研 究 速 報

UDC 666.982.4 : 678.017

Properties of Fiber Reinforced Plastic Rods for Prestressing Tendons of Concrete (4)
——Evaluation of CFRP Rods Optimum Strength Using Monte Carlo Simulation——
プレストレストコンクリート用FRP緊張材の特性(4)
——モンテカルロ法を用いたCFRPロッドの理論強度の推定——

Hosam HODHOD* and Taketo UOMOTO*
ホッサム ホドホド・魚 本 健 人

1. INTRODUCTION

As the FRP rods are coming into greater use in the field of concrete structures, the evaluation of their optimum strength becomes more important. It leads, through comparison with experimental results, to the determination of the effects of different parameters on their strength. Such parameters includes manufacturing defects, gripping methods.. etc.. The prediction of the strength was carried out using the rule of mixture that does not specify the characteristic value of fibers strength or at which fiber length it should be measured. Another approach is the bundle theory that specifies the significant fiber length as the transfer length, but assumes the matrix so rigid that it can distribute the loads of the cut fibers equally to all the sound fibers in the damaged section. On the contrary, in practical cases, the matrix is not a rigid medium, and usually possesses very small young's modules compared with that of the fibers. This limits the effect of the cut fibers to only one layer of the surrounding fibers. Hence, the effect of the relative positions of the fibers in the section becomes a governing factor for the damage accumulation of the fibers, and consequently composite strength.

In this research, the aforementioned effect is studied for the case of CFRP rods. A Monte Carlo work is done for the evaluation of rod strength and compared with the mean values available in the literature and with the predictions from the bundle theory.

*University of Tokyo, IIS, 5th. Division.

2. MAIN APPROACH

The fibers are distributed uniformly in a section that is considered the most critical section along the rod. Each fiber is assigned, at random, a strength and modulus according to the experimental results of the tensile testing of the actual fibers[3]. A strain increment is applied to the section and the resulting stresses in the fibers are calculated according to their moduli. The resulting stresses are compared with fibers strengths. Fibers with stresses higher than their strengths are coded as cut fibers and their loads are distributed to the surrounding, first layer of, fibers. Stresses after distribution are again compared with fibers strengths and the process is repeated till the stability of the number of damaged fibers is reached. Then, additional strain increment is applied and the above procedure is repeated. Rod failure is attained when fibers failure extends in unstable manner, and the critical percentage of failed fibers is taken, for certainty, 90%. The summation of the loads in the sound fibers, at the last strain increment, divided by the total section area is the strength. This is based on the assumption of neglected matrix stress. A computer program for the above process was developed and its flowchart is shown in Fig. 1.

3. CALCULATION OF NUMBER OF FIBERS INSIDE THE ROD

In order to have the fibers uniformly distributed in the rod section, equal spacings between them should be assured. The geometry of a hexagon suggests that

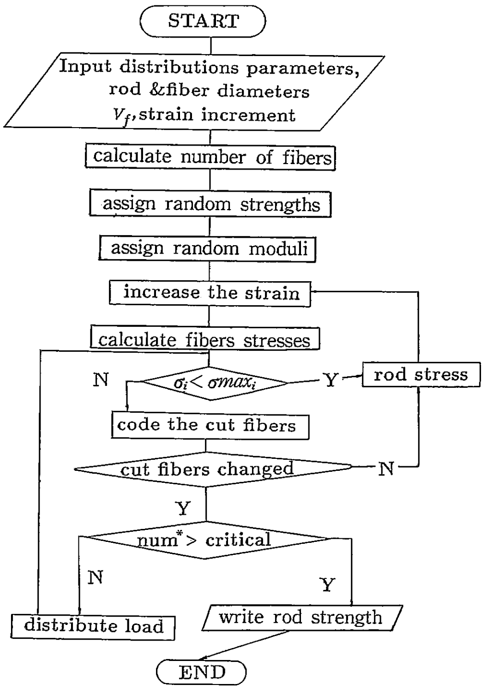

Fig. 1 Flow chart of Simulation Program
(*)num: number of cut fibers

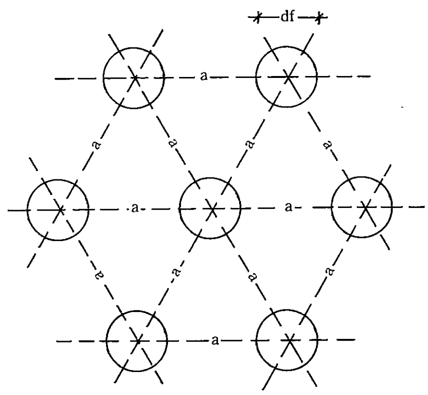

Fig. 2 Hexagon, the basic Unit in Uniform fiber Array

the arrangements of the fibers at its corners and center will fulfill this requirement, see Fig. 2. Repetition of this hexagon will give the fibers array in the rod. It also provides useful mean for the description of the relative positions of the fibers in the array.

