UDC 539.2.01：533.95

Analytical Study of One Dimensional Electron Gas Confined by a Harmonic Potential in a Magnetic Field

調和振動子型ポテンシャルに閉じ込められた磁場中の一次元電子状態の解析

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1. Introduction

Two–dimensional (2D) confinement of carriers in quantum wires (QWRs) is an important phenomenon in physics, and is expected to bring a significant improvement to lasers and other functional devices[1),2)]. Recently, several types of quantum wires of high quality have been realized by metal organic chemical vapor deposition (MOCVD)[3)~6)]. The QWRs of our group exhibited clear blue shift of photoluminescence (PL) peak with the reduction of the lateral width of the QWRs[6)]. In addition, an anisotropic effect of magneto-PL[7)] confirms the QWR effect. The experimental data of the magneto–PL can be analyzed by numerical calculations. However, the analytical solution is obtained only for isotropic harmonic potentials[8),9)].

In this paper, we describe the analytical solution of energy levels of a one-dimentional (1D) electron gas confined by an anisotropic 2D harmonic potential in the presence of a magnetic field.

2. Analytical Solution of the System

The Hamiltonian of a single particle in a harmonic potential and a magnetic field is

$$\hat{H}=-\frac{(\boldsymbol{P}+q\boldsymbol{A})^2}{2m^*}+\frac{1}{2}m^*\omega_x^2x^2+\frac{1}{2}m^*\omega_y^2y^2, \tag{1}$$

where $\omega_x(\omega_y)$ is the oscillator frequency of the harmonic potential along the x (y) direction, m^* is the effective mass and q is the electric charge.

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Here we discuss properties of a carrier along the (x, y)–plane with a magnetic field B along the z axis using a gauge with the vector potential

$$\boldsymbol{A}=B(-w^2y,\ v^2x,\ 0), \tag{2}$$

where

$$v=\sqrt{\omega_x/(\omega_x+\omega_y)}, \tag{3}$$

$$w=\sqrt{\omega_y/(\omega_x+\omega_y)}, \tag{4}$$

In case of $\omega_\xi=0$ and $\omega_\eta\neq 0$ (ξ, $\eta=x$, y), the analytical solution of the Schrödinger equation is known[10)]. In this case, the wavefunction is given by

$$\Phi=H_n\ (\zeta)\exp(-\zeta^2\ /\ 2+ip_\xi\xi/\hbar), \tag{5}$$

where $H_n(t)$ are the Hermite polynomials

$$H_n(t)=\sum_{r=0}^{[n/2]}(-1)^r(2r-1)!!\binom{n}{2r}2^r\,t^{\,n-2r}, \tag{6}$$

and

$$\zeta=\begin{cases}\sqrt{\dfrac{m^*\omega_0}{\hbar}}\,y+\dfrac{p_x\omega_c}{\sqrt{m^*\hbar\,\omega_0^3}}, & (\omega_x=0)\\[2ex] \sqrt{\dfrac{m^*\omega_0}{\hbar}}\,x-\dfrac{p_y\omega_c}{\sqrt{m^*\hbar\,\omega_0^3}}, & (\omega_y=0),\end{cases} \tag{7}$$

$$\omega_0=\sqrt{\omega_c{}^2+(\omega_x+\omega_y)^2}, \tag{8}$$

$$\omega_c=\frac{qB}{m^*}, \tag{9}$$

ω_c is the cyclotron frequency, and p_ξ is the momentum of plane wave along the ξ axis. The wavefunction has the eigenenergy

$$E_{n,p_\xi} = (n+\frac{1}{2})\hbar\omega_0+\frac{p_\xi^2}{2m^*}\frac{\omega_\xi^2}{\omega_0{}^2}, \qquad (10)$$

where n=0,1,2, . . . is the order of the Hermite polynomials.

