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Comparison between Resistances of Porous and Continuous Models of Saturated Soil for Plane Strain Case

飽和した平面ひずみ多孔質体と連続弾性体の動的剛性の比較

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Introduction

In dynamic analysis of soil-structure interaction, one of the key tasks is modeling of unbounded soil. In conventional study, this can be achieved, by considering soil as a continuous medium and solving a mixed-boundary value problem corresponding to semi-infinite region. If the geometry of the medium is assumed to be simple, analytical solution is available. While, the boundary element method in frequency domain will be an efficient tool (1, 2), when the boundary condition is not considered to be simple. The solution of a mixed-boundary value problem gives relation between force and displacement of semi-infinite medium or soil resistance. This relationship is called dynamic stiffness or impedance function in which the energy radiation to infinity is implicitly incorporated.

In actuality, however, soil is a porous medium and in saturated condition, is composed of solid and liquid phases. Thus, the resistance of soil will be modified due to relative motion between these two phases. This paper extends the approach by Novak (3), which determins the resistance of a rigid cylinder extending infinitity in an infinite medium to one for a porous medium based on Biot's theory (4), and the effects of pore pressure and mass coupling on resistance for vertical, torsional, rocking, and horizontal motion are discussed.

1. Equations of motion for porous medium in cylindrical coordinate.

Based on Biot's theory, the equation of a porous medium is rewritten in cylindrical coordinate. In his theory, the effect of damping of a porous medium is just considered to be due to diffusion of liquid phase through porous solid. In actual case, however, inelastic deformation of granular fabric causes hysteretic type damping. This effect can be taken into account, if the real values of Lame's constants for solid phase are replaced by the complex numbers. The equations in terms of displacement potentials can be written in the following form:

$$\begin{bmatrix} \lambda^*+2\mu^*+Q & Q \\ Q & Q \end{bmatrix} \begin{Bmatrix} \nabla^2\varphi \\ \nabla^2\psi \end{Bmatrix} = \begin{bmatrix} \rho & \alpha\rho_f \\ \rho_f & (1/n+\alpha-1)\rho_f \end{bmatrix} \begin{Bmatrix} \ddot{\varphi} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \begin{Bmatrix} \dot{\varphi} \\ \dot{\psi} \end{Bmatrix} \tag{1}$$

$$\begin{bmatrix} \mu^* & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \nabla^2 H_i \\ \nabla^2 G_i \end{Bmatrix} = \begin{bmatrix} \rho & \alpha\rho_f \\ \rho_f & (1/n+\alpha-1)\rho_f \end{bmatrix} \begin{Bmatrix} \ddot{H}_i \\ \ddot{G}_i \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \begin{Bmatrix} \dot{H}_i \\ \dot{G}_i \end{Bmatrix} \quad (i = 1, 2) \tag{2}$$

In the above equations φ, H_i, are related to the solid phase displacement potential and ψ, G_i, are related to the relative motion of liquid phase displacement potential. On the other hand φ and ψ denote volumetric waves and H_i, G_i, shear wave.

By finding the eigenvalues and eigenvectors of the coefficients of eq. (1) for steady state case, it is possible to decouple φ and ψ. By solving the uncoupled equations, the final results will be obtained as:

$$H_1 = K_m\,(i\beta_3 r)\,(C \sin m\theta + C_0 \cos m\theta)\exp(i\omega t) \tag{4–a}$$

$$H_2 = K_m\,(i\beta_3 r)\,(C_2 \sin m\theta + C_3 \cos m\theta)\exp(i\omega t)$$

$$G_i = \frac{\rho_f}{(ib/\omega - (1/n+\alpha-1)\rho_f)} H_i \quad (i = 1, 2) \tag{4–b}$$

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$$\begin{Bmatrix} \varphi \\ \psi \end{Bmatrix} = \begin{bmatrix} A_1 \sin m\theta + A_2 \cos m\theta & B_1 \sin m\theta + B_2 \cos m\theta \\ A_3 \sin m\theta + A_4 \cos m\theta & B_3 \sin m\theta + B_4 \cos m\theta \end{bmatrix}$$

$$\begin{Bmatrix} K_m (i\beta_1 r) \\ K_m (i\beta_2 r) \end{Bmatrix} \exp (i\omega t) \tag{5}$$

$\beta_i = \frac{\omega}{v_p^*}\lambda_i \ (i = 1, 2)$, $\beta_3 = \frac{\omega}{v_s^*}\lambda_3$, $\lambda^* = \lambda\ (1+ida)$,

