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An Asymptotic Solution for the Diffraction Problem of a Vertical Circle Cylinder in Short Ocean Waves

海洋波の直立円柱による散乱問題の短波長域の漸近解

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Abstract

An asymptotic solution for the diffraction problem of a circle cylinder in short waves is derived mathematically. Physically it represents the perfect reflection by the cylinder. Calculated results are in good agreement with those of the exact solution except in the 'shadowed area'.

I. Introduction

In the past decades, a lot of works have been done to obtain asymptotic solutions of radiation waves due to the oscillation of floating body. Among these works, Leppington [1], Davis [2], Takagi [3], Hermans [4], Bao and Kinoshita [5] derive asymptotic solutions for three-dimensional radiation problem by different methods. However, it seems that less effort has been made to the diffraction waves. In the present paper the diffraction problem of a circle cylinder in short plane waves is considered. An asymptotic solution is derived mathematically. It turns out that the first order approximation represents the perfectly reflected waves by the cylinder. It gives a quite good approximation in most directions of propagation except the shadowed area abehind the cylinder where no reflected waves can reach.

II. Diffraction Problem of a Circle Cylinder

When a train of plane waves hits a circle cylinder sitting at the sea bottom and piercing through water surface, the solution to the linearized diffraction problem is well known. [6] If the incident wave potential is given by

$$\phi_i=F(kz)e^{ikr\cos\theta}$$

$$=F(kz)\sum_{n=0}^{\infty}\varepsilon_n i^n J_n(kr)\cos n\theta$$

the diffraction potential is expressed as

$$\phi_d=-F(kz)\sum_{n=0}^{\infty}\varepsilon_n i^n\frac{J_n'(ka)}{H_n^{(1)\prime}(ka)}H_n^{(1)}(kr)\cos n\theta \qquad (1)$$

where

$F(kz)=\dfrac{\cosh k(z+h)}{\cosh kh}$ k=wave number; h=water depth; $(r,\ \theta)$=horizontal polar coordinates; z=vertical coordinate positive upwards; $H_n^{(1)}$=Hankel function of the first kind and n-th order; J_n=Bessel function of the first and n-th order; $\varepsilon_n=1$ as $n=0$, or 2 as $n>0$

superscript prime indicates derivative with respect to the argument.

The diffraction potential can be written as a series expansion in Hankel function $H_n^{(1)}(kr)$, i.e.

$$\phi_d=F(kz)\sum_{n=0}^{\infty}i^n a_n H_n^{(1)}(kr) \qquad (2a)$$

where

$$a_n=-\varepsilon_n\frac{J_n'(ka)}{H_n^{(1)\prime}(ka)}\cos n\theta \qquad (2b)$$

By means of the properties of Bessel function and Hankel function, it can be shown that

$$\overline{\lim_{n\to\infty}}\sqrt{n!|a|}=0$$

Then according to the theorem given by Von Jasef Meixner, [7] the diffraction potential has an asymptotic expansion when $kr\gg1$ as follows

$$\phi_d\sim F(kz)\sqrt{\frac{2}{\pi kr}}e^{i(kr-\frac{\pi}{4})}\sum_{m=0}^{\infty}\frac{b_m}{(-2ikr)^m} \qquad (3a)$$

The coefficient b_m is given by

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$$b_m=\sum_{n=0}^{\infty} =\frac{1}{m!}\frac{\Gamma(n+m+\frac{1}{2})}{\Gamma(n-m+\frac{1}{2})}a_n \qquad (3b)$$

Now, We are going to seek an asymptotic solution when the incident wave is very short or equivalently, when $ka \gg 1$. This can be done by obtaining an asymptotic expression of b_m for very large value of ka. To the first order of approximation, only the first coefficient b_o is considered in the present work. The other coefficients can similarly be obtained to get higher order approximation. Written explicitly, b_o is given by

$$b_o=\sum_{n=0}^{\infty} -\frac{\varepsilon_n J_n'(ka)}{H_n^{(1)\prime}(ka)}\cos n\theta \qquad (4)$$

This series can be related to a controur integral on the complex plane of ν i.e.

$$I=\frac{1}{i\pi}\int_c f(\nu)d\nu \qquad (5)$$

The integrand is defined as

$$f(\nu)=\frac{\pi J_\nu'(ka)}{H_\nu^{(1)\prime}(ka)}\frac{\cos\nu(\pi-\theta)}{\sin\nu\pi} \qquad (6)$$

