研 究 速 報
UDC 624.041/.047

A Note on Finite Element Synthesis of Structures (Part 8)
——Formulation for Homology Design——
有限要素法による構造シンセシスに関するノート（第8報）
——ホモロジー設計の定式——

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1. Introduction

Sophisticated design methods other than classical optimum design techniques have evolved to meet stringent demands of such new breed of structure as adaptive structures equipped with control mechanism. The sophisticated methods are characterized by that the attention is paid to geometrical quantities such as deformation of the structure in addition to mechanical quantities such as stress in it in order to fulfil delicate functions assigned to the structure, as is exemplified by compliance control design by making use of tailored anisotropy of fiber-reinforced plastic materials[1] and homology degisn discussed by Hoerner for huge radio telescope[2].

Hangai devised a method to obtain structural parameters which can afford prescribed homologous deformation[3]. His method starts from the necessary and sufficient condition for the existence of the solution of the modified stiffness equation with rectangular coefficient matrix. The derivatives of the generalized inverse with respect to design variables should be evaluated in his formulation, and it seems to consume long CPU time to compute the derivatives.

This note presents a new formulation for the homologous deformation. The generalized inverse is employed, but the computation of the derivatives is discarded, giving rise to simple formulation as in case of forming homologous eigenmode of vibration[4].

2. Description of Problem

Equation (1) expresses stiffness equation of an elastic structure under small displacement theory in static problems,

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$$[K]\{U\} = \{F\} \tag{1}$$

where $[K]$ denotes the symmetric, non-singular stiffness matrix, $\{U\}$ the unknown nodal displacement vector of N degrees of freedom, and $\{F\}$ the given nodal force vector which is assumed to be unchanged by design change. The matrix $[K]$ is determined by structural parameters whose value is given with a baseline design. Suppose that J nodal displacements denoted by $\{U_h\}$ are constrained to form homologous deformation by I independent nodal displacements denoted by $\{U_i\}$, that is, $\{U_h\}$ is a certain function of $\{U_i\}$. I is equal to N-J. Subscript h indicates the terms regarding homologous deformation, and i the independent terms hereafter. The constraint of $\{U_h\}$ due to $\{U_i\}$ thus defined is not satisfied in usual for the baseline design when $\{U\}$ is determined simply as the solution of Eq. (1). The problem dealt with in this note is how to obtain the structural parameters that satisfy the constraint to afford homologous deformation.

3. Partition of Stiffness Equation for Homologous Deformation

In this study we deal with the homologous deformation, for which the deformation constraint is expressed in the linear form of Eq. (2).

$$\{U_h\} = [C]\{U_i\} \tag{2}$$

The stiffness equation (1) can be partitioned in the form of Eq. (3),

$$\begin{bmatrix} K_{ii} & K_{ih} \\ K_{hi} & K_{hh} \end{bmatrix} \begin{Bmatrix} U_i \\ U_h \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_h \end{Bmatrix} \tag{3}$$

and separated by means of eliminating the term of $\{U_h\}$ in the form of Eqs. (4) and (5) according to the relationship of Eq. (2) and $\{U\}^T=\{U_h, U_i\}^T$, superscript

T denoting transpose.

$$[K_s]\{U_i\} = \{F_i\} \tag{4}$$
$$[K_r]\{U_i\} = \{F_h\} \tag{5}$$

where $[K_s]$ is $I \times I$ square, asymmetric matrix and $[K_r]$ is $J \times I$ rectangular one affected by the constraint matrix $[C]$ as given below.

$$[K_s] = [K_{ii}] + [K_{ih}][C] \tag{6}$$
$$[K_r] = [K_{hi}] + [K_{hh}][C] \tag{7}$$

If Eq. (5) holds when the displacement vector $\{U_i\}$ obtainted as the solution of Eq. (4) is substituted, it means that homologous deformation is realized by the original structural parameters employed in the baseline design. If not, the parameters should be changed so that Eqs. (4) and (5) hold simultaneously for the same $\{U_i\}$.