When $\omega_x\omega_y\neq 0$, we substitute variables as follows.

$$X = vx\sqrt{m^*\omega_0/\hbar}, \qquad (11)$$

$$Y = wy\sqrt{m^*\omega_0/\hbar}. \qquad (12)$$

Then, the Hamiltonian is described as

$$\hat{H} = E_0\left\{v^2\left(-\frac{\partial^2}{\partial X^2}+X^2\right)+w^2\left(-\frac{\partial^2}{\partial Y^2}+Y^2\right)\right\} -2ivw\,E_c\left(X\frac{\partial}{\partial Y}-Y\frac{\partial}{\partial X}\right), \qquad (13)$$

where

$$E_0=\hbar\omega_0/2, \qquad (14)$$

$$E_c=\hbar\omega_c/2. \qquad (15)$$

Moreover, we assume an eigenfunction of the form

$$\Psi=\Phi\exp(-X^2/2-Y^2/2), \qquad (16)$$

The Schrödinger equation $\hat{H}\Psi(x, y)=E\Psi(x, y)$ is transformed into $\hat{H}_1\Phi=(E-E_0)\Phi$, where

$$\hat{H}_1 = E_0v^2\left(-\frac{\partial^2}{\partial X^2}+2X\frac{\partial}{\partial X}\right)+E_0w^2\left(\frac{\partial^2}{\partial Y^2}+2Y\frac{\partial}{\partial Y}\right) -2ivwE_c\left(X\frac{\partial}{\partial Y}-Y\frac{\partial}{\partial X}\right). \qquad (17)$$

Furthermore, the wavefunction can be expanded by base functions

$$u_{k,m}=\frac{H_m(X)H_{k-m}(Y)}{\sqrt{m!(k-m)!}}, \qquad (18)$$

(k=1,2, . . . ; m=0,1,2, ... ,k). Then Hamiltonian $\hat{H}_1$ are given by matrices ($\langle u_{k,j}\mid\hat{H}_1\mid u_{k,j'}\rangle$) with dimension $(k+1)\times(k+1)$ as follows

$$\hat{H}_1 = E_0I_k-(v^2-w^2)E_0A_k-vwE_cB_k, \qquad (19)$$

where

$$I_k=\begin{bmatrix} k & & & & \\ & k & & & \\ & & k & & \\ & & & \ddots & \\ & & & & k \\ & & & & & k \end{bmatrix} \qquad (20)$$

$$A_k=\begin{bmatrix} k & & & & & \\ & k-2 & & & & \\ & & k-4 & & & \\ & & & \ddots & & \\ & & & & -k+2 & \\ & & & & & -k \end{bmatrix} \qquad (21)$$

$$B_k=\begin{bmatrix} 0 & i\sqrt{k} & & & & & \\ i\sqrt{k} & 0 & -i\sqrt{k-1}\sqrt{2} & & & & \\ & i\sqrt{k-1}\sqrt{2} & 0 & -i\sqrt{k-2}\sqrt{3} & & & \\ & & i\sqrt{k-2}\sqrt{3} & 0 & \ddots & & \\ & & & \ddots & \ddots & i\sqrt{2}\sqrt{k-1} & \\ & & & & i\sqrt{2}\sqrt{k-1} & 0 & -i\sqrt{k} \\ & & & & & i\sqrt{k} & 0 \end{bmatrix} \qquad (22)$$

The eigenergies $E_{k,m}$ of the Schrödinger equation can be found by diagonalizing the matrices:

$$E_{k,m} = (1 + k)E_0 + (k - 2m)E_1, \tag{23}$$

where $E_0=(\hbar/2)\sqrt{\omega_c^2+(\omega_x+\omega_y)^2}$, $E_1=(\hbar/2)\sqrt{\omega_c^2+(\omega_x-\omega_y)^2}$. The eigenergies can be written by quantum numbers $l=0,\pm1,\pm2,\ldots$, $n=0,1,2,\ldots$, as

$$E_{n,l} = (2n + 1 + |l|)E_0 + lE_1 \tag{24}$$

In the isotropic case of $\omega_x=\omega_y$, the eigenvalue is equal to the value given by Fock etc.[8),9)]

Figure 1 shows the configuration of energy levels, where $\Delta E_-=E_0-E_1$, $\Delta E_+=E_0+E_1$, and $\Delta E_0=2E_0$. Dependence of these energies E_0, E_1, ΔE_-, and ΔE_+ on ω_x, ω_y, and ω_c is shown in figure 2, where contours of normalized E_0, E_1, ΔE_-, and ΔE_+ are described. α, β, γ, and δ denotes $E_0/(\hbar\omega_c/2)$, $E_1/(\hbar\omega_c/2)$, $\Delta E_-/(\hbar\omega_c/2)$, and $\Delta E_+/(\hbar\omega_c/2)$, respectively.

