研 究 速 報
特 集 5
UDC 539.219.2 : 538.971

Inelastic Fracture Parameter, CED, for an Interface Crack
——1st Report, Fundamental Relations——
界面き裂の非弾性破壊パラメータ CED
——第1報, 基本的な関係——

Katsuhiko WATANABE*, Yutaka SATO* and Daisuke NAGASAWA**
渡 辺 勝 彦・佐 藤 裕・長 沢 大 介

1. Introduction

As are well-known, around an interface crack tip, different from around a crack tip in a homogeneous material, oscillations of stress component appear and the deformation becomes a mixed-mode one even under a load which causes mode I deformation for a crack in a homogeneous material[1)~4)]. These make the problem difficult to deal with and the fracture parameter of an interface crack has yet to be established even for an elastic crack[5)~8)].

The CED (Crack Energy Density), which is defined as the strain energy area density in the plane where fracture is considered, was proposed as a crack parameter which enables us to be free from the restrictions on the constitutive equation[9),10)], and it was shown recently that the behavior of a mixed-mode crack in a homogeneous material can be explained by it in a unified way from completely elastic fracture to fracture with large scale yielding[11),12)]. Therefore, the CED may also be applicable to an interface crack if the stress oscillation problem is cleared.

In this report, the concrete definition of the CED for an interface crack is given first and it is shown that the CED can be divided into mode I and mode II contributions that can be expressed by domain integrals without any restrictions on the constitutive equation. Subsequenqly, the relations to the conventional crack parameters of CED are discussed, and the results above are confirmed through the elastic finite element analyses of a bimaterial specimen with a center-crack in the interface.

*Institute of Industrial Science, University of Tokyo
**Central Japan Railway Co.

2. CED for an Interface Crack

2.1 Definition

Consider a semi-circular notch with a sufficiently small root radius ρ shown in Fig. 1 as an interface crack. The material constants in the Interface Layer are considered to change continuously from those of Material 2 to those of Material 1 with X_2. In this situation, no sigularity and discontinuity appear around the notch tip, and the CED defined as strain energy area density[9)] is concretely given by

$$\mathcal{E}(\rho) = \int_{\Gamma_0} W dX_2 \tag{1}$$

Here, W is the strain energy density given by

$$W = \int_0^t \sigma_{ij}\dot{\varepsilon}_{ij} d\tau \quad (\alpha, \beta = 1, 2) \tag{2}$$

and Γ_0 is the path along the notch tip in Fig. 1. σ_{ij} and ε_{ij} in Eq. (2) are stress and strain tensors respectively, and $(\dot{\ })$ $= \partial (\ \ ) / \partial \tau$.

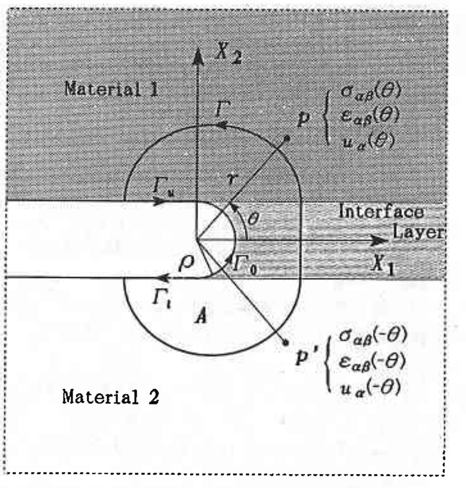

Fig. 1 Notch Model of an Interface Crack

The CED for a crack when $\rho = 0$ is defined as the limit when ρ goes to 0 by

$$\mathcal{E}^{(C)} = \lim_{\rho \to 0} \mathcal{E}(\rho) \tag{3}$$

2.2 Mode I and Mode II Contributions of CED

Let the quantities at p and p' points symmetric about X_1 axis in Fig. 1 denote as the functions of θ and $-\theta$ $(-\pi<\theta<\pi)$ and define the following quantities.

