研　究　速　報

特　集　7

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A Normal Stress Subgridscale Eddy Viscosity Model in Large Eddy Simulation

ラージ・エディ・シミュレーションにおけるノーマルストレス渦粘性係数モデル

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1. Motivation and objective

The Smagorinsky subgrid-scale eddy viscosity model (SGS-EVM) is commonly used in large eddy simulations (LES) to represent the effects of the unresolved scales on the resolved scales. This model is known to be limited because its constant must be optimized in different flows, and it must be modified with a damping function to account for near-wall effects. The recent dynamic model (Germano *et al.*, 1991) is designed to overcome these limitations, but is computationaly intensive as compared to the traditional SGS-EVM. In a recent study using direct numerical simulation date, Horiuti (1993) has shown that these drawbacks are due mainly to the use of an improper velocity scale in the SGS-EVM. He also proposed the use of the subgrid-scale normal stress as a new velocity scale that was inspired by a high-order anisotropic representation model. The testing of Horiuti (1993), however, was conducted using DNS data from a low Reynolds number channel flow simulation. It was felt that further testing at higher Reynolds numbers, and also using different flows (other than wall-bounded shear flows) are necessary steps needed to establish the validity of the new model. This is the primary motivation of the present study. The objective is to test the new model using DNS databases of high Reynolds number channel and fully developed turbulent mixing layer flows. The use of both channel (wall-bounded) and mixing layer flows is important for the development of accurate LES models because these two flows encompass many characteristic features of complex turbulent flows.

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2. Model assessment

The subgrid-scale stress tensor, τ_{ij}, consists of three terms (Bardina, 1983)

$$\tau_{ij}=L_{ij}+C_{ij}+R_{ij}, \tag{1}$$

$$L_{ij}=\overline{\bar{u}_i\bar{u}_j}-\bar{u}_i\bar{u}_j,\quad C_{ij}=\overline{\bar{u}_iu'j+u_i'\bar{u}_j},\quad R_{ij}=\overline{u_i'u_j'}$$

where $\bar{u}_i$ denote the filtered velocity component and $u_i'=u_i-\bar{u}_i$ denote the SGS component of u_i. L_{ij} is the Leonard term, C_{ij} is the cross term, and R_{ij} is the SGS Reynolds stress. The indices i=1, 2, 3, correspond to the directions x, y, and z, with x the streamwise coordinate ($u_1=u$), y the normal (cross-stream) coordinate ($u_2=v$), and z the spanwise coordinate ($u_3=w$).

The Leonard term in eq. (1) is not modeled but is treated explicitly by applying the filter, while the other two terms (C_{ij} and R_{ij}) need to be modeled. A successful model for the cross term is a model suggested by Bardina (1983) where,

$$C_{ij}=\overline{u_i'}\bar{\bar{u}}_j+\bar{\bar{u}}_i\overline{u_j'}$$

This model has been tested by Bardina (1983) for homogeneous flows and by Horiuti (1989) for the channel flow and was found to be a good model for the cross terms. This model will not be tested further in this work.

For the R_{ij} terms, the eddy viscosity model by Smagorinsky (Smagorinsky, 1963):

$$R_{ij}\sim\frac{2}{3}\bar{E}_G\delta_{ij}-\nu_e\left(\frac{\partial\bar{u}_i}{\partial xj}+\frac{\partial\bar{u}_j}{\partial x_i}\right), \tag{2}$$

$$\nu_e=(Cs\Delta)^2\left[\frac{1}{2}S_{ij}S_{ij}\right]^{1/2},\qquad S_{ij}=\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i},$$

and the Bardina model

$$R_{ij} \sim C(\bar{u}_i - \bar{\bar{u}}_i)(\bar{u}_j - \bar{\bar{u}}_j). \tag{3}$$

