研　究　速　報
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A Note on Finite Element Synthesis of Structures (Part 5)
——Shape Modification for Weight Minimization Based on Finite Element Sensitivity Analysis——
有限要素法による構造シンセシスに関するノート（第5報）
——有限要素感度解析による重量最小化の形状変更——

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1. Introduction

Optimal shape of a structure with minimum weight is rather new topic in problems of optimal design [1)~3)], though parametric optimization of structures has been discussed in detail so far [4),5)]. In the past the minimum weight design had been sought by changing such parameters as cross-sectional area and moment of inertia of members. When the finite element method has been made available, shape optimization can be devised by means of changing the nodal coordinates on the structure contour [6),7)].

As a matter of strutural synthesis, an attempt is made in this paper to formulate the minimum weight design which enables us to keep the generated stresses below allowable stress. Weight minimization techniques proposed so far do not predict in usual how much the weight of a structure can be reduced and how much the stress state in changed beforehand the computation. By a rule of thumb, stress state is expected to be stringent when the weight is reduced. It turns out that it is necessary to incorporate proper constraint conditions, stress limit in elastic design for instance, in weight minimization.

This paper deals with a formulation of weight minimization based on the Hessian matrix derived from finite element sensitivity analysis and equality constraint conditions incorporated by Lagrangian multipliers. The weight change is simulated by the second-order Taylor series expansion with respect to desigh variables chosen duly, while the stress change is approximated by the first-order Taylor series expansion. The validity and efficiency of the proposed formulation is examined by a numerical example concerned with a connecting rod of internal combustion engine.

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2. Statement of problem

Suppose that the change of design parameters X_n such as nodal coordinates on the structure contour are expressed by the design variables α_n chosen adequately and expressed by Eq. (1). The change of structural responses under interest Z_j near the baseline design are assumed in the linear form of Eq. (2), and the change of objective weight in nonlinear form of Eq. (3) with respect to the design variables α_n. The upper bar indicates quantities defined at the baseline design hereafter. The superfices $^{\mathrm{I}}$ and $^{\mathrm{II}}$ indicate the order of sensitivity. The Hessian matrix is constituted by means of arranging the second-order sensitivities in matrix form.

$$X_n=\overline{X}_n(1+\alpha_n) \tag{1}$$

$$Z_j=\overline{Z}_j+\sum_{n=1}^{N} Z_{jn}{}^{\mathrm{I}}\alpha_n \tag{2}$$

$$W=\overline{W}+\sum_{n=1}^{N} W_n{}^{\mathrm{I}}\alpha_n+\frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{N} W_{nk}{}^{\mathrm{II}}\alpha_n\alpha_k \tag{3}$$

The problem in this study is to determine the design variables which make the objective weight minimized and satisfy the equality constraint conditions posed for structural responses.

3. Formulation Based on Lagrangian Multiplier Method

The subsequent numerical example is carried out on the basis of the finite element analysis under plane strss state. Triangular, constant-strain finite elements are employed in this study. The stress sensitivities $Z_{jn}{}^{\mathrm{I}}$ and weight sensitivities $W_n{}^{\mathrm{I}}$ and $W_{nk}{}^{\mathrm{II}}$ in Eqs. (2) and (3) are calculated by the perturbation technique [8]. Particularly, W_{nn}^{II} is zero for any n in regard to the said triangular elements. Equivalent stresses in the elements in Mises' sense are taken as structural responses, for which equality constraint conditions are imposed.

A functional is constituted in the following form by the use of Lagrangian multipliers μ_j, so that the weight is to be minimized under the condition that the stresses in some chosen elements are increased to a limit value.

$$\Pi=\frac{1}{2}\sum_{n=1}^{N} K\alpha_n^2+\overline{W}+\sum_{n=1}^{N} W_n^{I}\alpha_n$$
$$+\frac{1}{2}\sum_{n=1}^{N}\sum_{k=1}^{N} W_{nk}^{II}\alpha_n\alpha_k$$
$$+\sum_{j=1}^{J}\mu_j(\overline{Z}_j+\sum_{n=1}^{N} Z_{jn}^{I}\alpha_n-Z_j^*) \qquad (4)$$

In the above, asterisk denotes the limit value. The first term of the right hand side of Eq. (4) is added artificially to the functional. The second and third terms stand for the weight change and stress shift of the limit value, respectively. The stationary condition of the functional with respect to the design variables and Lagrangian multipliers is summarized in the matrix form of Eq. (5), which is the governing equation of the unknown variables and multipliers.

