研 究 速 報

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Applicability of Inclined Strip Yield Superdislocation Model to Crack Problems

——2nd Report, A Study on Mixed-Mode Crack Problem——

傾斜超転位モデルのき裂問題への適用性
——第2報 混合モードき裂問題に対する検討——

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1. Introduction

The slip plane inclined to the crack plane in terms of dsilocation arrays was first studied by Bilby and Swinden[1]. Vitek[2] and Riedel[3] have obtained more comprehensive results for slip-plane inclined at various angles. Atkinson *et al.*[4] introduced the idea of a superdislocation to represent the net effect of the dislocation array. Watanabe and Sato[5] developed a discontinous model to analyze the plasticity near the crack tip by finite element method. In this report, a crack with mixed mode loading condition is considered. An alternative model with three yield strips is proposed.

2. General Analysis

Consider a crack of length of $2a$ in a homogenous material, with the remotely applied stress T normal to the crack plane, and S the shear stress parallel to the crack plane. (see Fig. 1). l_i and b_i denote the distance from the crack tip to the superdislocation, and the strength of the superdislocation, respectively. Here the index i = 1 and 2 and denote the upper half-plane S^+ and the lower half-plane S^-, respectively. Now introduce the nondimensional quantities as following

$$\begin{cases} \bar{T} = \dfrac{T}{G/(1+\kappa)} \quad \bar{S} = \dfrac{S}{G/(1+\kappa)} \\ \bar{b}_i = b_i / a \quad \bar{l}_i = l_i / a \end{cases} \tag{1}$$

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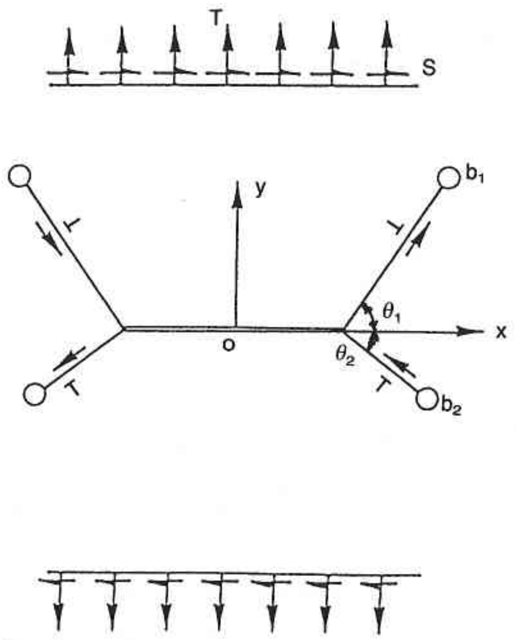

Fig. 1 Griffith crack under a mixed mode loading.

One of the key element for the superdislocation model is for the cancellation of the crack tip singularity. In the case

$$\frac{\bar{b}_1}{\sqrt{\bar{l}_1}}(\sin\ \theta_1/2+\sin 3\theta_1/2)-\frac{\bar{b}_2}{\sqrt{\bar{l}_2}}(\sin\ \theta_2/2+\sin 3\theta_2/2) = \frac{\sqrt{2}\,\pi\bar{T}}{3} \tag{2}$$

$$\frac{\bar{b}_1}{\sqrt{\bar{l}_1}}(\cos\ \theta_1/2+3\cos 3\theta_1/2)-\frac{\bar{b}_2}{\sqrt{\bar{l}_2}}(\cos\ \theta_2/2+3\cos 3\theta_2/2) = \sqrt{2}\,\pi\bar{S} \tag{3}$$

by using the results in reference 6). G is shear modulus and $\kappa = 3-4\nu$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress. θ_1 and θ_2 denote the angles between the crack plane and slip plane on the upper half-plane S^+ and the lower half-plane $S,^-$ respectively.