As the basic unit of this array is the hexagon, then the fiber volume fraction (V_f) becomes the tatio of the area of the fibers in the hexagon to the area of the hexagon

$$V_f = \frac{3\,\frac{\pi df^2}{4}}{\frac{3\sqrt{3}}{2}a^2} \qquad (1)$$

where d_f=fiber diameter

a=hexagon side length (fibers spacing)

(The number of fibers belonging to each hexagon is $\frac{1}{3} * 6 + 1$)

Hence, fibers spacing a is expressed as

$$a = \sqrt{\frac{\pi}{2\sqrt{3}\,V_f}}\;d_f \qquad (2)$$

In order to avoid fibers overlap, the geometric condition $a \leq d_f$ should be fulfilled. However, substituting $a = d_f$ in the above equation yields $V_f = 0.907$ which is larger than the practical maximum of V_f. In this research, the value of V_f is taken as 0.66 for later use in assessing the actual performance of the available rods. However, the application to any volume fraction is straight forward.

4. FIBERS STRENGTH AND MODULUS DISTRIBUTIONS

The distributions used for fibers strength and modulus are obtained from experiments conducted on individual PAN type carbon fibers each of length 25 mm[3]. When a fiber is cut at a certain section, it needs a transfer length l_c to retrieve its effectiveness in strengthening the section, and along this length its load is transmitted to the surrounding fibers. Hence, the strength distribution used should be that of the fibers of length=l_c. The length l_c can be calculated from the following formula

$$\frac{\sigma_f}{\tau} = \frac{2l_c}{d_f} \qquad (3)$$

where σ_f=normal stress in the fiber

τ=matrix shear strength

d_f=fiber diameter

The above equation yields l_c=0.350mm, using appropriate values for its parameters. The strength distribution of fibers with 0.35mm long can be predicted using the concept of weakest link described by Weibull[1]. Weibull distribution in the form

$$f(x) = bmx^{m-l}e^{-bx^m} \qquad (4)$$

considers the effect of the length in the parameter b where for two sets of specimens with lengths l_1 and l_2, the parameters b_1 and b_2 are related by $\frac{l_1}{l_2} = \frac{b_1}{b_2}$, and

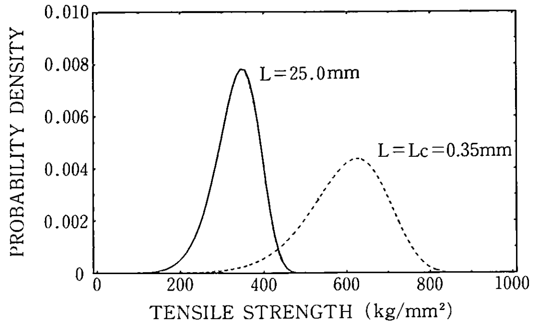

Fig. 3 Strength Distributions for Carbon Fibers at Standard Testing length (25mm) and at Transfer Length (0.35mm)

Weibull modulus m is a material constant independent of the length. Thus, the strength distribution at length 0.35mm can be obtained from that of 25mm long fibers. Both distributions are shown in Fig. 3. The modulus distribution is taken the same as that of 25mm long fibers as it is not affected by the length.

5. RANDOM NUMBERS GENERATION

In order to assign, randomly, each fiber a strength and modulus according to the experimental distributions obtained[3], uniform random numbers need be generated first. Uniform random numbers in the range (0→1) were generated using a computer routine[2]. The long cycle and randomness of these numbers were assured through their plot in 2D space. Fig. 4 shows a photograph for the generated numbers and, for comparison, another one for a short cycle random numbers. The randomstrengths and moduli can, then, be obtained by using the formula

$$X = F^{-1}(y) \tag{5}$$

where X = required strength or modulus

y = generated uniform random number

F = cumulative probability function according to which the value of X is to be generated

The employed distribution for both strength and modulus was Weibull distribution as its inverse, F^{-1}, can be easily obtained. Besides, it represents, conceptually, the variation of the strength.

6. RESULTS AND COMMENTS

The approach described above was applied to many sets of random numbers in order to get the strength

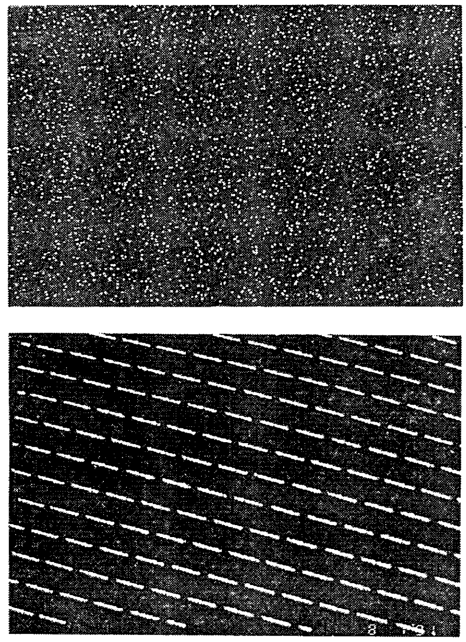

Fig. 4 upper) Generated Uniform Random Numbers. lower) Uniform Randon Numbers with Short Cycle.