$$\left[\begin{array}{cccc|ccc} K & W_{12}^{II} & \cdots & W_{1N}^{II} & Z_{11}^{I} & \cdots & Z_{J1} \\ W_{21}^{II} & K & & \vdots & \vdots & \ddots & \vdots \\ \vdots & & \ddots & \vdots & \vdots & & \vdots \\ W_{N1} & \cdots & & K & Z_{1N}^{I} & \cdots & Z_{JN}^{I} \\ \hline & \text{SYM.} & & & & 0 & \end{array}\right]\left\{\begin{array}{c}\alpha_1\\ \vdots\\ \alpha_N\\ \hline \mu_1\\ \vdots\\ \mu_J\end{array}\right\}=\left\{\begin{array}{c}-W_1^{I}\\ \vdots\\ -W_N^{I}\\ \hline Z_1^*-\overline{Z}_1\\ \vdots\\ Z_J^*-\overline{Z}_N\end{array}\right\} \qquad (5)$$

As a particular case, W_{nn}^{II}s are zero for the finite elements used in the numerical example, giving rise to that all the diagonal ingredients of the matrix in Eq. (5) are rendered to zero, unless the first term of right hand side of Eq. (4) is present. In presence of the first term with a weighting coefficient K, the stationary condition is converted into the minimization condition and the numerical solution of the governing equation are likely to be stabilized. To this end, the coefficient K equal to the largest absolute value of W_{nk}^{II} is found to be sufficient, after some numerical experiments, for the stabilization and yielding the design variables directly applicable to design change. When positive but small value is taken for K, the obtained design variables are likely to be large enough to distort the finite elements too much.

The design variables are determined as the solution of Eq. (5). The first-order and second-order approximation stated before is deficient to result in accurate design variables, however. It is therefore necessary to reiterate the determination by means of renewing the current shape and sensitivity analysis until the stresses attain the limit value.

4. Numerical Example

A rod, which connects crank shaft with piston pin of internal combustion engine is taken as the numerical example for the proposed formulation. The connecting rod is subjected to compressive force so that the flexural rigidity should be large to avoid buckling of the rod. for simplicity, the equality constratint conditions are imposed only for the stress limit, the cndition for the flexural rigidity being omitted. Only in-plane bending of the rod is taken into account.

Figure 1 shows the generic model of the connecting rod and idealized finite element division. The nodes, to whose coordinates the design variables are assigned, are indicated by solid circles in the figure. Young's modulus and Poisson's ratio are taken equal to 210 GPa and 0.3, respectively. The smaaller end of the connecting rod is fixed to eliminate rigid body motion. The loading is simulated by the distributed pressure applied to the inner surface of the larger end, he maximum being 550 MPa. The number of the design variables are fifteen, thirteen of which for the shape of the shank and neck regions and two for the outer radii of the right and left ends. The coordinates of internal nodes are changed by a simple rule the coordinate shift is almost proportional to the distance from the bold line in Fig. 1, in order to avoid excessive distorrtion of the finite element mesh.

Figure 2 shows the stress distribution along the rod shank before and after the shape modification. The initial stresses in the upper row (dotted) elements are

