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Modelling of Non-Linear of Stress-Strain Relations of Soils and Rocks ——Part 2, New Equation——

土と岩の非線型応力〜ひずみ関係のモデル化 ——その2，新しい式——

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INTRODUCTION

In the first part (Tatsuoka and Shibuya, 1991), it is shown that the original hyperbolic (OH) relation and other forms of hyperbolic relation with constant parameters of strength and initial stiffness can simulate only a part of a given observed stress-strain relation, or even cannot utterly. It is expected, therefore, that the hyperbolic relation with the coefficients of correction c_1 and c_2 either or both of which is (are) a function of strain x may simulate better a given observed stress-strain relation. In this second and last part, this point will be examined. In particular, a new form of hyperbolic equation is proposed, which can model most stress-strain relations of soils and rocks from very small strain levels to the peaks stress condition. In addition, modified forms of hyperbolic equation and other functions are also discussed.

HYPERBOLIC MODELS USING NON-CONSTANT PARAMETER(S)

(2-1) Method using the strength paramater as a function of strain:

Hardin and Drnevich (1972) has noticed that for a better degree of fitting, Eq. (5) is to be modified as:

$$G_{eq}/G_{max}=1/(1+\gamma_h),\quad \gamma_h=x\cdot\{1+a\cdot\exp(-b\cdot x)\},\ x=d(\gamma)_{SA}/\gamma_r \tag{10}$$

, which is equivalent to use Eq. (6): $y=x/(1/c_1+x/c_2)$ with $c_1=1.0$ and $c_2=1/\{1+a\cdot\exp(-b\cdot x)\}$. This relation with $a=-0.5$ and $b=0.16$, which were obtained for air-dried clean sands by Hardin and Drnevich (1972), is denoted as (γ_h-1) in Figs. 1 through 6. Obviously, this relation is even worse than the OH relation in simulating the ML PSC test data.

However, a much better degree of the fitting of Eq. (10) to the observed relation may be obtained by selecting more appropriate values for the parameters 'a' and 'b'. Namely, in Fig. 7, at the point A, the diagonal from the top right corner to the bottom left corner intersects with the observed relation of the ML PSC test. Note that this diagonal means $x=1.0$. When the relation of Eq. (10) passes the point A, we obtain $y(x=1)=1/\{2+a\cdot\exp(-b)\}$, or $a\cdot\exp(-b)=1/y(x=1)-2$. For the ML PSC data, $y(x=1)$ is equal to 0.215 and this value gives $a\cdot\exp(-b)=2.651$. On the other hand, the initial linear part of the observed ($y/x-y$) relation seen in Fig. 7 can be expressed by $y/x=1/\{1+x/c_2(0)\}$, or $y/x=1-y/c_2(0)$. The value of $c_2(0)$ by Eq. (10) is $1/(1+a)$, which is 0.142 for the ML PSC data. Then, $a=6.04$ and $b=0.824$ are obtained. The relation of Eq. (10) with these values of 'a' and 'b' are denoted as (γ_h-2) in Figs. 1 through 6. It may be seen that even this relation can model only the very initial part of the observed relation. Note particularly that, at large values of x, Eq. (10) with $a=6.04$ and $b=0.824$ collapses into the OH relation. Since the ML PSC test data is typical of the monotonic loading stress-strain relations of uncemented soils, it can be said that Eq. (10) is not able to model various stress-strain relations for a wide range of strain of uncemented soils.

(2-2) Method using the two parameters as a function of strain:

Letting y_e be the stress at the elastic limit, the equation with c_1 and c_2 as a function of x for $y\geqq y_e$ becomes:

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$$y=y_e+\frac{x-y_e}{1/c_1(x)+(x-y_e)/c_2(x)} \qquad (11)$$