distribution of the rods. The fiber volume fraction was taken as 0.66. The effect of the number of the fibers in the rod, for the same fiber volume fraction, was studied. This corresponds to the effect of changing the rod diameter. The simulation was made for five different fibers samples (number of fibers). For each, at least 100 runs were executed and the strength distributions were obtained. Those distributions are shown in Fig. 5 where the mean value and the standard deviation of the strength decreases with the increase of the number of fibers inside the rod. For the biggest three samples, the distribution seems to become insensitive to the number of fibers; where at this point the number of fibers constitute a sample representative to the fibers population. The stable mean strength (217kg/mm^2) agrees well with that obtained from the rule of mixture, employing the mean strength of the fibers at length 25mm, which is 220kg/mm^2. It is well accepted that the rule of mixture, as applied above, represents the actual strength of carbon fibers composites (in standard specimens)[4].

When the bundle theory is applied[1], the formula for rods mean strength becomes

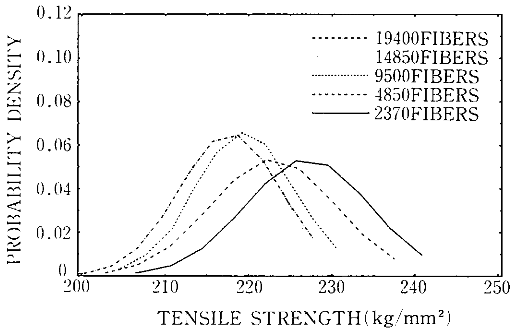

Fig. 5 Simulated Strength Distributions for CFRP Rods with Different Fibers Content

$$\bar{\sigma}=V_f(b_{lc}l_cme)^{-\frac{1}{m}} \qquad (6)$$

where m = Weibull modulus

b_{lc} = Weibull distribution constant for fibers of length $= l_c$

e = natural logatithm base ($\simeq 2.7$)

Equation (6) yields for carbon fibers the value of 281 kg/mm² that is about 30% higher than the actual one. This overestimate is a direct consequence of the assumption of rigid matrix assumed in this theory.

The randomness of the failure evolution in the simulated rods was assured by developing graphics program that takes the output of the simulation program at each loading step and plots it on the screen to show the damage accumulation in the section. The output of this program is shown in Fig. 6 for four different cases, and it is obvious that the failure evolutes from different positions in the section which confirms the randomness of the results.

7. CONCLUSIONS

1. The optimum strength of FRP rods can be estimated by simulating the accumulation of fibers failures within rod section. This needs knowledge of the fibers strength and modulus distributions in order to carry out a Monte Carlo work.
2. The above concept was applied for the case of CFRP rods and showed efficiency as it agreed well with the known strength of such composites.
3. The simulation approach is an improvement for the bundle theory that assumes the matrix as rigid medium capable of distributing the loads of the cut fibers equally to all the sound ones. Here, the load can be distributed only to the first layer of fibers surrounding the cut fibers. This is because of the big diffrence between the moduli of the fibers and the matrix that prevents the cut fibers loads from further traveling.
4. The same work is intended to be done for other types of FRP rods too.

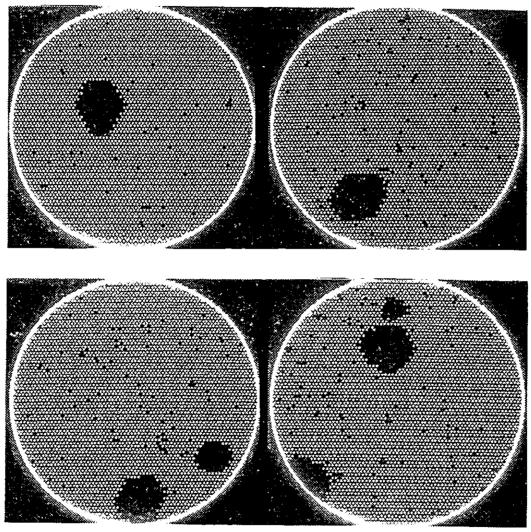

Fig. 6 Randomness of Failure Evolution in the Simulated Rods.
(Photographs Taken at Instability Stage)

(Manuscript received, November 27, 1991)

REFERENCES

1) Jayatilaka, A.S., "Fracture of Engineering Brittle Materials", Applied Science Publishers Ltd., London, 1979.
2) Joohnhang Ahn, "Introduction to Computer Analysis", Lectures Notes, University of Tokyo, 1991.
3) Hodhod, H. and Uomoto, T., "Experimental Model for the Ideal Tensile Failure of FRP Rods", JCI Proceedeings, Vol. 13 No. 1, 1991.
4) Watt, W and Perov, B.V., "Handbook of Composites", Vol. 1 "Strong Fibers", Elsevier Science Publishers B.V., Netherland, 1985.