are two of several models which are used in LES computations. In these models, Cs and C are model constants, and $E_G = \overline{u'_l u'_l}/2$ and ν_e are respectively the SGS turbulent kinetic energy and SGS eddy viscosity coefficient. Δ is the characteristic SGS length scale whose value is defined as $(\Delta x \Delta y \Delta z)^{1/3}$, $\Delta x, \Delta y$ and Δz are the grid intervals in the x, y, and z directions, respectively. In an eddy viscosity approximation, ν_e is written as the product of a characteristic time scale τ and a velocity scale $E^{1/2}$,

$$\nu_e = C_\nu \tau E \tag{4}$$

where C_ν is a model constant. τ is then expressed as (Horiuti, 1993)

$$\tau = \frac{\overline{E}_G}{\epsilon}, \quad \epsilon = \nu \overline{\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j}} = C_\epsilon \frac{\overline{E}_G^{3/2}}{\Delta}, \tag{5}$$

where ϵ is the dissipation rate of $\overline{E}_G$ and C_ϵ is a model constant. Smagorinsky model assumes that $E = \overline{E}_G$ in (4).

In the present study, we make use of the direct numerical simulation flow fields available at CTR to directly test the various approximations. The fields we consider are homogeneous in two-directions. To compute the large-eddy flow fields, we filter the DNS fields by applying a two dimensional Gaussian filter in the $i=1, 3$ directions. In the major gradient direction ($i=2$), a top-hat filter is applied to the channel flow fields. No filter was applied in this direction ($i=2$) to the mixing layer flow field. This is due to the fact that, occasionally, the doubly filtered ($\bar{\bar{()}}$) grid-scale variables were larger than the singly filtered ones ($\bar{()}$). This is due to the inaccuracy of a top-hat filter in regions where grid spacing is sparse.

The DNS databases we used, were the fully developed incompressible channel flows at Re_τ (Reynolds number based on the wall friction velocity u_τ and the channel height)=360 (Kim *et al.*, 1987) and 790 (Kim, 1992), and the incompressible mixing layer at Re_θ (the Reynolds number based on the momentum thickness and the velocity difference)=2400 (Moser and Rogers, 1992). We started with the low Reynolds number channel flow data as a confidence test. We found that the results obtained in this case are consistent with the previous work of Horiuti (1993) which used a different set of DNS data, but at the same Reynolds number.

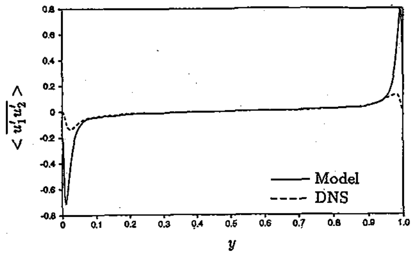

Fig. 1. y-distribution of the SGS-Reynolds shear stress. (*model with* $E = \overline{E}_G$)

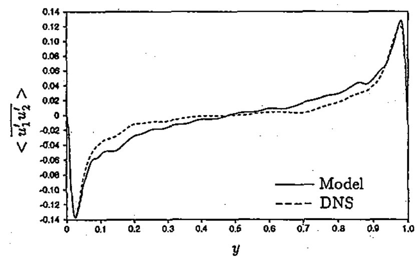

Fig. 2. y-distribution of the SGS-Reynolds shear stress. (*model with* $E = \overline{u'_2 u'_2}$)

The high Reynolds number channel flow field started with 256×193×192 grid points which was filtered to 64×97×48 grid points. The mixing layer flow field started with 512×210×192 and was filtered to 64×210×48 grid points. These LES grid point numbers were chosen so that the turbulent kinetic energy retained in the SGS components is still large. SGS model evalutions were conducted by comparing the y-distribution of the mean values averaged in the x–z plane (denoted by <()>), and also by comparing the y-distribution of the root-mean-square (rms) values of the exact terms with the model predictions. Only the y-distribution of the mean values are shown in the present report because the rms values were found to give similar results.