Let's consider a special case that $\theta_1 = -\theta_2 = \cos^{-1}(1/3)$, where the left side of eq. (3) will vanish. This means that the interaction between the crack and dislocations at these angles has no contributions on shear stress along the crack plane and, $\bar{b}_1/\sqrt{\bar{l}_1}$ and $\bar{b}_2/\sqrt{\bar{l}_2}$ become undetermined. Because the variable $\bar{b}_1$ and $\bar{b}_2$ must take positive value, it can be found, from eq. (2) and (3), that $\theta_1 < \cos^{-1}(1/3)$ when $-\theta_2 < \cos^{-1}(1/3)$ and, $\theta_1 > \cos^{-1}(1/3)$ when $-\theta_2 > \cos^{-1}(1/3)$. It seems that the original inclined strip yield model can not be directly extended to the mixed-mode fracture, even the shear stress being very small. The angles of $\theta_1 = -\theta_2 = \cos^{-1}(1/3)$ were used in tensile loading case for the calculation of the crack opening displacements[4], which give a very good agreement with other more rigrously obtained results. It may be concluded that i). In the inclined strip model, the slip angle should be taken only in values of $\cos^{-1}(1/3)$, where no contribution is added on the shear stress along the crack line, ii). Such the inclined strip yield model can only represent the plasticity induced by tensile loading.

3. Three-Strip Yield Model

We imagine that the superdislocations situated symmetrically on the inclined slips, at $\theta_1 = -\theta_2 = \cos^{-1}(1/3)$, but travelled to different distance l_i $(i = 1, 2)$ as is shown in Fig.2 The strengths of these superdislocations are assumed to be equal, i. e. $b_1 = b_2$. This means that the superdislocation placed on the inclined plane can only represent the opening crack deformation. In addition, another superdislocation of the strength b_3 is placed on the crack plane, at the distance l_3 ahead of the crack, to represent the shear effect. The equilibrium of the superdislocation and the cancellation of the singularity at crack tip give

$$\begin{cases} \bar{\tau}_y = \bar{T}f_1 + \bar{S}h_1 + \bar{b}_1 g_1 + \bar{b}_3 k_1 \\ -\bar{\tau}_y = \bar{T}f_2 + \bar{S}h_2 + \bar{b}_1 g_2 + \bar{b}_3 k_2 \\ \bar{\tau}_y = \bar{T}f_3 + \bar{S}h_3 + \bar{b}_3 g_3 + \bar{b}_3 k_3 \end{cases} \quad (4)$$

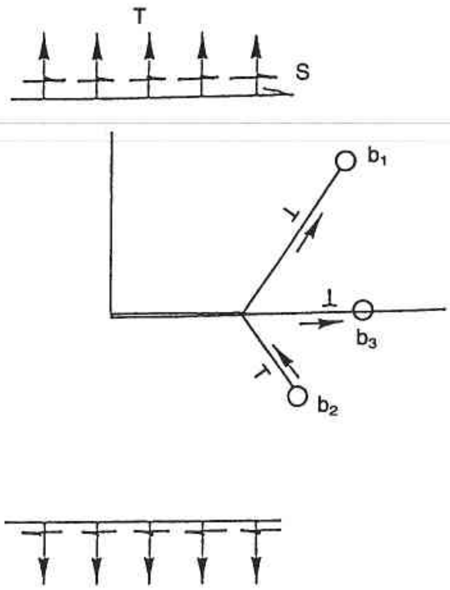

Fig. 2 Three-strip yield model.

$$\frac{\bar{b}_1}{\sqrt{\bar{l}_1}+\sqrt{\bar{l}_2}} = \frac{\sqrt{6}\,\bar{T}}{8} \quad (5)$$

$$\frac{\bar{b}_3}{\sqrt{\bar{l}_3}} = \frac{\sqrt{2}\,\bar{S}}{4} \quad (6)$$

where $\bar{\tau}_y = \tau/(G/(1+\kappa))$, τ denotes the shear yield (friction) stress and the variables f, h, g and k are the functions of $\bar{l}_i$ $(i = 1, 2, 3)$, related to the stresses acting on the dislocation due to the applied stress, the crack and the superdislocations, respectively. The five unknowns for the problems, b_i and l_i $(i = 1, 2, 3, b_1 = b_2)$, will be solved by five algebraic equations as indicated by eqs. (4), (5) and (6).