4. Governing Equation of Design Variables and Its Solution Using Generalized Inverse

Use is made of M structural parameters p_m to afford homologous deformation, and non-dimensional design variables α_m are assigned to the suructural parameters in the form of Eq. (8). The upper bar indicates quantities regarding the baseline design hereafter.

$$p_m = \overline{p_m}(1 + \alpha_m) \tag{8}$$

The governing equation of α_m is derived based on the first-order approximation method in following. It is rather easy to obtain the first-order sensitivities of the stiffness matrix $[K^I_{sm}]$ and $[K^I_{rm}]$. Then the change of the modified stiffness matrices $[K_s]$ and $[K_r]$ can be approximated in the first-order form of Eqs. (9) and (10). Subscript m stands for the design variable α_m and superscript I for the first-order. It is assumed that the change of the independent nodal displacement vector $\{U_i\}$ can be approximated in the same form of Eq. (11).

$$[K_s] = [\overline{K}_s] + \sum_{m=1}^{M} [K^I_{sm}]\,\alpha_m \tag{9}$$

$$[K_r] = [\overline{K}_r] + \sum_{m=1}^{M} [K^I_{rm}]\,\alpha_m \tag{10}$$

$$[U_i] = [\overline{U}_i] + \sum_{m=1}^{M} [U^I_{im}]\,\alpha_m \tag{11}$$

Substituting the expression of Eqs. (9) and (11) into Eq. (4) and applying the first-order perturbation technique, we have the following expression for any m.

$$\{U^I_{im}\} = [\overline{K}_s]^{-1}([K^I_{sm}]\{\overline{U}_i\}) \tag{12}$$

Equation (5) is rewritten in the form of Eq. (13) when the change of $[K_r]$ in the form of Eq. (10) and that of $\{U_i\}$ of Eq. (11), the sensitivity of which is computed in the form of Eq. (12), are substituted and the second-order term is truncated.

$$\sum_{m=1}^{M} ([K^I_{rm}]\{\overline{U}_i\} + [\overline{K}_r]\{U^I_{im}\})\,\alpha_m = \{F_h\} - [\overline{K}_r]\{\overline{U}_i\} \tag{13}$$

Equation (13) is further summarized as follows,

$$[A]\{\alpha\} = \{b\} \tag{14}$$

where $[A]$ is $J \times M$ rectangular matrix, and $\{\alpha\}$ is the design variable vector. It is easily seen that the design variables to afford homologous deformation can be obtained by means of solving Eq. (14) with rectangular coefficient matrix and overcoming the deficiency caused by the first-order approximation by renewing the baseline design iteratively.

Equation (14) is solved by use of the Moore-Penrose generalized inverse[5]. The necessary and sufficient condition for the solution $\{\alpha\}$ to exist is satisfied in the numerical example described in the following section. Then the design change is determined by the particular solution of Eq. (14) in this study.

When it is taken into account that the displacement vector $\{U_i\}$ determined by Eq. (4) is in proportion to the force vector $\{F_i\}$, Eq. (13) implies that the magnitude of loading is not decisive for the design variables, but the mode of loading is crucial to homologous deformation in the case that the constraint is given in the form of Eq. (2).

5. Numerical Example

Figure 1 illustrates a planar lattice frame simply supported at the four corners. Uniformly distributed load of 0.1 N/mm is applied vertically to the central member which connects the nodes indicated by triangles. All the members are circular in the cross-section, whose diameter is 50 mm for the initial baseline design. Then the central member sags down. The homologous deformation in this numerical example is chosen so that the central member is allowed to be displaced but has to be straight and parallel to the undeformed state.

