

Fig. 1 Decision tree for one crisp variable problem

ity.

In the present case, defining the utilities boils down to defining one utility function of the variable R for each alternative. The meaning of the utility function is rather subjective and it can be defined by asking some experts' advice. Examples of such utility functions $u_m(r)$ and $u_c(r)$ are shown in Fig. 2. For alternative 1 (maintain the supply), a high value of R gets a low rating because the situation is very dangerous, whereas for alternative 2 (cut the supply) a low value of R gets a low rating because it indicates that the supply has been interrupted although it was not necessary.

Once these two utility functions are defined, the problem is easy to solve. We can judge which of $u_c(R_0)$ and $u_m(R_0)$ is higher and choose the corresponding alternative (see Fig. 2).

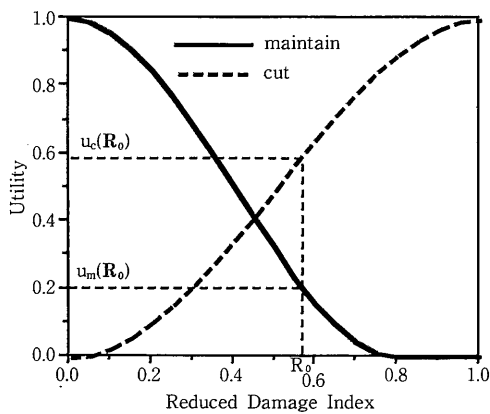


Fig. 2 Solution of one-variable crisp problem using utility functions

DECISION ANALYSIS FOR A FUZZY PROBLEM WITH ONE VARIABLE

Now, consider the case when the state of the system is still represented by a single variable R but instead of being known precisely, R is known through a fuzzy set R_0 . A fuzzy set (Dubois and Prade, 1980) is a set with no sharp distinction between membership and non-membership. It is represented by the membership function $\mu_{R_0}(r) \in [0,1]$. μ_{R_0} can be interpreted as the possibility distribution of R.

The concept of the utility function can still be applied without any change. But to compute the utility of each alternative, the information contained in the fuzzy set R_0 will have to be used. Instead of obtaining two real numbers $u_c(R_0)$ and $u_m(R_0)$ to characterize the two alternatives, two fuzzy sets U_c and U_m will be obtained by using the following formula (Jain, 1976):

$$\mu_{u_\alpha}(u) = \mu_{R_0}(u_\alpha^{-1}(u)) \quad \alpha = c, m \quad (1)$$

where u_α^{-1} represents the inverse function of u_α . The meaning of Eq. 1 is that the possibility that $U=u$ is the possibility of a damage ratio r such as $u_\alpha(r) = u$.

Even though U_c and U_m have been obtained, the problem is not completely solved because which is higher of the two fuzzy values still has to be determined. This is not a very easy task as exemplified by Fig. 3. The possible values for u (i.e., the values for which $\mu_{u_\alpha}(u) \neq 0$) should be as high as possible and at the same time their membership should be as high as possible. The following choice procedure is based on Jain's (1976) but with an additional normalization of the membership functions. The "value" $v_\alpha (\alpha = c, m)$ of each alternative is computed as follows:

$$v_\alpha = \text{Max}_u \min(\mu_{u_\alpha}(u), \mu_M(u)) \quad (2)$$

and

$$\mu_M(u) = \mu_{\text{max}}(u/u_{\text{max}}) \quad (3)$$

where μ_{max} is the highest value for μ_{U_m} and μ_{U_c} , and u_{max} is the highest possible utility i.e.,

$$u_{\text{max}} = \max\{u \mid \mu_{U_m}(u) \neq 0 \text{ or } \mu_{U_c}(u) \neq 0\} \quad (4)$$

The meaning of Eq. 2 is that the fuzzy set U_α is compared with the fuzzy set M which represents the ideal combination between high possible values for u and a high membership. The final decision is made by choosing the alternative having the highest value v_α .

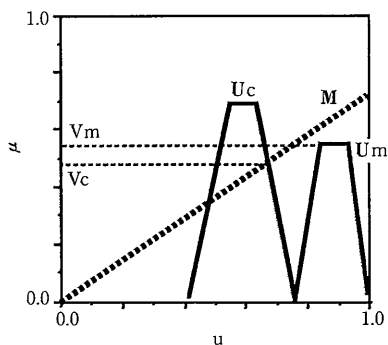


Fig. 3 Choice between two fuzzy utilities

DECISION ANALYSIS FOR A CRISP PROBLEM WITH TWO VARIABLES

Now, consider a crisp problem with two variables. Suppose that the damage state of the system is described by two variables R_b and R_p and that they are known as $R_b = R_b^0$ and $R_p = R_p^0$.