2.1. A proper eddy viscosity velocity scale

2.1.1 Channel flow

The y-distribution of the SGS Reynolds shear stress $<\overline{u'_1 u'_2}>$ obtained with $E = \overline{E}_G$ and $C_\nu = 0.1$, in (4) is compared with the DNS data in Fig. 1. While the agreement between the model and the term is good in the centeral portion of the channel, the agreement deteriorates near the wall where, the model predicts a very large peak relative to the actual data. This overprediction of the shear stress near the wall when $\overline{E}_G$ is used for E in (4) implies that a damping function is needed to account for the presence of the wall. This near wall overprediction of the stress is similar to the near-wall behavior of one-point closure models (see Rodi & Mansour, 1992). This behavior of one-point closure models is attributed to the rapid reduction of the Reynolds shear stress (as the wall is approached) due to the preferential damping of the normal stress (Launder, 1987, and Durbin, 1992). Horiuti (1993) reasoned that the same wall damping effects should hold true for the SGS field. Indeed, when the SGS normal stress $\overline{u_2' u_2'}$ is used for E (with C_ν=0.23, see Fig. 2), the model agrees well with SGS Reynolds shear stress near the wall without introducing an additional damping function. But, the model is less effective as compared to using the total energy in the core region of the channel. The main deficiency in the core region is attributed to excessive grid stretching in the y-direction due to the mapping used in conjunction with Chebyshev expansions. In an actual LES computation finite differences can be used in the y ditrection and a more uniform grid and therefore a more isotropic SGS energy distribution can be expected in this case.

The anisotropy effects of the grid can be evidenced by the y-distribution profile of the 'flatness parameter' A (Lumley, 1978) averaged in the x–z plane. In this case A is defined as

$$A=[1-\frac{9}{8}\{A_2-A_3\}], A_2=a_{ij}a_{ij},\ A_3=a_{ij}a_{jk}a_{ki}, \quad (6)$$

$$a_{ij}=\{\overline{u_i'u_j'}-\frac{1}{3}\delta_{ij}\overline{u_k'u_k'}\}/k,\ k=\frac{1}{2}\overline{u_k'u_k'}$$

We find (see Fig. 3) that in the core region of the channel, A~0.35, which is much smaller than the expected A=1 when the small scale turbulence is isotropic. In the region around y=0.1, A peaks around A~0.5, and then gradually decreases to 0.35 at the channel center. The y-distribution of A for the unfiltered DNS data does not show this overshoot and is close to A=1 around the centerline. The grid spacing in the central region of channel seems to be too sparse, therefore a considerable anisotropy exists in the SGS turbulence fluctuations. In fact, when the SGS-EVM model with $E=\overline{u_2'u_2'}$ in (4) was used in an actual LES channel flow calculations at high Reynolds number (Re_τ=1280) (Horiuti, 1993), good agreement with experimental data was found. The present comparisons for the high Re channel flow confirm the conclusions of Horiuti (1993) based on the low Re channel flow fields.

For the records, the y-distribution of the conventional Van Driest damping function ((1−exp(−y^+/26.0)) (normalized with value of A at the channel center) is included in Fig. 3. It should be noted that the 'flatness parameter' A has a similar distribution across the channel as the Van Driest function, suggesting that A may be used as an alternative method to damp the eddy viscosity near the wall (Horiuti, 1992).

2.1.2 Mixing layer

The y-distribution of $\overline{u_1'u_2'}$ obtained using $E=\overline{E}_G$ (C_ν=0.20) and $E=\overline{u_2'u_2'}$ (C_ν=0.26) in (4) are compared with the DNS data in Fig. 4 and 5, respectively. Both cases show good agreement of the model with the DNS data, indicating that the two models are equivalent in this case. It should be noted that the optimized C_ν values obtained for the $\overline{u_2'u_2'}$ model in the channel flow at lower Re (0.22), at high Re (0.23), and in the mixing layer (0.26) are very close. This implies that the model constant of the SGS normal stress model is rather universal independent on the type of flow, whereas the optimized C_ν values for the $E=\overline{E}_G$ model were respectively 0.11, 0.10, 0.20. This is further indication of the potential strength of the model.