$\mu^* = \mu\ (1+ida)$, $v_p^{*2} = \frac{(\lambda^*+2\mu^*)}{\rho}$,

$v_s^{*2} = \frac{\mu^*}{\rho}$, $\lambda_1 = \sqrt{0.5(-A+\sqrt{A^2-4\ (B+C)})\ /\ (1+ida)}$,

$v_s = \mu / \rho$

$\lambda_2 = \sqrt{0.5(-A+\sqrt{A^2-4\ (B+C)})\ /\ (1+ida)}$,

$\lambda_3 = \sqrt{\left(1+\frac{\alpha\rho_f^2}{\rho(-ib/\omega+(1/n+\alpha-1)\rho_f)}\right) / (1 + ida)}$

$A=-1+\frac{\rho_f}{\rho}+\alpha\frac{\rho_f}{\rho}-\left((1/n+\alpha-1)\frac{\rho_f}{\rho}-\frac{ib}{\omega\rho}\right)$
$\left(\frac{(\lambda+2\mu)\ (1+ida)\ n}{K_f}+1\right)$,

$B=\left(-1+\frac{\rho_f}{\rho}\right)\left(-\alpha\frac{\rho_f}{\rho}+\left((1/n+\alpha-1)\frac{\rho_f}{\rho}-\frac{ib}{\omega\rho}\right)\right.$
$\left.\left(\frac{(\lambda+2\mu)\ (1+ida)\ n}{K_f}+1\right)\right)\Bigg)$,

$C=\left(\alpha\frac{\rho_f}{\rho}-(1/n+\alpha-1)\frac{\rho_f}{\rho}-\frac{ib}{\omega\rho}\right)$
$\left(1-\left(-1+\frac{\rho_f}{\rho}\left(\frac{(\lambda+2\mu)\ (1+ida)}{K_f}+1\right)\right)\right)$, λ and μ=Lame's constants, K_m=the second modified complex Bessel function of order m, da=hystersis damping ratio, n=porosity, K_f=bulk modulus of fluid, ρ_s=density of solid, ρ_f=density of fluid, $\alpha=2/n-1$ is toruosity for sphere shapes, $b=gn^2\rho_f\ /k$ is diffusive coefficient, g=ground acceleration, k=permeability of soil, $\rho=(1-n)\ \rho_s+n\rho_f$ is density of soil, ω=frequency of motion, $a_i^*=i\beta_j r_0$, where J= 1, 2, 3, r_0=radius of the disk and p=pore pressure.

2. Comparsion between resistances of porous and continuous models of saturated soil for different motions

The material properties used for computation are as follows:

$\lambda=\mu=667$ kgf/cm², n=0.3, K_f=20000 kg/cm²,
$\rho_f=10^{-6}$, $\rho_s=2.2x10^{-6}$ kgf. s²/cm

2.1 Motion in axial direction

when medium undergoes this kind of axisymetrical motion: $u_\theta = u_r = 0$, $u_z = u\ (r, \omega)$, the dynamic stiffness will be:

$$K_{ep} = 2\pi\mu\,(1+ida)\,a_3^*\frac{K_1\,(a_3^*)}{K_0\,(a_3^*)} \tag{6}$$

Fig. 1 shows the ratio of the real part of eq. (6) to the one of a continuous medium for f = 1. The ratio is kept almost constant over a wide range of dimensionless frequency ($a=\omega r_0\ /\ v_s$), when damping ratio (da) is small, while it decreases with an increase of dimensionless frequency in case that (da) is fairly large.

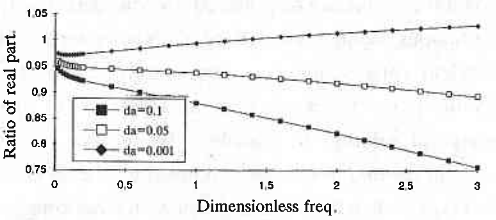

Fig. 1 Variation of ratio of real part factor of porous to continuous medium for motion in axial direction motion (f=1) with dimensionless freq.

2.2 Torsional motion

For a disk undergoing harmonic rotation around its vertical axis $u_r = u_z = 0$ and $u_\theta = u\ (r, \omega)$, the dynamic stiffness of porous medium is:

$$K_{ep} = 2\pi\mu r^2\,(1+ida)\left(2+a_3^*\frac{K_0\,(a_3^*)}{K_1\,(a_3^*)}\right) \tag{7}$$

The ratio of the imaginary part of eq. (7) to the one of a continuous medium for f =0.001 is shown in **Fig. 2**. These lines are steeply downward to the right, and converge to

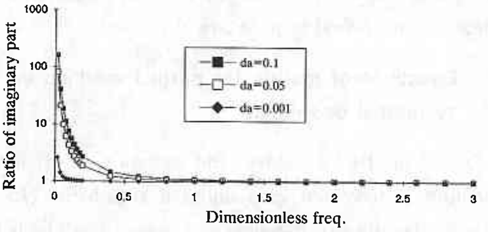

Fig. 2 Variation of imaginary part of porous to continuous medium with dimensionless freq. for torsion motion (f=0.001)

unity. The bigger the damping ratio is, the bigger the value is.

