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Asymptotic Solutions for the Interaction Problems of Multiple Bodies in Short Ocean Waves

海洋波の多列干渉問題の短波長域の漸近解

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Abstract

The asymptotic solutions for the diffraction and radiation problem of a vertical circle cylinder in short waves are applied to the interaction problem of multiple cylinders to obtain an estimation of hydrodynamic forces acting on cylinders. The interactions are represented by additional waves emitting from each cylinder towards the others. The hydrodynamic forces are evaluated by formulae at far field.

I. Introduction

Recently, with the development of large offshore structure supported by a number of cylinders, more attention has been paid to the problem of estimating hydrodynamic wave forces acting on multiple bodies. Much effort has been made to assess the effects of interactions among these cylinders. Kinoshita [1] deals with the problem in two-dimensional domain by a matrix method. Ohkusu [2] uses an interation method to account the interactive waves among multiple cylinders. Simon [3] approximates the cylindrical interactive waves by plane waves with appropriate amplitude. McIver and Evans [4] use the same approximation as Simon's to a higher order of expansion. Kagemoto and Yue [5] solves the problem by a matrix method which takes both the cylindrical and evanescent interactive waves into account.

However, when the oscillatory frequency is large or equivalently when the wave is short, to obtain reasonable accuracy, it is usually time consuming.

It is the purpose of this paper to present an approximate method to estimate hydrodynamic forces acting on multiple cylinders in short waves. To this end, asymptotic solutions for radiation and diffraction roblems of a single cylinder in short waves are used to represent the interaction waves among multiple cylinders. Since Simon's [3] plane wave approximation gives a satisfactory result, follwoing his method, the above cylindrical waves are decomposed to plane waves with appropriate amplitude. After solving the interactive problem, the hydrodynamic forces are estimated by wave amplitudes at far field.

Some numerical examples are also presented to verify the medhod. the results are fairly good if the cylinders are not arranged in line with the incoming waves.

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II. Interactions among multiple bodies

The interactions between bodies can be represented by some additional waves emitting frm one body to another. If the interactive effects from the j-th body to the i-th body are to be considered, these effects can be represented by a wave with wave amplitude C_{ij} coming from the j-th body toward the i-th body. In radiation problem, this wave consists of a radiation wave due to the forced oscillation of j-th body and diffraction waves due to the interaction between j-th body and others, i.e. the diffraction of waves emitting from other bodies by j-th body. In diffraction problem, instead of the radiation wave which no longer exists, diffraction wave by j-th body due to environmental incident waves should be included. Neglecting the effects of evanescent waves, the interaction waves are cylindrical waves which are expressed in the local coordinate system of j-th body. They have to be transferred to the coordinates corresponding to i-th body. At mean time, to make calculation easier, the cylindrical waves are decomposed to a plane wave with appropriate wave amplitude. The fact that C_{ij} is the sum of all those waves from j-th body to i-th body gives an equation relating C_{ij} to C_{jl} $(1 \neq j)$. Equations arising from every body yield a linear system to be solved for known wave amplitude C_{ij}. As long as C_{ij} is obtained, each body can be considered as isolated with these additional incident waves.

From the previous work, the radiation potential due to the forced oscillation of j-th body in otherwise calm water is given asymptotically by

$$\phi_{\mathrm{mj}} \sim \mathrm{A}_{\mathrm{mj}}(\theta_{\mathrm{j}}) H_o(kr_j) F(kz) \tag{1}$$

when the oscillating frequency tends to large value. As shown in the previous paper the diffraction potential due to an incident plane wave with unit wave amplitude and incident angle β has an asymptotic form as

$$\phi_{\mathrm{dj}} \sim \mathrm{A}_{\mathrm{dj}}(\theta_{\mathrm{j}}, \beta) H_o(kr_j) F(kz) \tag{2}$$

In egn. 1 and 2, (γ_j, θ_j) is local polar coordinates of j-th body in horizontal plane; m=1 to 6 indicates motion modes in conventional manner;

$$F(kz) = \begin{cases} e^{kz} & \text{for water with infinite depth;} \\ \dfrac{\cos hk(z+\lambda)}{\cos hkh} & \text{for water with depth } h; \end{cases}$$

$$k = \begin{cases} \dfrac{\omega^2}{g} & h \to \infty \\ \dfrac{\omega^2}{g \tanh kh} & h < \infty \end{cases} \quad \text{wave number;}$$

ω=wave frequency and g=acceleration of gravity; H_o is Hankel function of first kind and zeroth order. The time factor $e^{-i\omega t}$ is omitted. Referring to the sketch