The integral contour C consists of the imaginary axis with a small half circle around the origin and a half circle wit large radius R on the right part of the ν-plane (see Fig. 1). The integral is equal to the sum of residues at poles enclosed by the contour according to the Cauchy theorem. There are two kinds of poles to be taken into account. One comes from zeros of sin $\nu\pi$, i.e. $\nu=0, 1, 2, \ldots$, while the other one is zeros of $H_\nu^{(1)\prime}(ka)$. At each pole of the first kind, we have

$$\frac{1}{i\pi}\int_{C_n} f(\nu)d\nu=-2\mathrm{Res}[F(\nu)]_{\nu=n} \qquad (7)$$

$$=-2\frac{J_n'(ka)}{H_n'(ka)}\cos n\theta$$

$$(n=0,\ 1,\ 2,\ \ldots)$$

where C_n is a clockwise small circle around the real integer n.

If the radius R of the large circle tends to infinity so that all the poles on the real axis will be enclosed, the sum of the residues at these poles, together with the contribution from the small half circle around the origin, yields the series required for calculating b_o. Meanwhile the integrand $f(\nu)$ will vanish uniformly as $R\to\infty$ which means no contribution will be made by the integral along the path of large half circle Therefore the coefficient b_o is given by

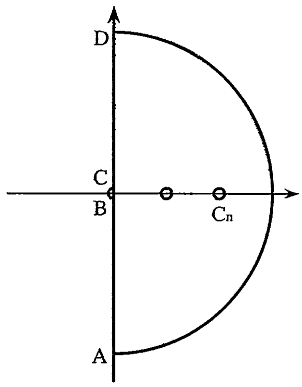

Fig. 1. Contour of the intégal on the ν-plane

$$b_o=2\sum \mathrm{Res}[f(\nu)]_{\nu=\nu_\nu} + \frac{1}{i\pi}\int_A^D f(\nu)d_\nu \qquad (8)$$

Here, ν_r represents solutions of the equation

$$H_\nu^{(1)\prime}(ka)=0$$

The contribution of ν_r to b_o is considered as a higher order of approximation. In the present work, we will concentrate on the work of obtaining an asymptotic value of integral along imaginary axis which is explained as a principle value of Cauchy type. The integral can be done by the method of steepest descent. To do so we first split the integrand into two parts, i.e.

$$f(\nu)=\frac{\pi J_\nu'(ka)}{2H_\nu^{(1)\prime}(ka)}\cdot\frac{e^{i\nu(\pi-\theta)}+e^{-i\nu(\pi-\theta)}}{\sin\nu\pi} \qquad (9a)$$

$$=f_1(\nu)+f_2(\nu)$$

where

$$f_{1,2}(\nu)=\frac{\pi}{2}\cdot\frac{J_\nu'(ka)}{H_\nu^{(1)\prime}(ka)\sin\nu\pi}\cdot e^{\pm i\nu(\pi-\theta)} \qquad (9b)$$

with upper (lower) sign corresponding to $f_1(f_2)$.

Substituting $-\nu$ for ν in $f_2(\nu)$, and by means of formulae

$$H_{-\nu}^{(1)\prime}(ka)=e^{i\nu\pi}H_\nu^{(1)}(ka) \qquad (10a)$$

$$i\sin\nu\pi H_\nu^{(2)\prime}(ka)=e^{i\nu\pi}J_\nu'(ka)-J_{-\nu}'(ka) \qquad (10b)$$

the integral can be manipulated to the following form, i.e.

$$\frac{1}{i\pi}\int_A^D f(\nu)d\nu=\frac{1}{2}\int_A^D \frac{e^{-i\nu\theta}H_\nu^{(2)\prime}(ka)}{H_\nu^{(1)\prime}(ka)}d\nu \tag{11}$$

To make this integral easier to be evaluated, the asymptotic expansions of Hankel functions are to be used. Referring to Waston [8], we have, to the first order of approximation,

$$H_\nu(z)\sim\frac{\exp\{\pm[\nu(\tanh\gamma-\gamma)-i\frac{\pi}{4}]\}}{\sqrt{-\frac{1}{2}i\pi\nu\tanh\gamma}} \tag{12}$$

where ν=zcosh γ is assumed and upper (lower) sign corresponds to the hankel function of the first (second) kind. Taking derivative with respect to the argument z, the following expansion can be obtained

$$H_\nu'(z)\sim\pm\sqrt{\frac{\sinh 2\gamma}{-i\nu\pi}}\exp\{\pm[\nu(\tanh\gamma-\gamma)-i\frac{\pi}{4}]\} \tag{13}$$

