研 究 速 報

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Applicability of Inclined Strip Yield Superdislocation Model to Crack Problems

——1st Report, A Study on Interface Crack Problem——

傾斜超転位モデルのき裂問題への適用性
——第１報　界面き裂問題に対する検討——

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1. Introduction

The approach that the macroscopic plasticity at a crack tip can be considered in terms of dislocation arrays was first introduced by Bilby, Cottrell and Swinden (BCS model)[1]. Shortly thereafter, Bilby and Swinden[2] generalized this approach to permit the dislocation slip plane to be inclined to the crack plane. The work of Atkinson *et. al.*[3] circumvented the mathematical difficulties in the inclined strip yield model by introducing the idea of a superdislocation to represent the net effect of the dislocation array. In this report, this approach is extended to study the plasticity at an interface crack between the dissimillar materials.

2. The Problem

A crack of length $2a$ along the interface, with a remotely applied stress σ acting normal to the crack plane, is considered. It will suffice to describe the location (l_λ, θ_λ) and the strength b_λ of the superdislocation on the right tip due to the symmetry (see Fig. 1). Here the index $\lambda=1$ and 2 and denote the upper half-plane S^+ and the lower half-plane S^-, respectively. The stress and displacement fields for the inclined strip yield superdislocation model are obtained by superposition using the solutions for the follwoing subproblems

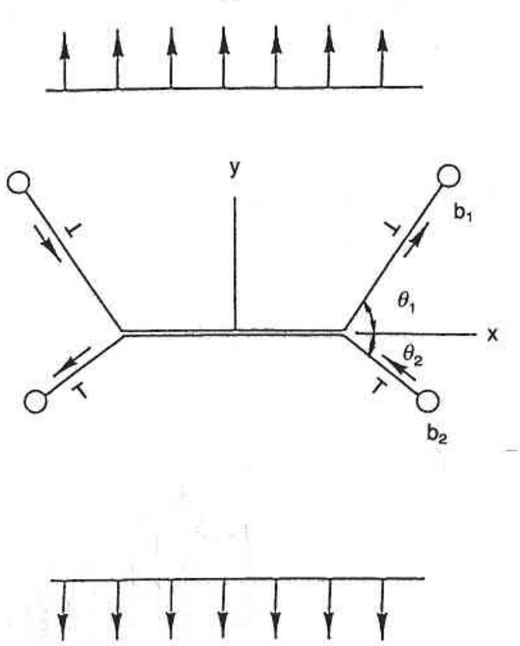

Fig. 1 Inclined strip yield zone model.

- Remotely applied tensile stress normal to the crack plane
- An interface crack with uniform traction equal and oppoiste to the applied stress
- Isolated superdislocations situated in different materials
- An interface crack with nonuniform surface tractions equal and opposite to the strsses of the superdislocations acting on the crack plane.

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3. The Stresses of A Dislocation Near An Interface

The stresses of a dislocation near an interface are determined by the Airy stress functions $\chi^{(\lambda)}$ indicated by Dundurs[4]. For a dislocation placed at $z_0 = r_0 e^{i\theta_0}$ in S^+, the expressions for $\chi^{(1)}$ will be indicated below

$$\begin{cases} \chi^{(1)}(b_x) = -\dfrac{2G_1 b_x}{\pi(\kappa_1+1)(1-\beta^2)}[(1-\beta^2)\rho_1 \ln \rho_1 \cos \gamma_1 \\ \quad +(\alpha+\beta^2)\rho_2 \ln \rho_2 \cos \gamma_2 - (1+\alpha)\beta\rho_2\gamma_2 \sin \gamma_2 \\ \quad -(\alpha-\beta)(1-\beta)\xi(2 \ln \rho_2 - \cos 2\rho_2 + 2\xi\dfrac{\cos \gamma_2}{\rho_2})] \\ \chi^{(1)}(b_y) = -\dfrac{2G_1 b_y}{\pi(\kappa_1+1)(1-\beta^2)}[(1-\beta^2)\rho_1 \ln \rho_1 \sin \gamma_1 \\ \quad +(\alpha+\beta^2)\rho_2 \ln \rho_2 \sin \gamma_2 + (1+\alpha)\beta\rho_2\gamma_2 \cos \gamma_2 \\ \quad -(\alpha-\beta)(1-\beta)\xi(\sin 2\gamma_2 - 2\xi\dfrac{\sin \gamma_2}{\rho_2})] \end{cases} \tag{1}$$

where α and β referred to Durdurs parameters

$$\alpha = \frac{G_2(\kappa_2+1)-G_1(\kappa_2+1)}{G_2(\kappa_1+1)+G_1(\kappa_2+1)} \tag{2}$$

$$\beta = \frac{G_2(\kappa_1-1)-G_1(\kappa_2-1)}{G_2(\kappa_1+1)+G_1(\kappa_2+1)} \tag{3}$$