4. Results

By subsitituting eqs. (5) and (6) into eq. (4), we can get two equations in two unknowns, l_2/l_1 and l_3/l_1, and can be expressed as

$$\begin{cases} F_1\ (l_2/l_1,\ l_3/l_1,\ S/T) = 0, \\ F_2\ (l_2/l_1,\ l_3/l_1,\ S/T) = 0 \end{cases} \quad (7)$$

It should be noted that l_2/l_1 and l_3/l_1 are only dependent on the values of S/T. Fig. 3 gives the results of l_2/l_1 and l_3/l_1 as the function of S/T. With the increase of the S/T, plastic strip size l_1 and l_3 should increase. However, it can be found, from Fig. 3, l_1 increases faster than l_2 when the ratio

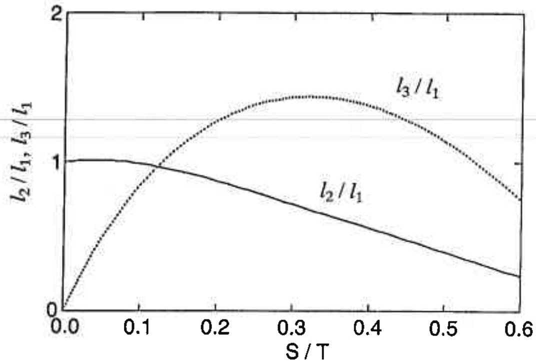

Fig. 3 Results of l_2 / l_1 and l_3 / l_1 (dashed line) as the function of S / T.

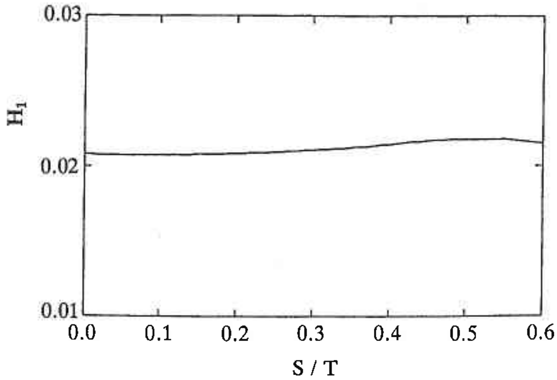

Fig. 4 Results of H_1 (S/T).

of S / T become large. Based on Eqs. (6) and (7), the crack opening displacement δ_t, and crack shearing displacement, δ_s can be indicated in the form

$$\begin{cases} \delta_t = 4\sqrt{2}\, b_1 / 3 = \dfrac{T^2(1+\kappa)}{G\tau_y} H_1 \ (S / T), \\ \delta_s = b_3 = \dfrac{T^2(1+\kappa)}{G\tau_y} H_2 \ (S / T) \end{cases} \tag{8}$$

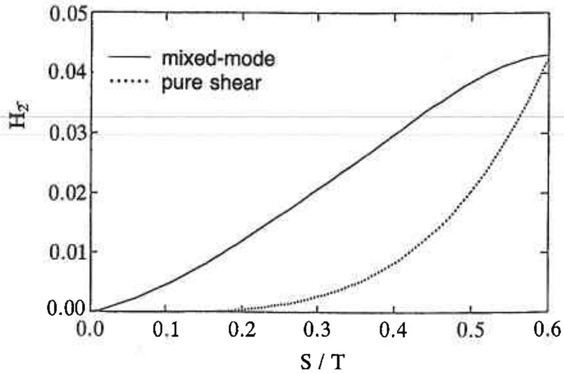

Fig. 5 Results of H_2 (S/T).

where the function H_1 and H_2 are plotted in Fig. 4 and Fig. 5 respectively. It can be found that the values of the opening displacement is not sensitive to the ratio of S / T. However, the crack shearing displacement, compared with that in the absence of the tensile loading shown in Fig. 5, is greatlty influnced by the ratio of S / T.

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