The first excited level is

$$E_{n=0,l=-1} = E_0+\Delta E_-. \tag{25}$$

If $\omega_x+\omega_y$ and ω_x are fixed, the first excited energy is smaller for the structure with more anisotropic confinement (i.e. either ω_x or ω_y is small). At the limit of high anisotropy, ΔE_- (γ) approaches zero. In addition, as the magnetic field B is increased compared to 2D confinement potential ($\omega_c \gg \omega_x$, ω_y), the energy levels are closer to Landau levels.

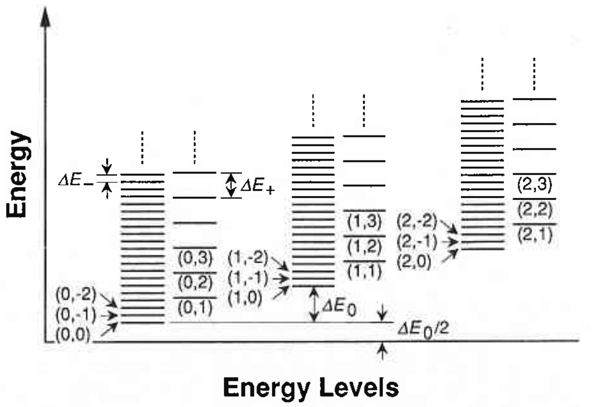

Fig. 1 Distribution of energy levels. (n, l) denotes quantum numbers which indicates $E_{n,l}$ of Eq. (24).

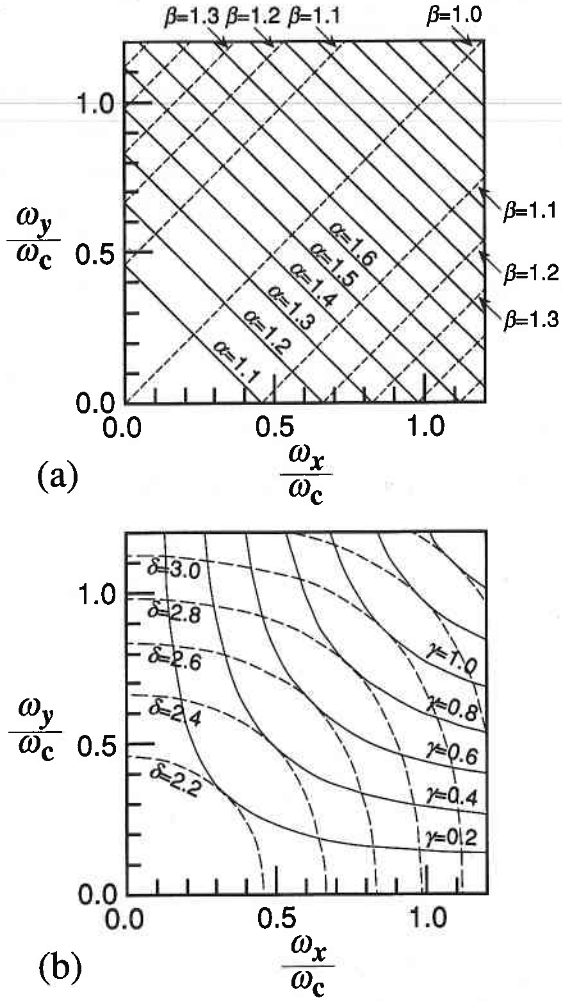

Fig. 2 Contours of (a) energy components of Eq. (24) E_0 (solid line), E_1 (dashed line), and (b) energy separations ΔE_- (solid line), ΔE_+ (dashed line), which depend on the oscillator frequency of the harmonic potentials (ω_x,ω_y) and the cyclotron frequency (ω_c). α,β,γ, and δ denotes normalized energy component and separation: $E_0/(\hbar\omega_c/2)$, $E_1/(\hbar\omega_c/2)$, $\Delta E_-/(\hbar\omega_c/2)$, and $\Delta E_+/(\hbar\omega_c/2)$, respectively.

3. Summary

In summary, we have analytically solved for the energy levels of a 1D carrier gas in an anisotropic 2D harmonic potential and a magnetic field. Moreover, we have discussed anisotropic effects of harmonic potentials. As the anisotropy increases, the excitation energy to the first excited level is closer to zero.

Acknowledgement

The authors express their thanks to Professors Y. Fujii, T. Ikoma, and H. Sakaki, University of Tokyo for their discussions and encouragement and Roger Helkey for his comments. This work is supported by the University-Industry Joint Project on Mesoscopic Electronics, TEPCO Research Foundation, and Japan Society for the Promotion of Science.

(Manuscript received, October 19, 1993)

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