

Non-Linearity in Stress-Strain Relations of a Wide Range of Geotechnical Engineering Materials

—Part II Effects of stress history—

各種地盤材料の応力ひずみ関係の非線型性

—その2 応力履歴の影響—

Satoru SHIBUYA, Fumio TATSUOKA, Fumihiro ABE and Supot TEACHAVORASINSKUN

澁谷 啓*・龍岡 文夫*・安部 文洋*・S.ティチャフォラスインスクン*

1. INTRODUCTION

In this second (and final) part, the effects of initial shear and overconsolidation on stiffness are discussed in the framework described in Part I (Shibuya et al., 1991a)⁵⁾, using the results of serieses of plane strain compression and torsional simple shear tests performed on Toyoura sand. A new method is proposed by which the nonlinear stress-strain relation of soils and soft rocks may be roughly estimated using results of routinely available laboratory tests.

2. EFFECTS OF INITIAL SHEAR AND OVERCONSOLIDATION ON STIFFNESS

The consolidation stress paths of a series of plane strain compression tests are shown in Fig. 8. All the

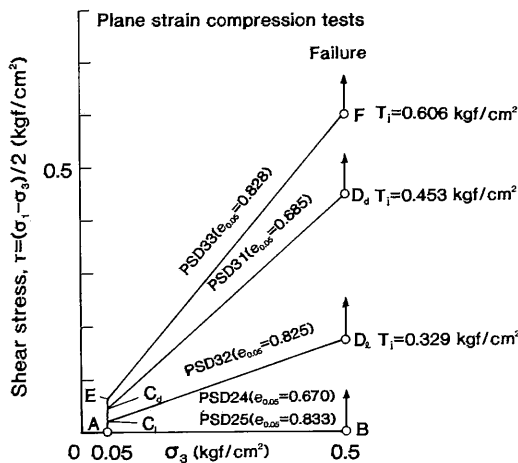
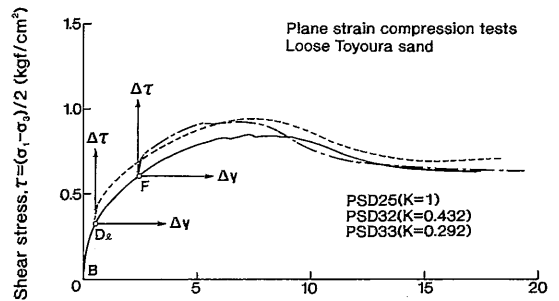
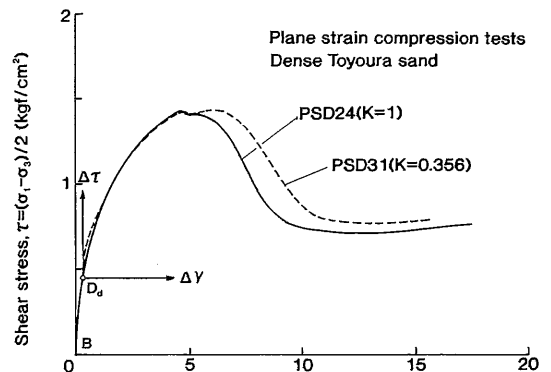


Fig. 8 Consolidation stress paths for plane strain compression (PSC) tests on Toyoura sand

specimens of Toyoura sand were consolidated to a common value of σ'_3 equal to 0.5kgf/cm² using different values of $K (= \sigma'_3/\sigma'_1)$. The results of isotropically consolidated (IC) specimens (i.e., tests PSD24 and PSD25) can be seen in Table 1 of Shibuya et al (1991a). The relationships of shear stress ($\tau = (\sigma'_1 - \sigma'_3)/2$) versus shear strain ($\gamma = \epsilon_1 - \epsilon_3$) are



(a) loose specimens



(b) dense specimens

Fig. 9 Shear stress versus shear strain for PSC tests on Toyoura sand

*Dept. of Building and Civil Engineering, Institute of Industrial Science, University of Tokyo.