and ξ, ρ_λ and γ_λ ($\lambda = 1, 2$) are defined in Fig. 2. b_x and b_y are x and y component of Burgers vector respectively, G_λ is shear modulus and $\kappa_\lambda = 3-4\nu_\lambda$ for plane strain and $(3-\nu_\lambda)/(1+\nu_\lambda)$ for plane stress where ν_λ is Poisson's ratio. Based on the above expressions the stresses along the interface induced by this dislocation can be obtained as

$$\sigma_{yy} - i\tau_{xy} = B\frac{1+\alpha}{1+\beta}\frac{1}{x-\bar{z}_0} + B\frac{1+\alpha}{1-\beta}\frac{1}{x-z_0} + \bar{B}\frac{1+\alpha}{1+\beta}\frac{z_0-\bar{z}_0}{(x-\bar{z}_0)^2} \tag{4}$$

with

$$B = \frac{G_1}{\pi i(\kappa_1+1)}(b_x + ib_y) \tag{5}$$

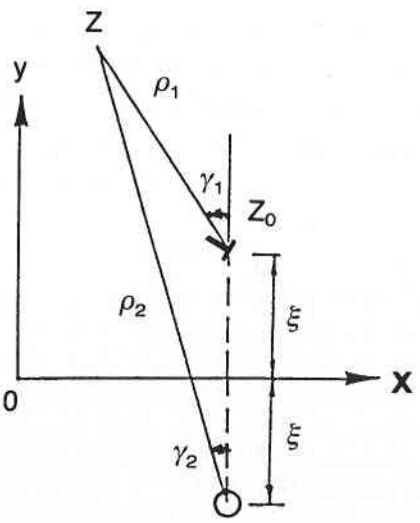

Fig. 2 An edge dislocation in dissimilar material.

4. The Complex Potentials of The Interface Crack

It can be proved[6] that for the crack problems, the stresses in an inhomogenous material can be expressed only by one complex potential $\phi(z)$, i.e.,

$$\begin{cases} \sigma_{yy}^{(1)} - \sigma_{xx}^{(1)} + 2i\tau_{xy}^{(1)} = (\bar{z}-z)(1-\beta)\phi''(z) + (1+\beta)\bar{\phi}'(z) - (1-\beta)\phi'(z) \\ \sigma_{yy}^{(2)} - \sigma_{xx}^{(2)} + 2i\tau_{xy}^{(2)} = (\bar{z}-z)(1+\beta)\phi''(z) + (1-\beta)\bar{\phi}'(z) - (1+\beta)\phi'(z) \end{cases} \tag{6}$$

where

$$\phi'(z) = \frac{1}{2\pi i}X(z)\int_{-a}^{a}\frac{p(t)}{(t-z)X(t)}dt \tag{7}$$

$$X(z) = \left(\frac{z+a}{z-a}\right)^{i\epsilon}\frac{1}{(z^2-a^2)^{1/2}} \tag{8}$$

$$\epsilon = \frac{1}{2\pi}\ln\frac{1-\beta}{1+\beta} \tag{9}$$

In the case of the crack with uniform traction equal and opposite to the applied stress, i. e., $p(x)=\sigma$, the complex potential $\phi'_0(z)$ can be deduced from eq. (7)

$$\phi'_0(z) = \sigma[1-(z-2ai\epsilon)X(z)] \tag{10}$$

For the crack with the stresses induced by dislocation, for instance, by the dislocation in the upper half-plane indicated by eq. (4) with opposite sign, the complex potential $\phi'_\omega(z)$ can be written as

$$\begin{aligned} \phi'_\omega(z) = {} & B\frac{1+\alpha}{1-\beta}\left(\frac{1}{z-z_0}\left[\frac{X(z)}{X(z_0)}-1\right]+X(z)\right) + B\frac{1+\alpha}{1+\beta}\left(\frac{1}{z-\bar{z}_0}\left[\frac{X(z)}{X(\bar{z}_0)}-1\right]+X(z)\right) \\ & + \bar{B}\frac{1+\alpha}{1+\beta}\frac{z_0-\bar{z}_0}{(z-\bar{z}_0)^2}\left[\frac{X(z)}{X(\bar{z}_0)}-1+\frac{(z-\bar{z}_0)(\bar{z}_0+2ai\epsilon)X(z)(\bar{z}_0-a)^{i\epsilon-1/2}}{(\bar{z}_0+a)^{i\epsilon+1/2}}\right] \end{aligned} \tag{11}$$

