研 究 速 報
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A Note on Finite Element Synthesis of Structures (Part 7)
——Formation of Homologous Vibration Mode——
有限要素法による構造シンセシスに関するノート（第7報）
——ホモロガス振動モードの形成——

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1. Introduction

Hoerner stated that "the deformation of a structure shall be called homologous, if a given geometrical relation holds, for a given number of structural points, before, during and after the deformation"[1]. The concept of the homologous deformation was applied by Morimoto et al. to structural design of huge radio telescope, the mirror surface of which is to be finished in parabolic form at any tilt angle[2]. A methodology was proposed by Hangai et al. to form homologous deformation based on existence condition of the solution of the formulation by use of generalized inverse[3]. The concept of homologous deformation is considered applicable to dynamic problems as well as static problems. It can be devised to make use of homologous vibration mode in order to control or mitigate the effect of vibration, for instance.

This note proposes a formulation of how to form homologous vibration mode based on generalized inverse technique and finite element discretization. Linear and undamped eigenvalue problem is dealt with. The sensitivity analysis of the eigenpair derived from asymmetric matrices is exploited to approximate the behavior change. The validity of the proposed method is examined through formation of the homologous mode in problem of the out-of-plane vibration of a lattice structure.

2. Description of problem

Suppose that a baseline structure vibrates with the modes governed by the following eigenvalue problem (1),

$$([K]-\lambda[M])\{\phi\}=\{0\} \tag{1}$$

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where $[K]$, $[M]$, λ and $\{\phi\}$ denote the stiffness matrix, mass matrix, eigenvalue and eigenvector, respectively. We partition the N components of $\{\phi\}$ into I independent ones denoted by $\{\phi_i\}$ and J dependent ones by $\{\phi_d\}$, which is governed by the following equation (2) through $J\times I$ matrix $[C]$, which expresses homologous constraint. N is equal to $I+J$.

$$\{\phi_d\}=[C]\{\phi_i\} \tag{2}$$

When such a constraint is imposed on the eigenvector, the original eigenvalue problem cannot hold anymore in general. The problem is how to change the baseline structure to satisfy the homologous constraint.

3. Formulation Based on Generalized Inverse

The eigenvalue problem is partitioned in the form of Eq. (3) according to the partition of the eigenvector components, while attention is paid to an eigenmode.

$$\left(\left[\begin{array}{c:c} K_{ii} & K_{id} \\ \hdashline K_{di} & K_{dd} \end{array}\right]-\lambda\left[\begin{array}{c:c} M_{ii} & M_{id} \\ \hdashline M_{di} & M_{dd} \end{array}\right]\right)\left\{\begin{array}{c} \phi_i \\ \hdashline \phi_d \end{array}\right\}=\left\{\begin{array}{c} 0 \\ \hdashline 0 \end{array}\right\} \tag{3}$$

The upper part and lower one of the above equation can be rewritten in the separate form as given below, respectively,

$$([K_s]-\lambda[M_s])\{\phi_i\}=\{0\} \tag{4}$$
$$([K_r]-\lambda[M_r])\{\phi_i\}=\{0\} \tag{5}$$

where $[K_s]$ is $I\times I$ square, asymmetric matrix, and $[K_r]$ is $J\times I$ rectangular matrix defined as follows.

$$[K_s]=[K_{ii}]+[K_{id}][C] \tag{6}$$

$$[K_r]=[K_{di}]+[K_{dd}][C] \tag{7}$$

Similar formulae are applied to $[M_s]$ and $[M_r]$. Equation (4) constitutes modified eigenvalue problem, and yields new eigenpair of λ and $\{\phi_i\}$, which take different values from those governed by Eq. (1). No constraint is provided for the new eigenpair, because $\{\phi_i\}$ is independent. On the other hand, Eq. (5) does not hold when the eigenpair thus obtained is substituted together with the original matrices of $[K_r]$ and $[M_r]$. This means that $[K_r]$ and $[M_r]$ have to be changed so as to make Eq. (5) hold.