Along the integral path of AD, i.e. the imaginary axis, ν is a pure imaginary variable while the argument ka is a real number, therefore the integral dummy variable ν is replaced by γ such that $\nu=ka\cosh\gamma$, where γ runs from $-\infty+i\frac{\pi}{2}$ to $+\infty+i\frac{\pi}{2}$ corresponding to ν varying from $-i\infty$ to $+i\infty$. The integral is then transferred to

$$\frac{1}{i\pi}\int_A^D f(\nu)d\nu\sim\frac{1}{2}\int_{-\infty+i\frac{\pi}{2}}^{\infty+i\frac{\pi}{2}} \exp\{-2ka[\sinh\gamma-(\gamma-\frac{i\theta}{2})\cosh\gamma]\}ka\sinh\gamma d\gamma \tag{14}$$

According to the method of steepest descent, main contributions come from the saddle points which is the zeros of the derivative of exponential index. Here the index is given by

$$g(\gamma)=-[\sinh\gamma-(\gamma-\frac{\theta}{2})\cosh\gamma] \tag{15a}$$

Equating its derivative to zero yields following equation:

$$g'(\gamma)=2(\gamma-\frac{i\theta}{2})\sinh\gamma=0 \tag{15b}$$

One solution is located at $\gamma=\frac{i\theta}{2}$

By some tedious derivation, the integral and the coefficient b_o as well is finally given by

$$b_o\sim\frac{1}{i\pi}\int_A^D f(\nu)d\nu \sim\frac{1}{2}\sqrt{\pi ka\sin\frac{\theta}{2}}\exp(-i2ka\sin\frac{\theta}{2}+i\frac{\pi}{4}) \tag{16}$$

Physically this represents the wave amplitude at far field caused by the perfect reflection of the incident wave by the circle cylinder. It gives a quite good approximation in the most of the region. Nevertheless, in the direction of incident wave, i.e., $\theta=0$, it fails to give a reasonable solution. The reason for this is that as $\theta=0$, the zero of $g'(\gamma)$, i.e. $\gamma=0$, is a zero of order two and the method of steepest descent gives a result to the order of $(ka)^{1/3}$ which is neglected in the present work. The region around $\theta=0$ is so-called "shadowed area" behind the cylinder and no reflected wave can reach it. This explains the failure of present work physically.

The diffraction potential is asymptotically expressed as

$$\phi_d\sim b_o F(kz)\sqrt{\frac{2}{\pi kr}}e^{i(kr-i\frac{\pi}{4})} \tag{17}$$

with b_o given in the above expression.

If the incident angle of the incoming wave is β, then the θ in the above should be replaced by θ-β.

III. Numerical Results and Discussion

To verify the asymptotic solution for the diffraction problem of circle cylinder, numerically calculated results are compared with those of exact solution for $\theta=180°$, 100° and 60° shown in Fig. 2 through 5. For $\theta=180°$, i.e. opposite to the direction of income wave, the asymptotic solution is in good agreement with the exact solution even when ka (nondimensional wave number) and kS (distance form the cylinder) are not very large. For the other direction, the asymptotic solution is a good approximation as ka and kr are large enough. It should be pointed out that as θ becomes smaller, i.e. close to the direction of incoming wave, the values of ka and kS to make the asymptotic solution valid, become larger.

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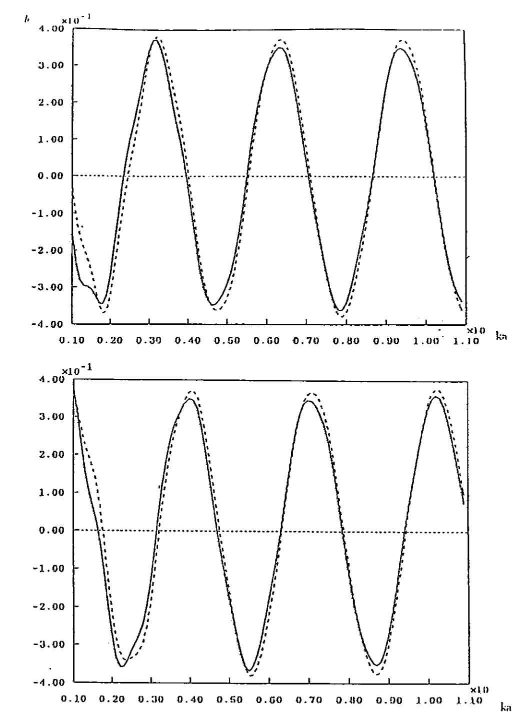