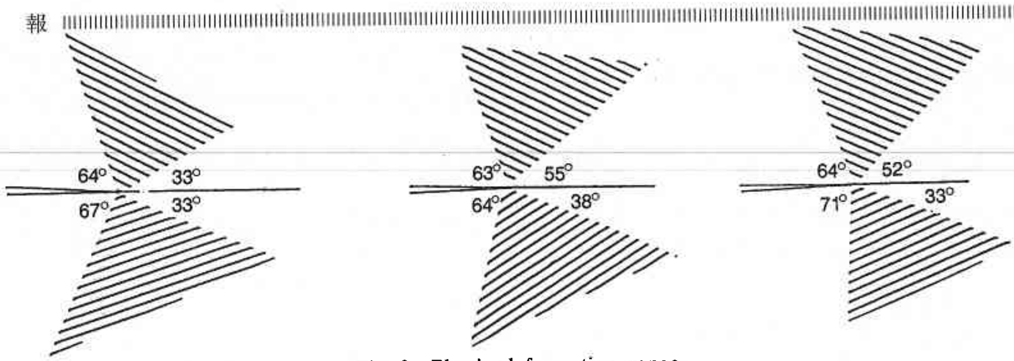

Fig. 3 Plastic deformation range.

5. The Governing Equations

The basic equations of the superdislocation inclined strip yield model are for the equilibrium of the superdislocation representing the crack-tip plasticity and for the cancellation of the crack tip singularity. These are

$$\begin{cases} \tau_y^{(1)} = \sigma h^{(1)} + \dfrac{G_1 b_1}{\pi(\kappa_1+1)} g_1^{(1)} + \dfrac{G_2 b_2}{\pi(\kappa_2+1)} (g_2^{(1)} + k_2^{(1)}) \\ \tau_y^{(2)} = \sigma h^{(2)} + \dfrac{G_2 b_2}{\pi(\kappa_2+1)} g_1^{(2)} + \dfrac{G_1 b_1}{\pi(\kappa_1+1)} (g_1^{(2)} + k_1^{(2)}) \end{cases} \quad (12)$$

$$\begin{cases} \sigma a = \dfrac{G_1 b_1}{\pi(\kappa_1+1)} f_2^{(1)} + \dfrac{G_2 b_2}{\pi(\kappa_2+1)} f_1^{(2)} \\ 2\epsilon\sigma a = \dfrac{G_1 b_1}{\pi(\kappa_1+1)} f_2^{(1)} + \dfrac{G_2 b_2}{\pi(\kappa_2+1)} f_2^{(2)} \end{cases} \quad (13)$$

where $\tau_y^{(i)}$ denotes the shear yield stress in two phases. The variables h, g, k and f can be calculated from eqs. (10), (11) and (1) respectively. The four unknowns, b_λ and l_λ (λ=1, 2), will be solved by four algebraic equations. In the case of small-scale yielding where $l_\lambda / a \ll 1$, and with the special combinations of material properties that ϵ=0 (β=0), the problem finally can be reduced to one equation in one unknown l_1 / l_2, i. e.,

$$F\,(\sigma / [G_1 / (1+\kappa_1)],\ l_1 / l_2,\ \theta_1,\ \theta_2) = \tau_y^{(1)} / \tau_y^{(2)} \quad (14)$$

6. Results and Discussions

With $\epsilon = 0$, from eq. (14) it is found that the distance of the superdislocation from the crack tip, l_λ, is independent of α. In other words, the size of plastic strip will be independent of the values of α. However, the plastic zone size is strongly depend on the values of the yield stress ratio of two materials. This conclusion also holds in the case of $\epsilon \neq 0$. According to the concept of the yield strip model, the crack tip displacement, δ, can be given by

$$\delta = \frac{\sigma^2 (1+\kappa_1)}{G_1 \tau_y^{(1)} (1+\alpha)} H\,(\theta_1,\ \theta_2,\ \tau_y^{(1)} / \tau_y^{(2)}) \quad (15)$$

and it can be found that δ varies with α in the form of $1/(1+\alpha)$.

In some ranges of θ_1 and θ_2, numerical results of eq. (14) give no real solution. This implies that the inclined slip plane can not exist in any direction with any combination of the angles θ_1 and θ_2. This situation is quite different from that in the homogenous material case. For the $\tau_y^{(1)} / \tau_y^{(2)}$ = 0.1, 0.5 and 0.8, the approximate ranges where plastic deformation may occur are given in Fig. 3.

7. Conclusions

The basic equations of the representation of plasticity at an interface crack by inclined strip yield superdislocation model are derived. With a special combination of the material properites and small-scale yielding case, the problem is reduced to one algebraic equation in one unknown, the ratio of plastic zone size. The relation of the crack opening displacement and material properties is given.

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References

1) Bilby, B. A. et. al. *Proc. Roy. Soci.*, A272 (1963), p. 304.
2) B. A. Bilby and K. H. Swinden, *Proc. Roy. Soci.*, A279 (1964), p. 1.
3) C. Atkinson and M. F. Kanninen, *Int. J. Fracture*, 13 (1977), p. 151.
4) J. Dundurs, in *Mathematical Theory of Dislocations*, T. Mura (ed.), (1969), p. 70.
5) A. H. England, *Complex Variable Methods in Elasticity.*, Wiley-Interscience, (1971).