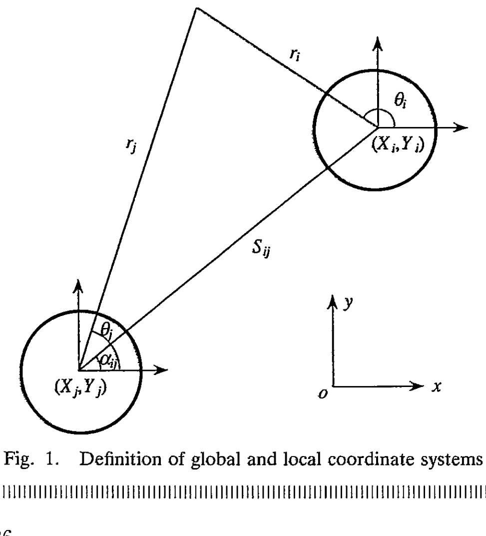

Fig. 1. Definition of global and local coordinate systems

for the definition of local coordinate systems and a global one (see Fig. 1), we have following geometric relations:

$$r_j = [r_i^2 + S_{ij}^2 - 2S_{ij}\cos(\pi - \theta_i + \alpha_{ij})]^{\frac{1}{2}} \tag{3}$$

where (S_{ij}, α_{ij}) is the distance and direction of i-th body referring to j-th body. By means of Grab addition theorem [6] for cylindrical functions,

$$H_o(kr_j) = \sum_{n=-\infty}^{\infty} H_n(kS_{ij}) J_n(kr_i) e^{in(\pi - \theta_i + \alpha_{ij})} \tag{4}$$

where H_n is the first kind Hankel function of n-th order. Since we are going to represent waves near i-th body, it can be expected that

$$r_i < S_{ij}$$

which is required by the validity of the theorem. At mean time, $i^n H_n(kS_{ij})$ is approximated by $H_0(kS_{ij})$ with an error of order $O\left(\frac{1}{kS_{ij}}\right)$, the radiation potential can be written as

$$\phi_{mij} \sim A_{mj}(\alpha_{ij}) H_o(kS_{ij}) F(kz) \sum_{n=-\infty}^{\infty} i^n J_n(kr_i) e^{-in(\theta_i - \alpha_{ij})} \tag{5}$$

which is the potential of a plane wave with wave amplitude

$$a_{ij} = A_{mj}(\alpha_{ij}) H_o(kS_{ij}) \tag{6}$$

and propagating in the direction of α_{ij} referring to the i-th body. The diffraction potential can be transferred and then approximated in a similar way, i.e.

$$\phi_{dij} \sim A_{dj}(\alpha_{ij}, \beta) H_o(kS_{ij}) F(kz) \sum_{n=-\infty}^{\infty} i^n J_n(kS_{ij}) e^{-in(\theta_i - \alpha_{ij})} \tag{7}$$

If this diffraction by j-th body is caused by waves emitting from other body, say l-th body, the incident angle will be α_{jl} referring to j-th body. Therefore the wave amplitude of approximated plane wave is given by

$$a_{ijl} = A_{dj}(\alpha_{ij}, \alpha_{jl}) H_o(kS_{ij}) \tag{8}$$

On the other hand, if the diffraction is the respond to the environmental incident wave whose potential is written as

$$\phi_I = F(kz)e^{ik(x\cos\beta + y\sin\beta)} \quad (9)$$

in the global system, to obtain appropriate wave amplitude, this incident wave potential is to be expressed in the j-th local system as follows

$$\phi_{Ij} = F(kz)e^{ikr_j\cos(\theta_j-\beta)+ik(X_j\cos\beta+Y_j\sin\beta)} \quad (10)$$

$$= I_j(\beta)F(kz)e^{ikr_j\cos(\theta_j-\beta)}$$

where

$$I_j(\beta) = e^{ik(X_j\cos\beta+Y_j\sin\beta)} \quad (10a)$$

with $(X_j,\ Y_j)$ being the coordinates of the origin of j-th local system in a global one.

After decomposing to a plane wave the diffracted wave amplitude due to this incident wave is given by

$$\tilde{a}_{ij} = I_j(\beta)A_{dj}(\alpha_{dj},\ \beta)H_o(kS_{ij}) \quad (11)$$

From the above expressions, it can be seen that these wave amplitudes of approximated plane waves are nothing else but the waves emitting from j-th body evaluated at the origin of i-th local system.

The wave amplitude C_{ij} at i-th body caused by j-th body's interaction is the sum of all the waves emitting from j-th body propagating toward i-th body. Therefore we have, for the radiation problem,

$$C_{ij} = a_{ij} + \sum_{\substack{l=1 \\ l\neq j}}^{L} C_{jl}a_{ijl} \quad (12)$$

where L=total number of bodies. Each body will receive interactive waves from the other L-1 bodies, so it has L-1 such kind of equation. Totally, for L bodies, there are $L(L\text{-}1)$ equations which form a linear system for $L(L\text{-}1)$ unknowns C_{ij} and can readily be solved. For diffraction problem, the only difference is that a_{ij} is replaced by $\tilde{a}_{ij}$.