$$\left.\begin{aligned}
\sigma_{11}^{\mathrm{I}}(\theta) &= \{\sigma_{11}(\theta) + \sigma_{11}(-\theta)\}/2\\
\sigma_{22}^{\mathrm{I}}(\theta) &= \{\sigma_{22}(\theta) + \sigma_{22}(-\theta)\}/2\\
\sigma_{12}^{\mathrm{I}}(\theta) &= \{\sigma_{12}(\theta) - \sigma_{12}(-\theta)\}/2\\
\varepsilon_{11}^{\mathrm{I}}(\theta) &= \{\varepsilon_{11}(\theta) + \varepsilon_{11}(-\theta)\}/2\\
\varepsilon_{22}^{\mathrm{I}}(\theta) &= \{\varepsilon_{22}(\theta) + \varepsilon_{22}(-\theta)\}/2\\
\varepsilon_{12}^{\mathrm{I}}(\theta) &= \{\varepsilon_{12}(\theta) - \varepsilon_{12}(-\theta)\}/2\\
u_1^{\mathrm{I}}(\theta) &= \{u_1(\theta) + u_1(-\theta)\}/2\\
u_2^{\mathrm{I}}(\theta) &= \{u_2(\theta) - u_2(-\theta)\}/2\\
T_\alpha^{\mathrm{I}}(\theta) &= \sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\cdot n_\beta
\end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned}
\sigma_{11}^{\mathrm{II}}(\theta) &= \{\sigma_{11}(\theta) - \sigma_{11}(-\theta)\}/2\\
\sigma_{22}^{\mathrm{II}}(\theta) &= \{\sigma_{22}(\theta) - \sigma_{22}(-\theta)\}/2\\
\sigma_{12}^{\mathrm{II}}(\theta) &= \{\sigma_{12}(\theta) + \sigma_{12}(-\theta)\}/2\\
\varepsilon_{11}^{\mathrm{II}}(\theta) &= \{\varepsilon_{11}(\theta) - \varepsilon_{11}(-\theta)\}/2\\
\varepsilon_{22}^{\mathrm{II}}(\theta) &= \{\varepsilon_{22}(\theta) - \varepsilon_{22}(-\theta)\}/2\\
\varepsilon_{12}^{\mathrm{II}}(\theta) &= \{\varepsilon_{12}(\theta) + \varepsilon_{12}(-\theta)\}/2\\
u_1^{\mathrm{II}}(\theta) &= \{u_1(\theta) - u_1(-\theta)\}/2\\
u_2^{\mathrm{II}}(\theta) &= \{u_2(\theta) + u_2(-\theta)\}/2\\
T_\alpha^{\mathrm{II}}(\theta) &= \sigma^{\mathrm{II}}_{\alpha\beta}(\theta)\cdot n_\beta
\end{aligned}\right\} \tag{5}$$

It can be easily known that the quantities with superscript I and II are symmetric and antisymmetric components, respectively, about X_1 axis. Therefore, the quantities by Eqs. (4) and (5) can be related to Mode I and Mode II deformations respectively. Here, u_α is displacement, and T_α and n_β are traction force and normal unit vector, respectively, at the point p or p' when an arbitrary path that passes through p or p' is considered. It is noted that, although the quantities by Eqs. (4) and (5) satisfy the equilibrium equation and the relation between strain and displacement, they do not satisfy the constitutive equation, different from the case of homogeneous material[11], even in liner elastic case.

When the symmetric and atisymmetric quantities by Eqs. (4) and (5) are applied to Eq. (2), the strain energy is expressed as

$$\begin{aligned}
W(\theta) &= \int_0^t \{\sigma_{\alpha\beta}^{\mathrm{I}}(\theta) + \sigma_{\alpha\beta}^{\mathrm{II}}(\theta)\}\{\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(\theta) + \dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(\theta)\}\, d\tau\\
&= \int_0^t \sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(\theta)\, d\tau + \int_0^t \sigma_{\alpha\beta}^{\mathrm{II}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(\theta)\, d\tau\\
&\quad + \int_0^t \sigma_{\alpha\beta}^{\mathrm{II}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(\theta)\, d\tau + \int_0^t \sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(\theta)\, d\tau
\end{aligned} \tag{6}$$

and, considering the relations as

$$\left.\begin{aligned}
\sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(\theta) &= \sigma_{\alpha\beta}^{\mathrm{I}}(-\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(-\theta)\\
\sigma_{\alpha\beta}^{\mathrm{II}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(\theta) &= \sigma_{\alpha\beta}^{\mathrm{II}}(-\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(-\theta)\\
\sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(\theta) &= -\sigma_{\alpha\beta}^{\mathrm{I}}(-\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}(-\theta)\\
\sigma_{\alpha\beta}^{\mathrm{I}}(\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(\theta) &= -\sigma_{\alpha\beta}^{\mathrm{I}}(-\theta)\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}(-\theta)
\end{aligned}\right\} \tag{7}$$