Table 2 Summary of PSC tests on Toyoura sand

Tests	$e_{0.01}$	σ'_3 (kgf/cm ²)	τ'_i (kgf/cm ²)	τ_{max} (kgf/cm ²)	G_{max} (kgf/cm ²)
PSD25	0.833	0.5	0.0	0.850	618
PSD32	0.825	0.5	0.329	0.937	726
PSD33	0.828	0.5	0.606	0.921	676
PSD24	0.670	0.5	0.0	1.420	720
PSD31	0.685	0.5	0.453	1.423	828

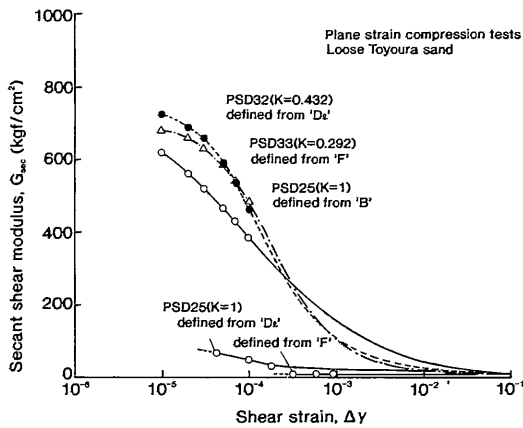


Fig. 10 Secant shear modulus versus shear strain for PSC tests on loose Toyoura sand

shown in Fig. 9, in which those of anisotropically consolidated (AC) specimens were plotted with their origins shifted on the stress-strain curve of the corresponding IC specimens. Note that the strength, τ_{max} , was scarcely affected by the initial shear (Table 2). As the secant shear modulus, $G_{sec} = \Delta\tau / \Delta\gamma$, is defined from the stress point at the end of consolidation (i.e., points B, D_i and F for tests PSD25, PSD32 and PSD33), those at $\Delta\gamma = 10^{-5}$ were roughly similar to each other, independent of the degree of initial shear (Fig. 10). The differences were marginal when the differences in $(\sigma'_1 + \sigma'_3) / 2$ amongst these tests were considered (Shibuya et al., 1991b). The values of G_{sec} for the IC specimens defined from points D_i and F were substantially smaller than those of AC specimens, particularly at small strains; i.e., the stress-strain relationship of AC specimens cannot be predicted from that of IC specimens. The variation of the tangent shear modulus, $G_{tan} = d\tau / d\gamma$, in relation to τ exhibited a tendency that the stiffness curves between IC (K=1) and AC specimens joined together, hence the effect of initial shear disappeared, at certain values of τ before reaching τ_{max} (Fig. 11). Figures 12 and 13 show these results examined in terms of

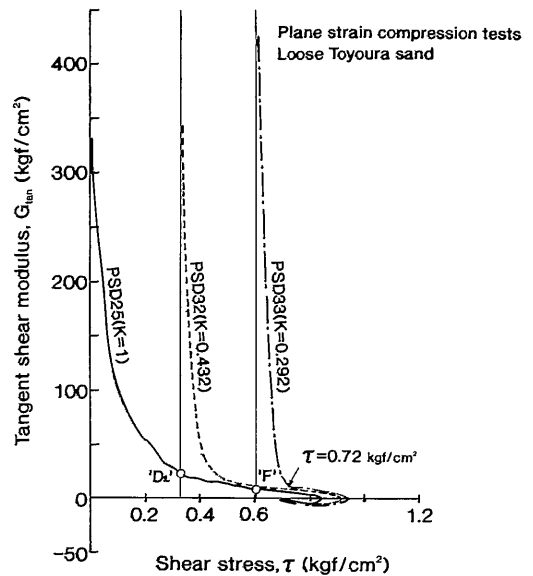


Fig. 11 Tangent shear modulus versus shear stress for PSC tests on loose Toyoura sand

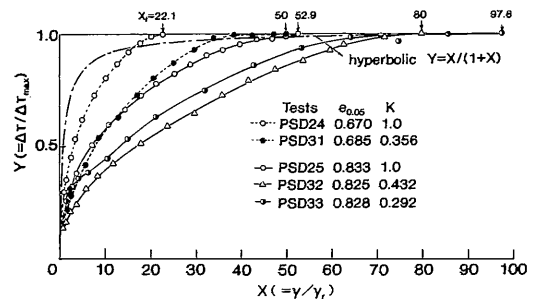
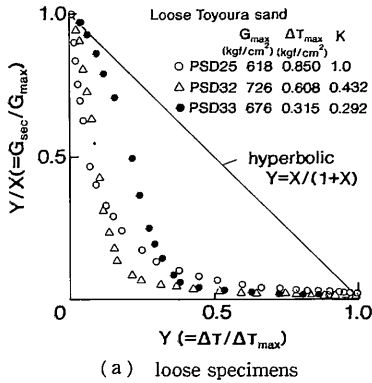