We assign M design variables α_m for judiciously chosen parameters P_m to change the baseline structure to form homologous vibration mode as given below,

$$P_m=\bar{P}_m(1+\alpha_m) \tag{8}$$

where the upper bar means current value of the parameters. The rates of change of the eigenpair λ_m and $\{\phi_{im}\}$ can be calculated with respect to the design variables and based on Eq. (4), when the first-order approximation is applied to the change of $[K_s]$, $[M_s]$ and so forth in the following form.

$$[K_s]=[\bar{K}_s]+\sum_{m=1}^{M}[K_{sm}]\alpha_m \tag{9}$$

$$[M_s]=[\bar{M}_s]+\sum_{m=1}^{M}[M_{sm}]\alpha_m \tag{10}$$

It is necessary to calculate both the right and left eigenvectors for the evaluation of $\{\phi_{im}\}$, because $[K_s]$ and $[M_s]$ are asymmetric. In doing so, we employ the method proposed by Nelson[4]. The change of the eigenpair is approximated in the following form of Taylor series expansion when the rates of change of the eigenpair are calculated.

$$\lambda=\bar{\lambda}+\sum_{m=1}^{M}\lambda_m\alpha_m \tag{11}$$

$$\{\phi_i\}=\{\bar{\phi}_i\}+\sum_{m=1}^{M}\{\phi_{im}\}\alpha_m \tag{12}$$

The matrices $[K_r]$ and $[M_r]$ are changed also by the design change. The first-order approximation is employed for the change as follows.

$$[K_r]=[\bar{K}_r]+\sum_{m=1}^{M}[K_{rm}]\alpha_m \tag{13}$$

$$[M_r]=[\bar{M}_r]+\sum_{m=1}^{M}[M_{rm}]\alpha_m \tag{14}$$

Equation (5), which ie required to hold always, is rewritten as Eq. (15) by means of substituting the first-order approximate formulae given above,

$$\begin{aligned}&\sum_{m=1}^{M}(\{[K_{rm}]-\bar{\lambda}[M_{rm}]-\lambda_m[\bar{M}_r]\}\{\bar{\phi}_i\}\\&\quad+\{[\bar{K}_r]-\bar{\lambda}[\bar{M}_r]\}\{\phi_{im}\})\\&\quad=\{[\bar{K}_r]-\bar{\lambda}[\bar{M}_r]\}\{\bar{\phi}_i\}\end{aligned} \tag{15}$$

and is summarized further in the following form,

$$[A]\{\alpha_m\}=\{b\} \tag{16}$$

where $[A]$ is $J\times M$ rectangular matrix. Equation (16) is the governing equation for the design variables. The eigenvalue analysis and sensitivity analysis are to be carried out by current use of the renewed parameters at each renewal. The coefficient matrix $[A]$ of Eq. (16) is rectangular. The unknown design variables can be determined by use of the generalized inverse $[A]^-$. We employ the particular solution of Eq. (16) calculated by the Moore-Penrose generalized inverse[5] as follows.

$$\{\alpha_m\}=[A]^-\{b\} \tag{17}$$

The design variables thus determined are affected by deficient first-order approximation so that the design change has to be renewed to overcome the deficiency until the right hand of Eq. (16) is made equal to nil vector.

4. Numerical Example

Figure 1 illustrates a lattice structure, flat in the x-y plane, supported simply at four points marked by black triangle. The structure is discretized by beam elements, a member being represented by an element, to analyse its out-of-plane vibration. The cross-section of all members is circular. The first mode shape of a base-line design is found to be warped in saddle-shape as shown in Fig. 2. The section diameter of the baseline design is 0.05 m and Young's modulus and Poisson's ratio are 70 GPa and 0.3, respectively. The first eigenvalue is 826.8.

Suppose that we set a homologous vibration mode that the deflection of the nodes arrayed in the y direction is the same. Such a constraint can be expressed easily by Eq.