The concept of utility function can still be used, but this time, two-variable utility functions $u_c(r_b, r_p)$ and $u_m(r_b, r_p)$ must be defined (see Fig. 4). The utility functions become more difficult to construct because it is difficult to assess a situation simultaneously in terms of r_b and r_p . If our preference structure is assumed to be additive, variables can be separated and the utility function can be written as (Keeney and Raiffa, 1976):

$$u_\alpha(r_b, r_p) = \lambda_\alpha^b \hat{u}_\alpha^b(r_b) + \lambda_\alpha^p \hat{u}_\alpha^p(r_p) \quad (5)$$

with

$$\lambda_\alpha^b + \lambda_\alpha^p = 1, \quad \alpha = c, m \quad (6)$$

where \hat{u}_α^b and \hat{u}_α^p are one-variable functions similar to u_α in Fig. 2.

Once the utility functions are defined, the procedure to find the largest utility is very simple. $u_c(R_b^0, R_p^0)$ and $u_m(R_b^0, R_p^0)$ should be compared and the alternative corresponding to the highest value should be selected as depicted in Fig. 4.

DECISION ANALYSIS FOR A FUZZY PROBLEM WITH TWO VARIABLES

Consider a system whose state is defined by two variables R_b and R_p that are known through fuzzy sets R_b^0 and R_p^0 . As before, two-variable utility functions $u_c(r_b, r_p)$ and $u_m(r_b, r_p)$ are defined for the two

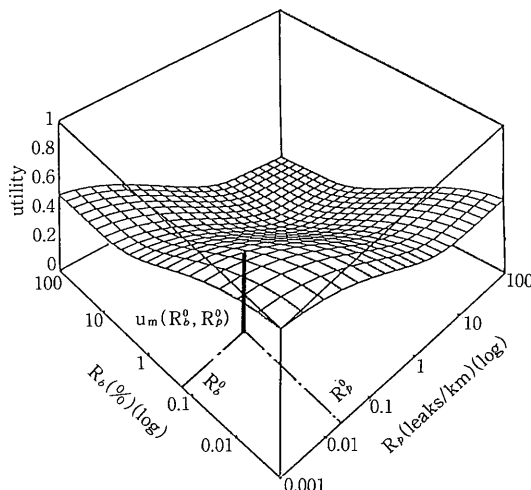


Fig. 4 Solution of two-variable crisp problem using utility functions (example of the alternative "maintain the supply")

alternatives. Two fuzzy utilities U_c and U_m for the two alternatives will be obtained by using the information contained in the fuzzy sets R_b^0 and R_p^0 . Equation 1, proposed by Jain, is easily extended to the two-variable case ($\alpha = c, m$):

$$\mu_{u_\alpha}(u) = \text{Max}_{(r_b, r_p) \in u_\alpha^{-1}(u)} \min(\mu_{R_b}(r_b), \mu_{R_p}(r_p)) \quad (7)$$

where $u_\alpha^{-1}(u) = \{(r_b, r_p) \mid u_\alpha(r_b, r_p) = u\}$ is the constant value curve in the damage space corresponding to the value u of the utility. Figure 5 illustrates the procedure corresponding to Eq. 7.

Once U_c and U_m are obtained, the same procedure as in Fig. 3 is used to determine the better alternative.

NUMERICAL EXAMPLE

The present decision procedure was applied to a real case: the Chibaken-Toho-Oki earthquake of December 17, 1987 with magnitude 6.7. For a given control block in the supply area (Fig. 6), the two damage indices R_b and R_p shown in Fig. 7 were evaluated by fuzzy reasoning using recorded ground motion characteristics (Cret et al., 1991). The estimated damage indices were non-negligible.

