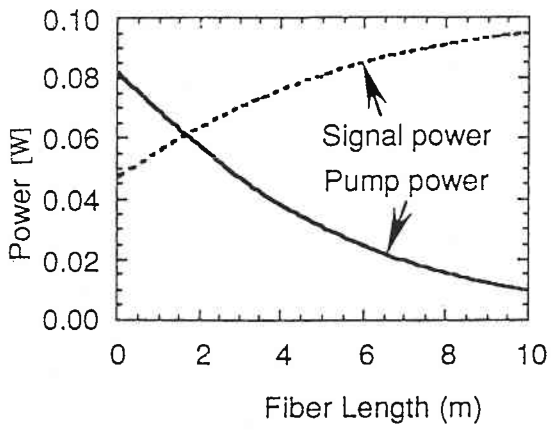


Fig. 5 Distribution of pump power and signal power along the fiber cavity. The pump power and the coupling efficiency are 100 mW ane 0.5 respectively.

power decreases from 82 mW to 10 mW, and the signal power increases from 47 mW to 95 mW. The output signal power is the same as that at the start of the cavity, i.e, 47 mW.

Fig. 6 plots the spectrum of the laser output with a cavity length of 10 m. The pump power and the coupling efficiency are 100 mW and 0.5 respectively. The lasing wavelength is 1.532 μm and the spectral width of the output signal is about 2 nm.

Fig. 7 shows the output power as a function of input pump power with different coupling efficiencies. The cavity length is assumed as 30 m. As the coupling coefficient (reflectivity) of signal increases, the slope efficiency decreases although the threshold input pump power becomes low. So it is necessary to use a low coefficient coupler for generating high-power output signal.

Fig. 8 plot the output power as function of coupling coefficient of signal with different cavity lengths. The input pump power is 100 mW. As expected, the optimum value of coupling coefficient exists, for a given cavity length, to get the highest output signal power. The optimum coupling coefficient increases as the cavity length decreases.

Fig. 9 plots the output power as a function of cavity length with different coupling efficiencies. The input pump power is 100 mW. The optimum fiber length of cavity exists, for a given coupling coefficient and input pump power, to get the highest output signal power. The optimum cavity length decreases as the coupling coefficient increases.

It is clear that there exists only one pair of value of cavity length and coupling coefficient by using which the highest output signal power can be obtained for a given pump power. These values can be predicted from Figs. 8 and 9. It seems that for a input pump power of 100 mW, the values may be in the range from 0.2 to 0.4 and the range of from 10 m to 20 m respectively.

Chen *et al.*[7] has analyzed by adopting the rate-equation model introducing the counterpropagating light wave modes and the pumping light. His results are essentially similar to the results obtained here where the ring laser cavity is used. The similar approach is done in this analysis but the consideration for the fiber coupler is done. The effect of the coupling coefficient of the fiber coupler is taken into account.

Many experimental works are reported. Wyatt[8] has also discussed experimentally the output power of the mirror-type fiber-optic laser. His observations are agrees with the analytical results obtained here. The pumping power can be negligible when the fiber length of the ring resonator or its attenuation is large.

6. CONCLUSION

The lasing characteristics of erbium-doped fiber lasers with ring cavities have been analyzed theoretically by using the rate equation and propagation equations.

A small signal model is proposed for analyzing the lasing threshold. There exists an analytic solution in the case of small signal. It is shown that threshold input pump power is relational to the cavity length and coupler efficiency. Iptimum cavity length and coupling efficiency exist for the minimum threshold.

Also the case of big signal is analyzed by solving the

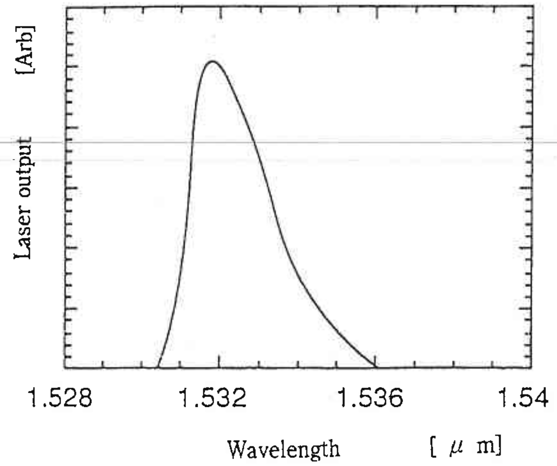


Fig. 6 Spectrum of the laser output with a cavity length of 10 m. The pump power and the coupling efficiency are 100 mW ane 0.5 respectively.

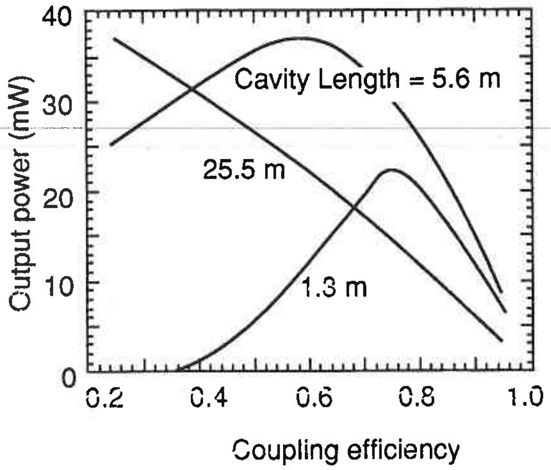


Fig. 8 The output power as function of coupling efficiency of signal with different cavity length. The input pump power is 100 mW.

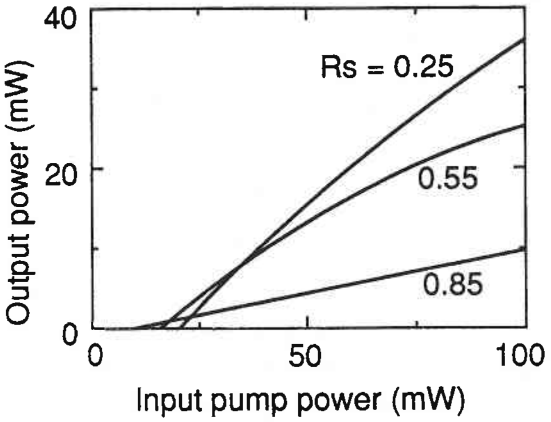


Fig. 7 The output power as a function of input pump power with different coupling efficiencies. The cavity length is assumed as 30 m.

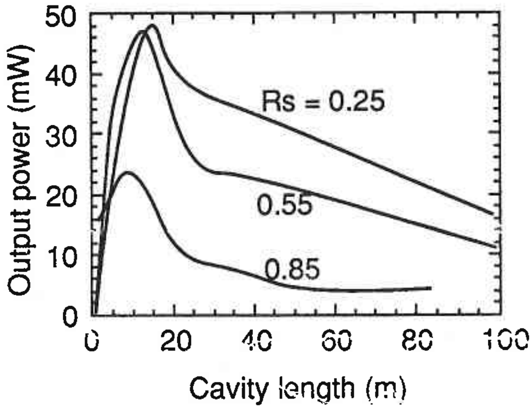


Fig. 9 The output power as a function of cavity length with different coupling efficiency.

equations numerically. The output spectrum is given, which shows that the spectral width of the output is approximately 2 nm.

The effects of the cavity length, coupler efficiency, and input pump power on the output characteristics are discussed. It has been shown that there exists only one pair of value of cavity length and coupling coefficient by using which the highest output signal power can be obtained for a given pump power.

It is believed that the theory provided here will help to design the parameters of a cw rare-earth doped especially a Er^{3+}-doped fiber laser with a ring cavity.

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