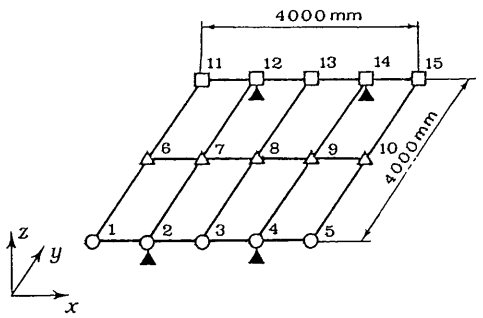

Fig. 1. Finite element model of lattice structure

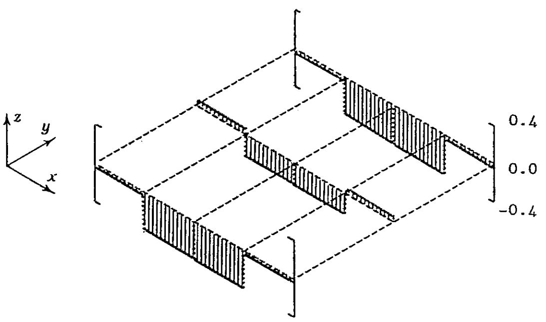

Fig. 4. Distribution of section diameter at the third renewal (x direction)

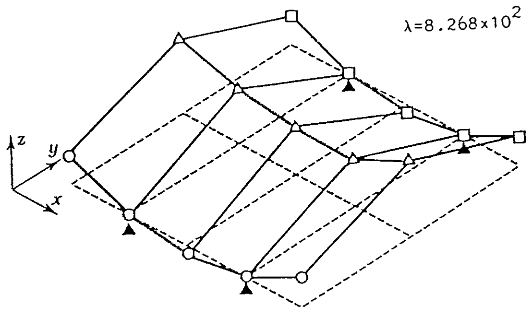

Fig. 2. Warped mode shape of baseline design

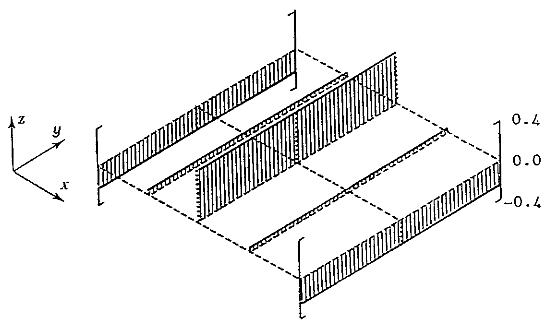

Fig. 5. Distribution of section diameter at the third renewal (y direction)

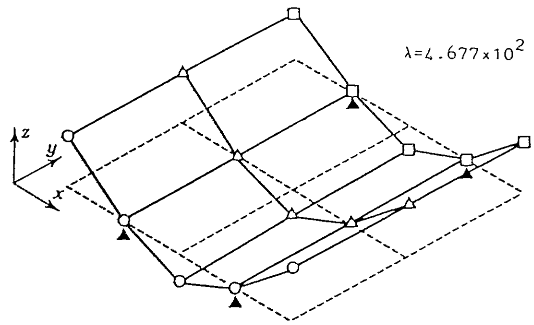

Fig. 3. Flattened mode shape at the third renewal

(2). Figure 3 shows the mode shape obtained at the third renewal, in which the design variables are assigned to all the section diameters. The mode shape is flattened, partly homologous to the flat lattice plane, as is aimed at. The increase or decrease of the section diameters after the design change with respect to the initial values are given in Figs. 4 and 5. The eigenvalue is reduced to 467.7 by the design change, however. When we make all the members arrayed in the y direction so stiff that any deformation does not take place, the hogging pattern of the warped mode shape is anticipated to be flattened in the y direction. Different from the anticipation, the proposed formulation reveals that the flat pattern can be formed by means of increasing the stiffness of the members in the y direction a little and decreasing the stiffness in the x direction also a little.

5. Concluding Remarks

The proposed formulation is proved to succeed in forming a homologous vibration mode for a structure. The existence and uniqueness of homologous vibration mode is still left unknown for various structures. Validity of a particular choice of design variables is also to be examined in future. (Manuscript received, June 9, 1992)

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