hold, it is derived that the CED can be divided into Mode I contribution $\mathcal{E}^{\mathrm{I}}(\rho)$ and Mode II contibution $\mathcal{E}^{\mathrm{II}}(\rho)$ as

$$\begin{aligned}
\mathcal{E}(\rho) &= \int_{\Gamma_0} W dX_2\\
&= \int_{\Gamma_0} W^{\mathrm{I}} dX_2 + \int_{\Gamma_0} W^{\mathrm{II}} dX_2\\
&= \mathcal{E}^{\mathrm{I}}(\rho) + \mathcal{E}^{\mathrm{II}}(\rho)
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\mathcal{E}^{\mathrm{I}}(\rho) &= \int_{\Gamma_0} W^{\mathrm{I}} dX_2, \quad W^{\mathrm{I}} = \int_0^t \sigma_{\alpha\beta}^{\mathrm{I}}\,\dot\varepsilon_{\alpha\beta}^{\mathrm{I}}\, d\tau\\
\mathcal{E}^{\mathrm{II}}(\rho) &= \int_{\Gamma_0} W^{\mathrm{II}} dX_2, \quad W^{\mathrm{II}} = \int_0^t \sigma_{\alpha\beta}^{\mathrm{II}}\,\dot\varepsilon_{\alpha\beta}^{\mathrm{II}}\, d\tau
\end{aligned} \tag{9}$$

2.3 CED by Domain Integrals

Let Γ and A be an arbitrary path surrounding the notch tip path Γ_0 and the area surrounded by the closed path $\Gamma + \Gamma_u - \Gamma_0 + \Gamma_l$ in Fig. 1 respectively. As all the quantities in the area A are continuous with X_1 and X_2, both the equilibrium equation and the relation between strain and displacement are satisfied and Cauchy's formula can be applied. Therefore, it is easily shown in the same way as for isotropic and homogeneous materials[11] that the domain integrals given by

$$\begin{aligned}
\mathcal{E}_J &= \int_\Gamma (W dX_2 - T_\alpha u_{\alpha,1}\, d\Gamma)\\
&\quad - \int_A \int_0^t (\sigma_{\alpha\beta,1}\,\dot\varepsilon_{\alpha\beta} - \varepsilon_{\alpha\beta,1}\,\dot\sigma_{\alpha\beta})\, d\tau dA
\end{aligned} \tag{10}$$

$$\begin{aligned}
\mathcal{E}_J^M &= \int_\Gamma (W^M dX_2 - T_\alpha^M u_{\alpha,1}^M\, d\Gamma)\\
&\quad - \int_A \int_0^t (\sigma_{\alpha\beta,1}^M\,\dot\varepsilon_{\alpha\beta}^M - \varepsilon_{\alpha\beta,1}^M\,\dot\sigma_{\alpha\beta}^M)\, d\tau dA
\end{aligned} \tag{11}$$

become path-independent without any restrictions on constitutive equation and the relations as

$$\mathcal{E}(\rho) = \mathcal{E}_J \tag{12}$$

$$\mathcal{E}^M(\rho) = \mathcal{E}^M{}_J \tag{13}$$

hold. Here, M = I, II.

When $\sigma_{\alpha\beta} = \partial W / \partial\varepsilon_{\alpha\beta}$ holds, the area integral in Eq.

(10) vanishes in the same way as in Ref. (11) and Eq. (10) coincides with the so-called J-integral.

$$\mathcal{E}_J = \int_\Gamma (W dX_2 - T_\alpha u_{\alpha,1} d\Gamma) = J \tag{14}$$

However, the area integral in Eq. (11) does not vanish even in linear elastic case.

3. Relations to Other Crack Parameters

3.1 Relation to J-integral

When $\sigma_{\alpha\beta} = \partial W / \partial \varepsilon_{\alpha\beta}$ holds, the path-independent integral J exists. Therefore, the relation as

$$\begin{aligned}\mathcal{E}^{(c)} &= \lim_{\rho \to 0} \mathcal{E}(\rho) \\ &= \lim_{\rho \to 0} \int_\Gamma (W dX_2 - T_\alpha u_{\alpha,1} d\Gamma) = J^{(c)}\end{aligned} \tag{15}$$

is obtained, taking Eq. (14) into consideration.