The utility function shown in Fig. 4 as well as a complementary one for the other alternative were employed. The obtained fuzzy utilities for the two alternatives and the resulting choice are shown in

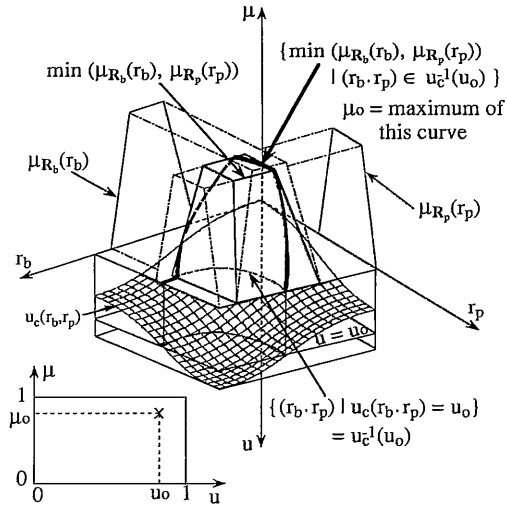


Fig. 5 Schematic view to obtain the fuzzy utility membership for an alternative

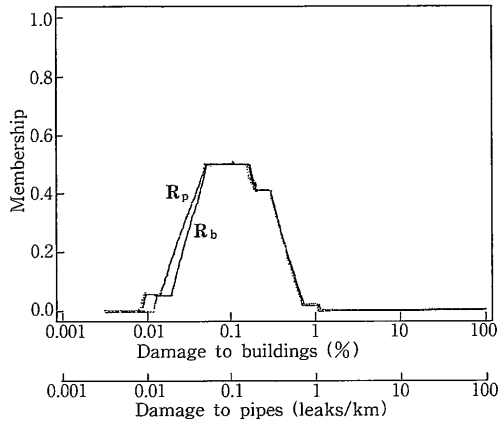


Fig. 7 Fuzzy damage indices for the considered block

Fig. 8. These fuzzy utilities have an intersection but it is rather easy to find the better utility. The values v_a for the two alternatives reflect it. The decision in that case is non trivial but clearly in favor of alternative 1 (maintain the supply), which is consistent with the decision that was actually made.

CONCLUSION

When a destructive earthquake strikes a large city with an extensive gas supply system, it is necessary to shut off the gas supply to avoid secondary dis-

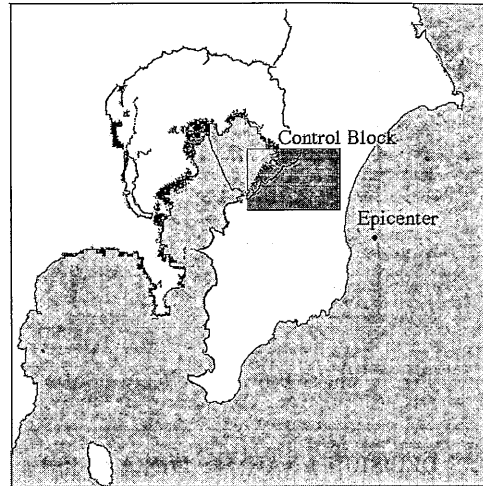


Fig. 6 Position of the considered control block in south Kanto area

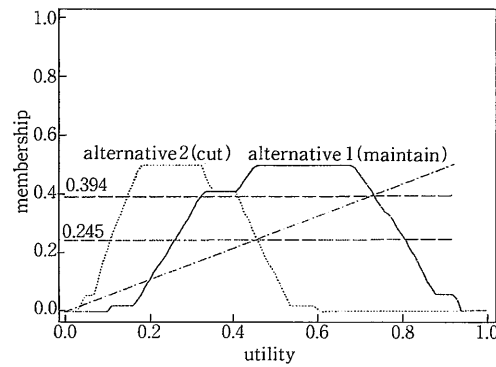


Fig. 8 Results of fuzzy decision analysis

ters. From that perspective, a system to estimate earthquake damage in the supply area from measured ground motion characteristics has been devised. But due to the uncertainty involved in the estimation, the results are obtained as fuzzy numbers. This paper shows how to use this fuzzy information when deciding whether to cut or maintain the gas supply.

A decision analysis including several fuzzy variables must be performed. The methods are explained from one crisp variable case to two fuzzy variable case. A multi-variate utility function is defined for each alternative which the decision maker is confronted with. The fuzzy values of the predicted damage indices are then combined with the utility

functions and yield one fuzzy utility for each alternative. The best alternative is the one giving the highest fuzzy utility. To illustrate and assess the method, it was applied to a real earthquake event as a numerical example.

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