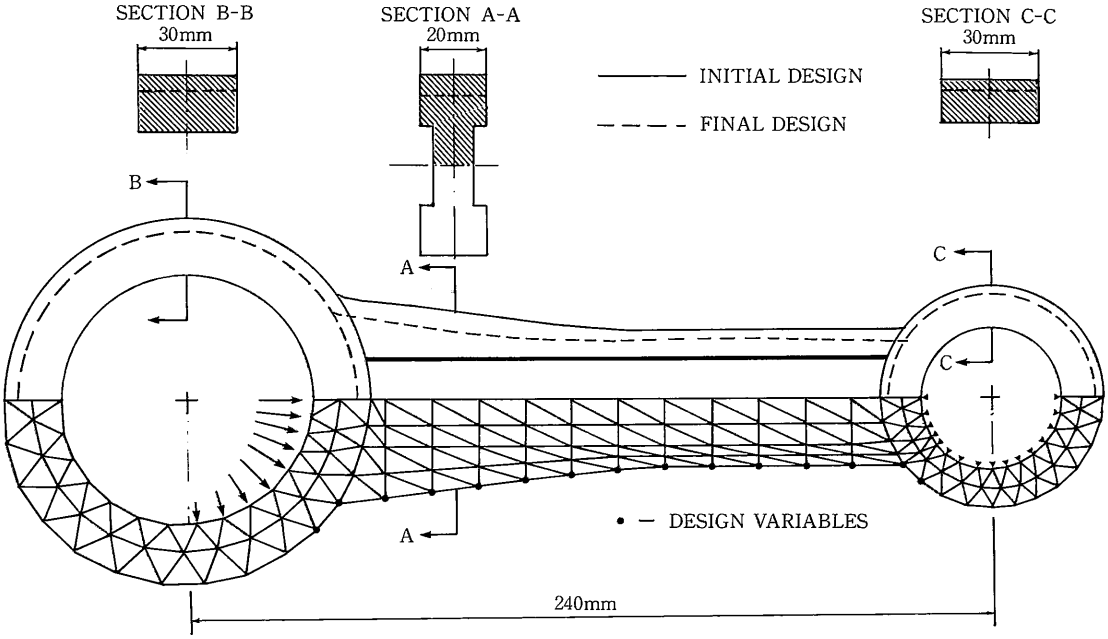

Fig. 1 Finite element representation of connecting rod

largest, the maximum being about 220 MPa. The stress limit is set equal to 230 MPa in order to suppress the stress increase due to the shape modification. The stresses in the black elements are increased when the width of the rod shank is decreased and the larger stresses are mitigated by the stress limit. The same stress is imposed also to the stresses in the black elements. The shape modification is stopped when the margin to the stress limit is judged exhausted. It is seen in Fig. 2 that most of the dotted elements and black elements are almost fully-stressed up to the stress limit, indicated by bold line, after twenty four renewals of the coordinate change. The shapes of the connecting rod before and after the shape modification are compared in Fig. 1.

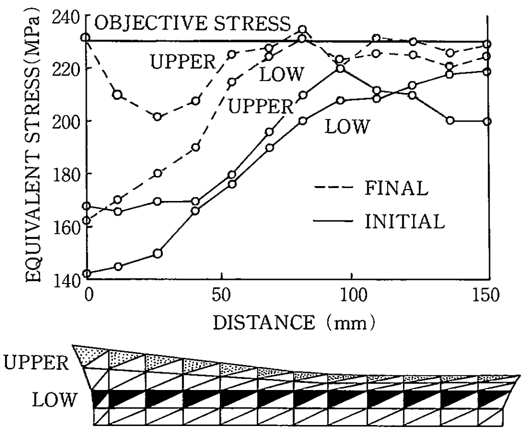

Fig. 2 Stress distribution along shank before and after shape modification

Figure 3 illustrates the iteration history of the weight reduction, showing that 20 % of the initial weight can be spared. The change of the maximum stress with progress of the iteration is plotted in Fig. 4. It means that the maximum stress violates the stress limit only by 3 % (defined by the stress limit). Such offset, arising from the deficient first-order apporoximation, is left for the constraint conditions. The monotonic weight reduction shown in Fig. 3 indicates that the artificial addition of the first term to the functional is effective to obtain the design variables resulting in moderate weight redution and gradual shape change.

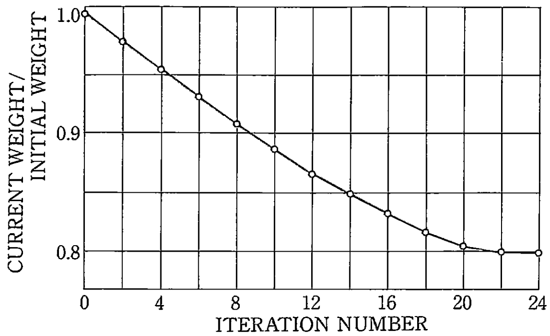

Fig. 3 Iteration history of weight reduction

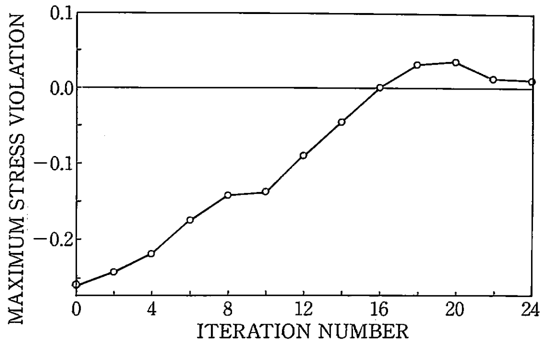

Fig. 4 Iteration history of maximum stress

5. Conclusion

A formulation is presented for design change aiming at he attainment of the mitigated stress state and the weight reduction at the same time. The stress shift is approximated by the first-order Taylor series expansion and the weight change by the second-order expansion with respect to the design variables assigned to the coordinates of the nodes on the shape contour. The squared sum of the design variables with a weighting coefficient is added to the functional in order to moderate the solution of the design variables. The weighting coefficient taken equal to the absolute value of the largest second-order sensitivity in the Hessian matrix gives such design variables that are employed directly in the design change.

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