The frame is modeled by the finite beam elements with

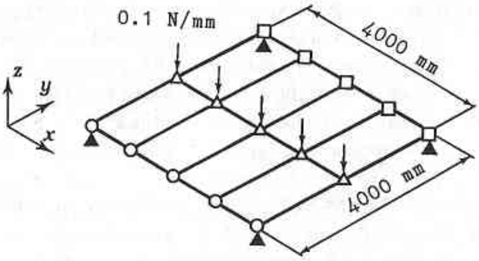

Fig. 1 Planar Lattice Frame

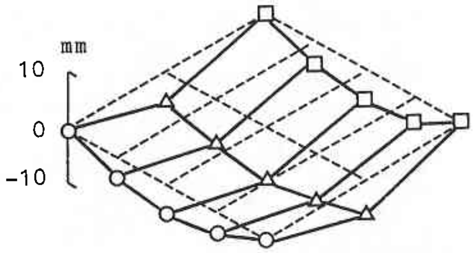

Fig. 2 Deformation of the Initial Baseline Design

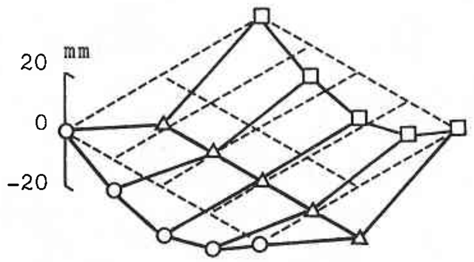

Fig. 3 Homologous Deformation

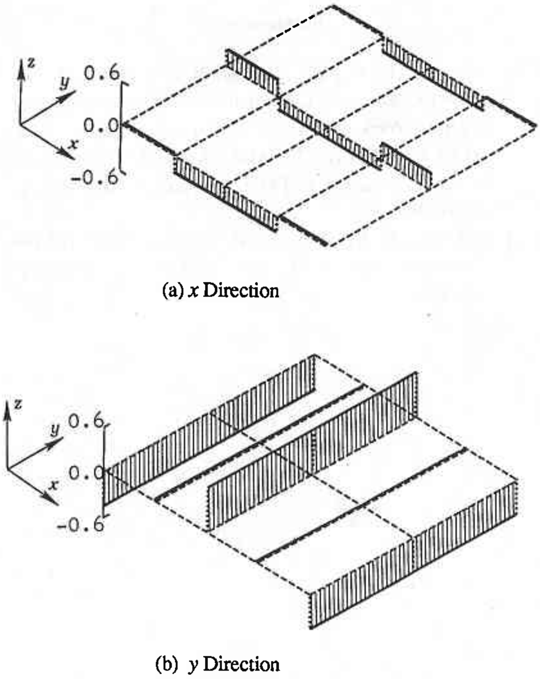

Fig. 4 Required Diameter Change to Afford the Homologous Deformation

twist. Each section of the lattice frame is represented by a finite element, whose Young's modulus and modulus of rigidity are taken equal to 70.0 GPa and 26.9 GPa, respectively. The design variables are assigned to the diameter of all sections. Figure 2 shows the sagging deformation of the initial baseline design and Fig. 3 the homologous deformation of the central member, as is aimed at and judged homologous sufficiently, obtained after the baseline design is renewed five times.

Figure 4 depicts the required diameter change to afford the homologous deformation, the ordinates being taken as the ratio to the initial diameter. The figure shows that the maximum change of the diameter remains smaller than about 0.6, when the diameter of all sections is changed adequately based on the formulation for homologous deformation. It is inferred that a way to afford the same homologous deformation is to design the member with large flexural rigidity. This way is simple, but seems to give rise to inevitable increase of the weight of the lattice frame. It should be noted that the weight is hardly changed by the homology design as shown in Fig. 4. Figure 4 implies that deformation can be controlled by subtle change of structural parameters obtained by the homology design without drastic change of weight.

Concluding Remark

A formulation is presented for homology design, and its validity is examined through the numerical example of a lattice frame. The prescription of homologous deformation and the formulation to determine design change are a key to success of homology design. Choice of the baseline design and design variables also governs the success. Compliance control design will be devised as an application of the proposed formulation. It can be said at present that homology design is likely to succeed when the number of design variables M is larger than the degrees of freedom of displacements under the homologous deformation J.

(Manuscript received, March 4, 1993)

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