Fig. 12 Relationship of X versus Y for PSC tests on Toyoura sand

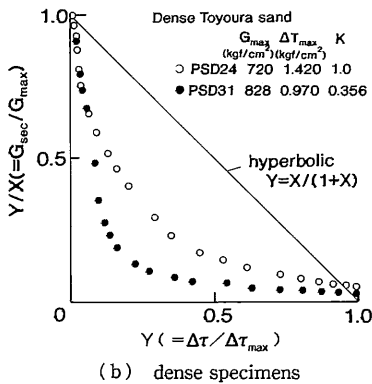
normalized shear stress, $Y = \Delta\tau / \Delta\tau_{max}$, and strain, $X = \gamma / \gamma_r$, where $\Delta\tau = \tau - \tau_i$, $G_{max} = G_{sec}$ at $\Delta\tau = 10^{-5}$ and $\gamma_r = \Delta\tau_{max} / G_{max}$ (see Table 2). In these figures, the original hyperbolic relation (Eq. 1') is also shown for comparison. Note that (i) as already observed in Figs. 3-5, the original hyperbolic model overestimated the stiffness of the sand, regardless of the degree of initial shear, and (ii) the X-Y relation was not unique, but affected by the degree of initial shear. Namely, except for the initial portion of PSD33 (K=0.292) with Y less than about 0.35, the degree of nonlinearity was larger for the AC specimens than for the IC specimens.

The effect of overconsolidation can be seen in Fig. 14, in which the results of torsional simple shear tests

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(a) loose specimens



(b) dense specimens

Fig. 13 Relationship of Y/X versus Y for PSC tests on Toyoura sand

on AC specimens of Toyoura sand were examined in terms of the normalized stress and stiffness. As reported in detail by Teachavorasinskun et al. (1991), the G_{max} -value was scarcely influenced however the X-Y relation (also Y/X-X relation) was influenced to a great extent by the overconsolidation ratio of four; i.e., the degree of nonlinearity was less pronounced as the OCR increased.

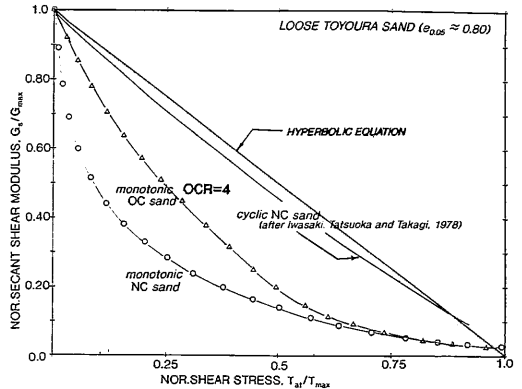


Fig. 14 Relationship of Y/X versus Y for torsional simple shear tests on dense Toyoura sand

3. EVALUATION OF NON-LINEARITY FROM ROUTINE TESTING

In a broad sense, the aspect of stress-strain relations of various geotechnical engineering materials (Figs. 3-6) showed the tendency that the degree of nonlinearity became more pronounced for the weaker and initially softer materials with smaller q_{max} and E_{max} (Fig. 15a). To explain it in a quantitative way, herein introduced is a non-linearity index, which is given as;

$$(X)_{Y=0.5} = \frac{(\epsilon_1)_{Y=0.5}}{(\epsilon_1)_r} = \frac{q_{max}}{2E_{50}(\epsilon_1)_r} = \frac{E_{max}}{2E_{50}} \quad (2)$$

The non-linearity index, $(X)_{Y=0.5}$ is the value of 'X' at $Y=0.5$, which in turn implies the ratio of E_{max} to $2E_{50}$ (Eq. 2). It takes the values of 0.5 and 1.0 for a perfectly linear material and for the original hyperbolic function, respectively. Note also from Figs. 3-6 that the degree of non-linearity increased as the $(X)_{Y=0.5}$ value increased (Figs. 15 a and c). Figure 16