3.2 Relation to Stress Intensity Factor

In linear elastic case, the relation

$$\mathcal{E}^{(c)} = \frac{1}{16 \cosh^2(\alpha\pi)} [\aleph] (K_1{}^2 + K_2{}^2) \tag{16}$$

is derived through Eq. (15) and the relation between J-integral and stress intensity factor. Here,

$$\alpha = \frac{1}{2\pi} \ln \left\{ \left(\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2} \right) \Big/ \left(\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1} \right) \right\} \tag{17}$$

$$[\aleph] = \frac{\kappa_1 + 1}{\mu_1} + \frac{\kappa_2 + 1}{\mu_2} \tag{18}$$

$$\kappa_j = \begin{cases} 3 - 4\nu_j & : \text{plane strain} \\ (3 - \nu_j) / (1 + \nu_j) & : \text{plane stress} \end{cases} \tag{19}$$

μ_j and ν_j are shearing modulus and Poisson's ratio respectively, and subscript j (= 1, 2) corresponds to Material 1 or 2 in Fig. 1. Moreover, K_1 and K_2 are stress intensity factors for an interface crack corresponding to $K_{\rm I}$ and $K_{\rm II}$ for a homogeneous material.

3.3 Relation to Energy Release Rate

Through the relation between energy release rate $\mathcal{G}$ and stress intensity factors K_1 and K_2[13], the following relation holds.

$$\mathcal{G}^{(c)} = \mathcal{G} \tag{20}$$

4. Finite Element Analyses of an Interface Crack in an Infinite Plate

Bearing an interface crack in an infinite plate in mind, the elastic finite element analyses of the part encircled by broken line in Fig. 2 were carried out by applying the forced displacements to the boundary evaluated from the singular solution[5], that is,

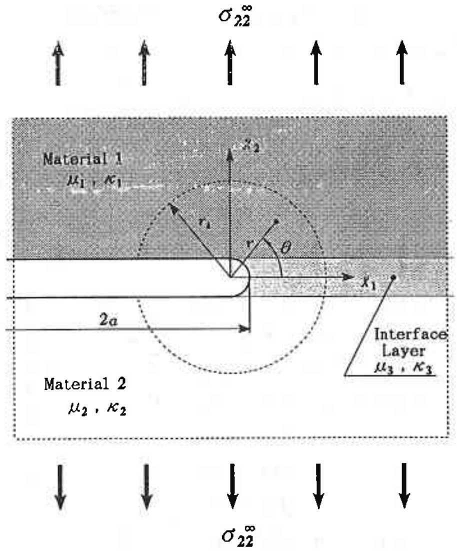

Fig. 2 Interface Crack in an Infinite Plate under Uniform Tension

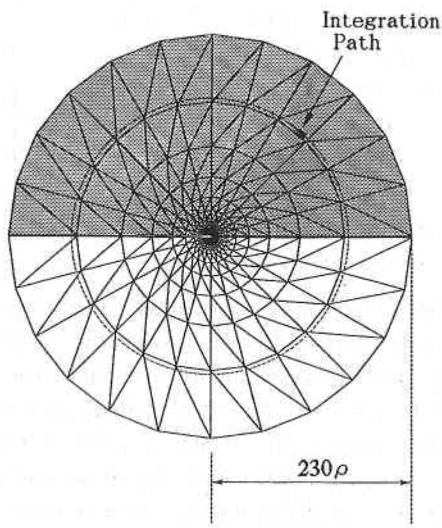

Fig. 3 Mesh Pattern

$$(u_i)_j = \sum_{m=1}^{2} \frac{\sqrt{2r/\pi}}{4\mu_j} K_m f_m \left(\theta, \alpha \ln \frac{r}{2a}, \alpha\right) \tag{21}$$

When the condition that $\rho \ll r_A \ll a$ is satisfied, it is thought that the analyses here correspond to the analyses of an interface crack in an infinite plate in Fig. 2. Figure 3 shows the mesh pattern used. r_A / ρ is fixed at 230.0 and