When the term y_e is ignored for simplicity in modelling a stress-strain relation excluding the very initial parts at very small strains, we obtain:

$$y=\frac{x}{1/c_1(x)+x/c_2(x)} \qquad (12)$$

From the condition that $dy/dx=1.0$ at $x=0.0$, we obtain that $c_1(0)=1.0$, and from the condition that $dy/dx=0$ at $x\rightarrow\infty$, we obtain that $dc_2/dx(x\rightarrow\infty)=0$. Among various other possible functions which satisfy these conditions, the following functions will be examined:

$$c_1(x)=\frac{1+c_1(x=\infty)}{2}+\frac{1-c_1(x=\infty)}{2}\cdot\cos\{\frac{\pi}{(\alpha/x)^m+1}\} \qquad (13)$$

$$c_2(x)=\frac{c_2(0)+c_2(x=\infty)}{2}+\frac{c_2(0)-c_2(x=\infty)}{2}\cdot\cos\{\frac{\pi}{(\beta/x)^n+1}\} \qquad (14)$$

Using $m=n=1.0$ as the first approximation, the other parameters $c_1(x=\infty)$, $c_2(0)$, $c_2(x=\infty)$, α and β can be determined as follows. For the $y/x-y$ plot of the data shown in Fig. 7, $c_2(0)$ is the intersect at the y axis of the linear relation fitted to the initial part of the observed relation. $c_1(x=\infty)$ and $c_2(x=\infty)$ are the intercepts at the y/x axis and the y axis, respectively, of the linear relation fitted to the observed stress-strain relation at large strains, which can be obtained from the linear fitting as shown in Fig. 5. In Fig. 7, at the point A, the diagonal for which $x=1$ intersects with the observed relation. Next, draw the line which is tangent to the observed relation at the point A. The intersects of this line with the y/x axis and the y axis are $c_1(x=1)$ and $c_2(x=1)$, respectively. The values of α and β are obtained by substituting these values of $c_1(x=1)$ and $c_2(x=1)$ together with $x=1$ into Eqs. (13) and (14). Fig. 8 shows the functions $c_1(x)$ and $c_2(x)$ with the parameters obtained by these procedures described in the above. This modified hyperbolic relation is denoted as (MH) in Figs. 1 through 6 in the first part (Tatsuoka and Shibuya, 1991). It may be seen that this relation fits very well the ML PSC test data for a range from very small to large strains. It has been found that this equation can model satisfactorily all the stress-strain relations which have been obtained so far in the authors' laboratory.

The value of y at the point A, which is y at $x=1.0$, is the important parameter which determines the value of α and β. Since the value of $y(x=1)$ increases as the linearity of stress-strain relation increases, this parameter can be called the linearity

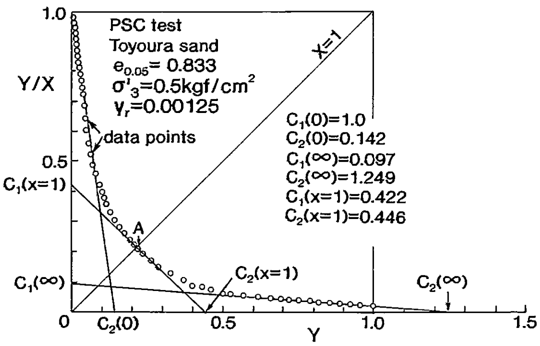

Fig. 7 Parameters for hyperbolic relation using coefficients as a function of x

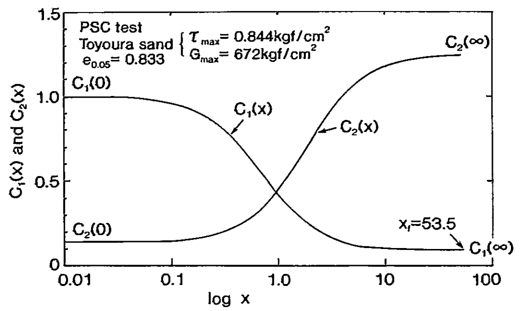

Fig. 8 Coefficients c_1 and c_2 as a function of x

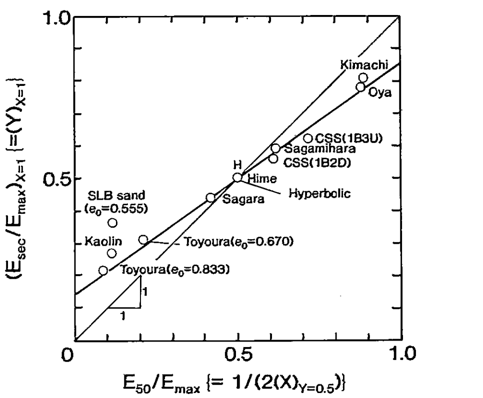

Fig. 9 Linearity index "y at $x=1$"="E_{sec} at $x=1.0$"/E_{max} versus E_{50}/E_{max} for a wide variety of geotechnical engineering materials