After solving these equations for C_{ij}, each body can be considered as isolated with additional incident waves. The radiation wave potential of i-th body accounting for the interaction can then be written asymptotically as

$$\phi_m^{(i)} \sim \Big[A_{mi}(\theta_i) + \sum_{\substack{j=1 \\ j\neq i}}^{L} C_{ij}A_{dj}(\theta_i,\ \alpha_{ij})\Big]H_o(kr_i)F(kz) \quad (13)$$

Similarly, the diffraction potential of i-th body due to incident wave ϕ_I with interaction taken into account is given by

$$\phi_d^{(i)} \sim \Big[I_i(\beta)A_{di}(\theta_i,\ \beta) + \sum_{\substack{j=1 \\ j\neq i}}^{L} C_{ij}A_{dj}(\theta_i,\ \alpha_{ij})\Big] \cdot H_o(kr_i)f(kz) \quad (14)$$

The quantities in the brackets in the above expressions are wave amplitude at far fild and are denoted as $a_m^{(i)}(\theta_i)$ and $a_d^{(i)}(\theta_i,\ \beta)$ respectively for radiation and diffraction waves. With these wave amplitudes we can evaluate wave exciting forces, wave damping coefficients and wave drifting forces as follows.

Wave exciting force:

$$\frac{F_m^{(i)}}{\rho g\zeta_a ah} = \frac{-4iCC_g}{gh}a_m^{(1)}(\beta+\pi) \quad (15)$$

Wave damping coefficient:

$$\frac{\lambda_{mn}}{\rho\omega a^2 h} = \frac{2CC_g}{\pi gh}\mathrm{Re}\int_0^{2\pi} a_m^{(i)}(\theta)a_n^{(i)*}(\theta)d\theta \quad (16)$$

Wave drifting force:

$$\frac{F_m^{(i)}}{\rho g\zeta_a^2 h} = \frac{C_g}{\pi khC}\int_0^{2\pi}(\cos\beta - n_m)|a_d^{(i)}(\theta,\ \beta)|^2 d\theta \quad (17)$$

where

$C=\dfrac{\omega}{k}$	wave celerity;
$C_g=\dfrac{d\omega}{dk}$	group velocity;
ζ_a	is amplitude of incident wave;
a	is a typical dimensional scale of the body;
h	is water depth (or draft in the case of infinitely deep water);
n_m	components of unit normal direction in m-th direction.

III. Numerical Results and Discussion

Some numerical calculations are carried out to verify the present method. Wave exciting force acting on a group of two cylinders in oblique wave (β=45°) is shown

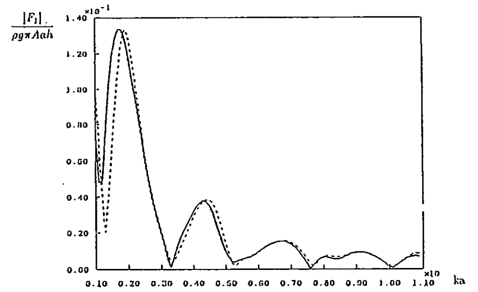

Fig. 2. Wave exciting force in surge direction acting on a group of two cylinders. Distance between cylinder centers S=4a and wave incident angle $\beta=45deg.$ — asymptotic solution; --- exact solution.

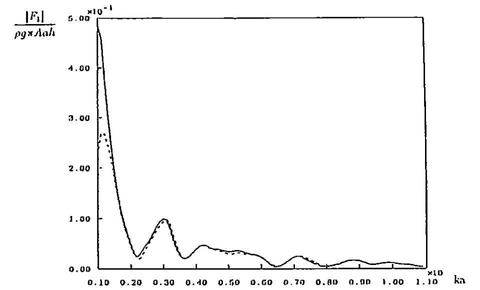

Fig. 3a. Wave exciting force in surge direction acting on a group of three cylinders. Distance between cylinder centers S=5a and wave incident angle $\beta=0deg.$ — asymptotic solution; --- exact solution.

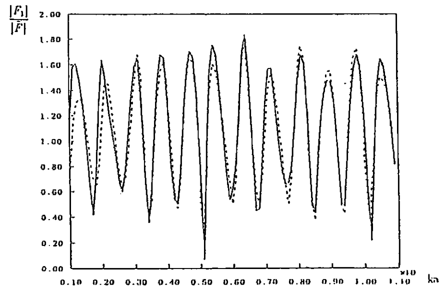

Fig. 3b. Wave exciting force in surge direction acting on the first cylinder in the above case. The force is normalized by the force experienced by an isolated cylinder, i.e.

$$\bar{F}=\frac{4\,\rho g\pi A\tanh kh}{k^2 H_1'(ka)}$$

— asymptotic solution; --- exact solution.

in Fig. 2 and ompared with the results by Simon's method. Presented in Fig. 3 are results of an array of three cylinders which consist of a triangle.

From these figures, it can be seen the present results are in fair agreement with those by Simon's method.

From the comparison giving above, we can see that the present method gives a good approximation to the estimation of hydrocynamic forces acting on a group of cylinders if the cylinders are not arranged in line with the direction of incoming wave. The accuracy largely depends on the behavior of the asymptotic solution for the single body. Since in the present work, it fails to give a reasonable approximation in the shadowed area, i.e. $\theta=0$, the method can not be applied to the cylinder array arranged in line with the incoming wave. Therefore it is required to obtain an asymptotic solution valid in all directions so that this method can be applied to more general cases. This will be our work of next step.

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