Fig. 2. Real and imaginary part of diffracted wave by a circle cylinder at θ=180deg and r=4a. — asymptotic solution; --- exact solution.

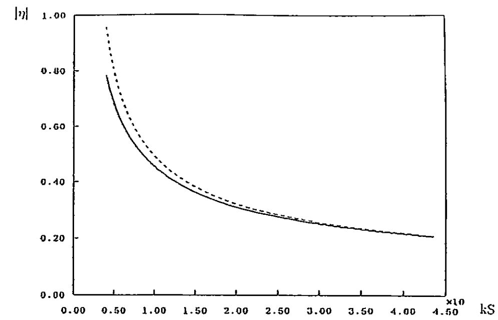

Fig. 3. Amplitude of diffracted wave with wave number ka=4 at θ=180 deg. kS is nondimensional distance from the cylinder center. — asymptotic solution; --- exact solution.

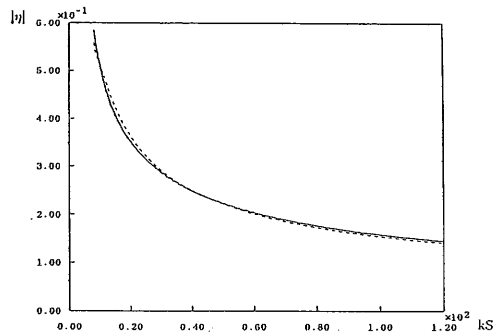

Fig. 4. Amplitude of diffracted wave with wave number ka=8 at θ=100 deg. kS is nondimensional distance from the cylinder center. — asymptotic solution; --- exact solution.

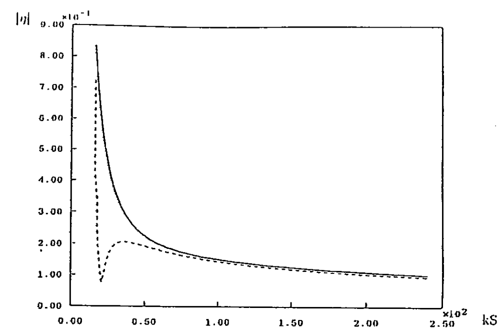

Fig. 5. Amplitude of diffracted wave with wave number ka=16 at θ=60 deg. kS is nondimensional distance from the cylinder center. — asymptotic solution; --- exact solution.

References

[1] Leppington, F.G.: On the radiation and scattering of short surface waves (part 3), J. Fluid Mech., Vol. 59, No. 1 (1973) pp. 129–146.

[2] Davis, A.M.J.: On the short surface waves due to an oscillating, partially immersed body, M. Fluid Mech., Vol. 75 (1976) pp. 791–807.

[3] Takagi, M. and Saito, K.: On the description of non-harmonic problems in the frequency domain (2nd report), J. Kansai Soc. N.A., Japan, No. 184 (1973) pp. 85–93.

[4] Hermans, A. J.: A perturbation method for the radiation of short surface waves in three dimensions, J. Eng. Math., Vol. 7, No. 1 (1973) pp. 75–84.

[5] Bao, W. and Kinoshita, T.: Asymptotic solution of wave-radiating damping at high frequency, Report of Ocean constructure Seminar (1991).

[6] Mc Camy, R.C. and Fuchs, R.A.: Wave forces on a pile, a diffraction theory, Tech. Memo., U.S. Army Board, U.S. Army Corp. of Eng., No. 69 (1954).

[7] Meixner, Von J.: Uber das asymptotisch verhalten von funktionen, die durch reihen nach zylinderfunkfionen dargestellt werden können, Math. Nach, Vol. 3 (1949), pp. 9–13.

[8] Watson, G.N.: A treatise on the theory of Bessel functions, Cambridge Univ. Press (1922).