index. For example, the value of $y(x=1)$ is 1.0 and 0.5 for the linear relation and the OH relation, respectively. Fig. 9 shows the relationship between the linearity index and the ratio E_{50}/E_{max} for a wide variety of geotechnical materials. The value of E_{50} can be obtained from the conventional testing method using undisturbed sample and the in situ value of E_{max} can be obtained from the in situ shear wave velocity.

MODIFIED FORMS OF HYPERBOLIC EQUATION AND OTHER FUNCTIONS

Prevost and Keane (1989) and Griffiths and Prevost (1990) proposed a modified version of hyperbolic equation which satisfies the condition that the stress-strain relation has zero tangent modulus at the peak stress at a finite strain x_f: i.e., $dy/dx(x_f)=0$. It can be shown readily that for their proposed equation, y is always larger than $x/(1+x)$. Therefore, referring to Figs. 1 through 6, it is obvious that their equation does not simulate utterly the ML PSC test data, even worse than the OH relation, except in simulating the stress-strain relation at and near the peak.

One may consider that the other forms of equation rather than the hyperbolic may be used. One of the functions which satisfy the conditions: $y(x=0)=0$, $dy/dx(x=0)=1$, $dy/dx>0$ and $d^2y/dx^2<0$ for $y=0\sim1$, and $y_{max}=1.0$ is:

$$dy/dx=(1-y/c_2)^{m+1} \quad (15)$$

, in which $c_2\geqq1$ and the condition $dy/dx=0$ at $y=1.0$ is satisfied only when $c_2=1$. Eq. (15) becomes the original hyperbolic when $c_2=1$ and $m=1$. By integrating Eq. (15), we obtain the following equations, each of which is valid up to $y=1$:

(a) When $m=-1$, the material is linear elastic;

$$y=x \quad (16)$$

(b) When $-1<m<0$; $y=c_2\{1-(m\cdot x/c_2+1)\}^{-1/m}$ (17)
in which $y=c_2$ when $x=-c_2/m$.

(c) When $m=0$; $y=c_2\{1-\exp(-x/c_2)\}$ (18)

(d) When $0<m$; $y=c_2[1-\{c_2/(m\cdot x+c_2)\}^{1/m}]$ (19)

It has been found that even when the appropriate value is selected for m, Eq. (15) can model a given stress-strain relation only in an approximated way.

For a better fitting, particularly at small strain levels, one may introduce a correction function in the form of derivative, $dr(x)/dx$, as:

$$dy/dx=(1-y/c_2)^{m+1}\cdot dr(x)/dx \quad (20)$$

in which $dr(x)/dx$ is 1.0 when $x=0$, has a large rate of change at a small value of y and converges to 1.0 when $x\rightarrow\infty$. For example, the following function, r(x), satisfies these conditions:

$$r(x)=x+(c/b)\{1-\exp(-b/x)\cdot(bx+1)\}$$
$$dr(x)/dx=1+c\cdot x\cdot\exp(-b/x) \quad (21)$$

in which b and c are the specimen constants. Then, the integrated equations of Eq. (20) are obtained from Eqs. (16) through (19) by replacing x with r(x). One of disadvantages of this method is that it has many parameters which can be determined only by the method of try and error. It has also been found that Eq. (20) cannot be better than the newly proposed model (i.e., Eq. (12) with Eqs. (13) and (14)).

SUMMARY

The modified hyperbolic equation proposed by Hardin and Drnevich (1972), for which the strength parameter is a function of strain, was found not able to model the overall stress-strain relation of sand similarly to other versions of hyperbolic equation having constant parameters. Some other models also were found unsatisfactory. A new form of hyperbolic equation, which has the parameters of initial stiffness and strength as a function of strain, is proposed. It was found that this new model can simulate a given stress-strain relation very well.

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