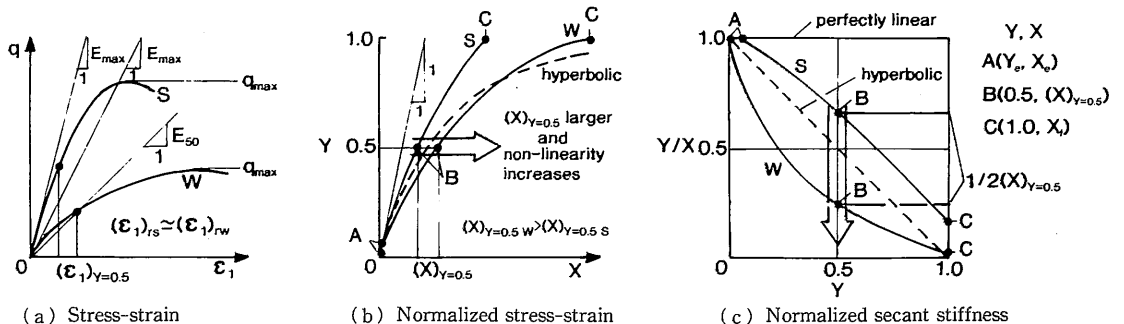


Fig. 15 General tendency for nonlinear stress-strain relations

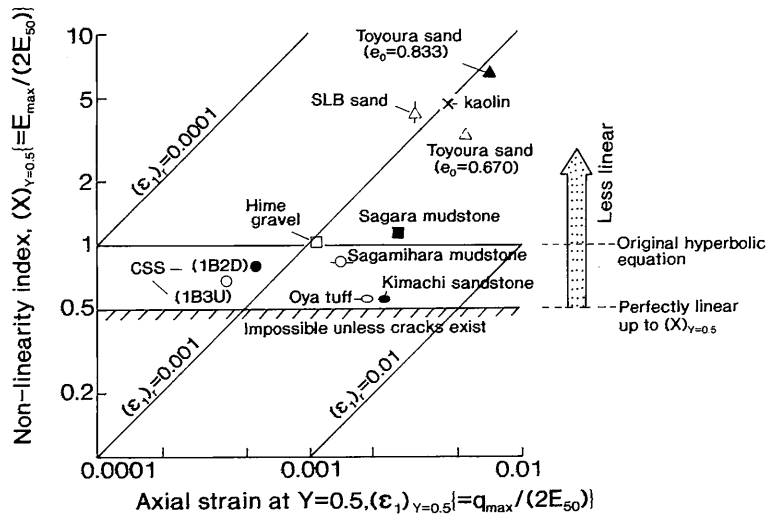


Fig. 16 Nonlinearity index versus axial strain at Y equal to 0.5 for materials without initial shear.

shows the relationship between $(X)_{Y=0.5}$ and ϵ_1 at $Y=0.5$, $(\epsilon_1)_{Y=0.5} = (q_{max} / (2E_{50}))$ for specimens without initial shear (see Eq. 2). It can be seen that this index reasonably depicts the observation that the softer material having larger $(\epsilon_1)_{Y=0.5}$ exhibited the larger nonlinearity. Note also that, except for hard rocks (Oya tuff and Kimachi sandstone, Noma et al., 1987), the values of reference strain were close to 1×10^{-3} , although it was slightly larger for natural soft rocks.

An approximate prediction of the stress-strain relation is possible from the results of routine testing in a sequence described in the followings.

- (i) Obtain E_{50} , q_{max} and the axial strain at failure, $(\epsilon_1)_r$, in the routine testing, if possible, also E_{max} from the in-situ seismic wave velocity.
- (ii) As sketched in Fig. 15, determine (Y, X) of three points of 'A', 'B' and 'C' at Y equal to Y_e , 0.5 and 1.0. If E_{max} is not known, refer to Fig. 16 to estimate the $(X)_{Y=0.5}$ value from the obtained $(\epsilon_1)_{Y=0.5}$ value.
- (iii) Draw smoothly an approximate X-Y relation through points 'A', 'B' and 'C', and
- (iv) Convert the X-Y relation to the $q-\epsilon_1$ relation using the measured or estimated q_{max} and E_{max} .

4. CONCLUSIONS

- 1) The stress-strain relation expressed in terms of $X = (\epsilon_1) / (\epsilon_1)_r$ (or $(\gamma) / (\gamma)_r$) and $Y = q / q_{max}$ (or $\tau /$

τ_{max}) was affected to a certain extent by initial shear and overconsolidation. However, the general tendency observed for different materials shown in Figs. 3-6 was unchanged.

- 2) For a specific geotechnical engineering material amongst sands, clays, gravels and soft rocks, a new method is proposed by which the nonlinear stress-strain relation may be roughly estimated using the results of routine testing.

5. ACKNOWLEDGEMENTS

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