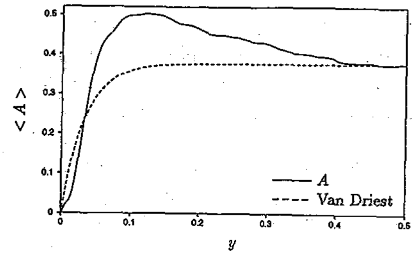

Fig. 3. y-distribution of the flatness parameter A and the Van Driest function

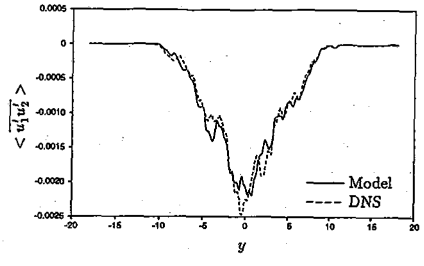

Fig. 4. y-distribution of the SGS-Reynolds shear stress. (*model with* $E=\overline{E}_G$)

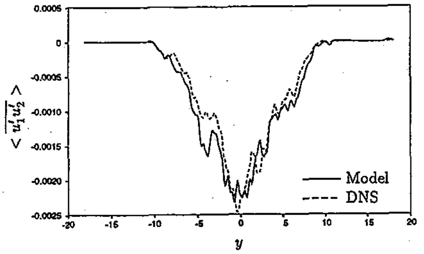

Fig. 5. *y*-distribution of the SGS-Reynolds shear stress. *(model with* $E=\overline{u_2'u_2'}$*)*

2.2. *Approximation method of the SGS turbulent energy*

To effectively use the model advocated in the previous sections a model for the normal stresses is needed. We can either carry equations for the normal stress or estimate the energy in the subgrid-scales from the energy in the large scales. In testing the scale-similarity model of Bardina (1983), Horiuti (1993) found good correlation between the model and the data. The model reads,

$$\overline{E}_G \sim C_K(\overline{u}_l-\overline{\overline{u}}_l)(\overline{\overline{u}}-\overline{\overline{u}}_l)/2, \qquad (8)$$

$$\overline{u_2'u_2'}=C_N(\overline{u}_2-\overline{\overline{u}}_2)(\overline{\overline{u}}_2-\overline{\overline{u}}_2),$$

where a constant different from unity was needed. It was pointed out that the optimized model constants C_K and C_N were not equal to unity because the scale-similarity model provides a partial estimate of the whole SGS fluctuations which resides in the vicinity of the cutoff-wave number ($=\pi/\Delta$). The poor performance of the model when these coefficients are set equal to unity can be evidenced by the fact that in this case the SGS flatness parameter A becomes identically zero (purely two-dimentional state). We have optimized C_k/C_N for the low-*Re*, the high-*Re* channel, and the mixing layer flows and found 7.0/12.0, 7.0/9.0, and 9.0/12.0 to be representative values for these flows. We note that they are slightly (but tolerably) sensitive to the type of flow field, and that they are generally close to each other.

3. Summary

An ultimate goal of the present study is to develop an SGS model which yields good predictions of turbulent flows in a complex geometry. Of particular interest is the flow over a backward-facing step. In this case, while the flow is bounded by the walls, the internal mixing layer present in this flow plays a major role in setting the turblence levels. In this work, a proper velocity scale for the SGS-EVM viscosity was determined for the fully developed channel and the mixing layer flows. In the channel, a clear advantage over more conventional treatments was shown by using the normal stress. It was also shown that the SGS normal stress is equally useful as the total SGS turbulent energy for modeling in mixing layers. The model constant in the normal stress model was found to be fairly independent on the type of flow. The generalized normal stress model will be tested in the backward-facing step flow in both *'a priori'* and *'a posteriori'* manner in the future. Although the Bardina model constants C_K and C_N in Eq. (8) are rather consistent in three different flow fields, some variance was noticed. An attempt to determine these coefficients more accurately using the Dynamic scale model approach is currently underway.

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