2.3 Rocking motion

When the disk vibrates in the vertical direction, in an antisymetric manner $u_r = u_\theta = 0$ and $u_z = u\,(r, \theta, \omega)$, the dynamic stiffness is:

$$K_{rp} = \pi\mu^2\,(1+ida)\left(1+a_3{}^*\,\frac{K_0\,(a_3{}^*)}{K_1\,(a_3{}^*)}\right) \tag{8}$$

Eq. (8) is very similar to eq. (7) and the ratios show similar variation.

2.4 Horizontal motion

When a disk undergoes harmonic horizontal motion in drain boundary condition:

$u_r = u\,(r, \theta, \omega)$, $u_\theta = v\,(r, \theta, \omega)$, $u_z = 0$, $p\,(r_0, \theta, \omega) = 0$, the dynamic stiffness of porous medium is:

$$K_{hp} = \pi\mu\,(a_3{}^*)^2\,R_2\,/\,R_3 \tag{9}$$

Where:

$R_1 = -\lambda_1 K_1\,(a_1{}^*)\ T\,/\,(\lambda_2 K_1\,(a_2{}^*)$, $T = (1+t_1)\,/\,(1+t_2)$

$t_i = (\rho_f\,/\,\rho - 1 - \lambda_i^2)\,/\,((\alpha\rho_f\,/\,\rho - ib\,/\,\omega\rho + (1\,/\,n + \alpha - 1)\,\rho_f\,/\,\rho)$ (i = 1, 2)

$R_2 = K_1\,(a_3{}^*)\,(4K_1\,(a_1{}^*) + a_1{}^* K_0\,(a_1{}^*) + 4R_1 K_1\,(a_2{}^*) + R_1 a_2{}^* K_0\,(a_2{}^*) + a_3 K_0\,(a_3{}^*)\,(K_1\,(a_1{}^*) + R_1 K_1\,(a_2{}^*))$

$R_3 = a_3{}^* K_0\,(a_3{}^*)\,(K_1\,(a_1{}^*) + R_1 K_1\,(a_2{}^*) + a_1{}^* K_0\,(a_1{}^*) + R_1 a_2{}^* K_0\,(a_2{}^*)\,) + K_1\,(a_3{}^*)\,(a_1{}^* K_0\,(a_1{}^*) + R_1 a_2{}^* K_0\,(a_2{}^*)\,)$

Fig. 3 shows the ratios of imaginary and real parts of eq. (9) to those of continuous medium, respectively, for f=0.001 (undrained condition). Their ratios increase almost linearly with increasing value of dimensionless frequency. On the other hand, the ratios changes drastically, when f=1 (drain condition), as shown in **Fig. 4**. As for the real part, the ratio decreases with frequency and becomes even negative, Concerning the imaginary part, the ratio tends to a constant value.

conclusion

Based on Biot's theory, the dynamic stiffness of porous medium for four kinds of wave front shapes are computed in closed form. The results obtained through parametric study shows that the pore pressure generation has a major effect on the dynamic stiffness and the mass coupling plays a secondary role. These finding are particularly important in the estimation of dynamic soil stiffness associated with radiation of waves with various front shapes. (Manuscript received, May 14, 1993)

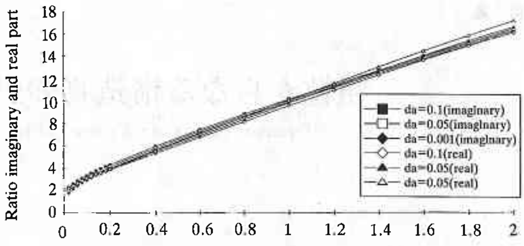

Fig. 3 Variation of ratio of imaginary and real part factor of porous to continuous medium for horizontal motion (f=0.001) with dimensionless freq.

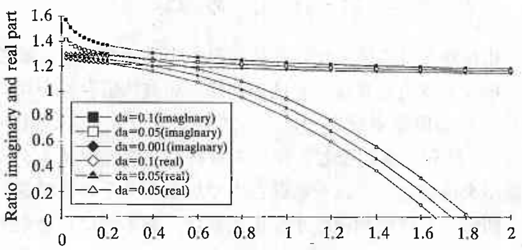

Fig. 4 Variation of ratio of imaginary and real part factor of porous to continuous medium for horizonal motion (f=1) with dimensionless freq.

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