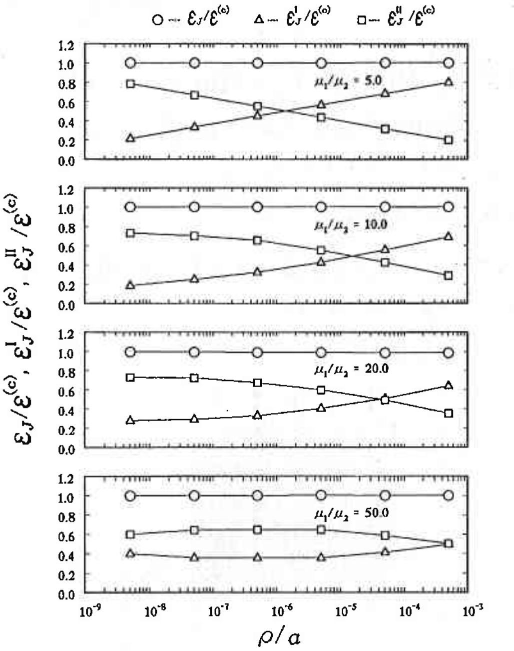

Fig. 4 ρ-dependencies of CED and its Mode I and Mode II Contributions

the plane strain analyses were carried out varing the value of ρ. Crack lengh a is supposed to be 20.0mm. As to the material constants of Material 1 and Material 2, five kinds of μ_1/μ_2 are taken (μ_1/μ_2 = 1.0, 5.0, 10.0, 20.0, 50.0) and $\nu_1 = \nu_2 = 0.3$. In the interface layer, let the material constants be constant for simplicity and let us take them at the geometrical mean values, that is, $\mu_3 = \sqrt{\mu_1 \mu_2}$ and $\nu_3 = \sqrt{\nu_1 \nu_2} = 0.3$.

The CED and its Mode I and Mode II cotributions were evaluated by $\mathcal{E}_J$-integral (Eq. (10)) and $\mathcal{E}_J^M$-integral (Eq. (11)) and J-integral was also evaluated. $\mathcal{E}_J$-integral agrees well with J-integral, so, the relation of Eq. (14) is confirmed. The path-independencies of domain integrals were also confirmed numerically (an example of integration path Γ employed is shown in Fig. 3).

The ρ-dependencies of CED and its Mode I and Mode II contributions are shown in Fig. 4. Here, $\mathcal{E}^{(c)}$ is evaluated from Eq. (16). From this figure, it is known that $\mathcal{E}_J$, that is, $\mathcal{E}(\rho)$ is not sensitive to ρ and the relation as

$$\mathcal{E}(\rho) \cong \mathcal{E}^{(c)} = \frac{1}{16\cosh^2(\alpha\pi)}[\aleph](K_1^2 + K_2^2) \qquad (22)$$

holds when ρ is sufficiently small. This fact means that, when ρ is taken sufficiently small, $\mathcal{E}(\rho)$ can be a crack parameter which represents some situation around a crack tip. On the other hand, $\mathcal{E}_J^{I}$ and $\mathcal{E}_J^{II}$, that is, $\mathcal{E}^{I}(\rho)$ and $\mathcal{E}^{II}(\rho)$ depend on the value of ρ. This is thought to correspond to the fact that each mode contribution of energy release rate becomes indeterminate[6]. It is considered that $\mathcal{E}^{I(c)}$ and $\mathcal{E}^{II(c)}$ are also indeterminate and $\mathcal{E}^{I}(\rho)$ and $\mathcal{E}^{II}(\rho)$ can not be crack parameters at least in linear elastic case without fixing ρ at some value for convenience' sake.

5. Conclusion

The fundamental relations were derived about the CED for an interface crack and, through the elastic finite elemennt analyses of an example, its some fundamental matters were studied to explore the possibility of CED as an interface fracture parameter.

(Manuscript received, June 23, 1993)

References

1) Dundurs, J. and Gautesen, A. K., Int. J. of Fracture, 3G (1988), p. 151.
2) Erdogan, F., ASME J. of App. Mech., 32 (1965), p. 199.
3) Hong, C. H. and Stern, M., J. of Elasticity, 8 (1978), p. 21.
4) Hamoush, S. A. et al., Eng. Fracture Mech., 33–3 (1989), p. 421.
5) Rice, J. R. and Sih, G. C., ASME J. of App. Mech., 32 (1965), p. 418.
6) Sun, C. T. et al., Eng. Fracture Mech., 28 (1987), p. 13.
7) Yuuki, R. and Cho, S. B., Trans. of JSME, 55–510, A (1989), p. 340.
8) Shih, C. F. and Asaro, R. J., J. of App. Mech., 55 (1988), p. 299.
9) Watanabe, K., Bulletin of JSME, 24, 198 (1981), p. 2059.
10) Watanabe, K., Bulletin of JSME, 28, 235 (1985), p. 26.
11) Watanabe, K. and Utsunomiya, T., Trans. of JSME, 53–491 (1987), p. 1285.
12) Watanabe, K. and Utsunomiya, T., Int. J. of Pressure Vessel and Piping, 44 (1990), p. 175.
13) Malyshev, B. M. and Salganik, R. L., Int. J. Fracture Mech., 1 (1965), p. 114.