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Non-linearity in Stress-Strain Relations of a wide Range of Geotechnical Engineering Materials
——Part I Experimental results——
各種地盤材料の応力ひずみ関係の非線型性
——その1 実験結果——

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INTRODUCTION

The small strain (say, less than 0.1%) as well as large strain behaviour of soils and rocks is important when evaluating deformation and stress conditions of ground, soil structures and foundations. In a single test, however, the examination of the stiffness from an extremely small strain range to the peak is technically difficult so that, for a variety of geotechnical engineering materials, the general picture of their non-linearity has not been fully understood. When the small strain is not measured, the maximum Young's modulus, E_{max}, is usually extrapolated by means of the hyperbolic fitting method using a part of data having a range of strain larger than say, 0.1% (Kondner, 1963). It is unknown whether or not the extrapolated E_{max} coincides with the true E_{max} and how it is linked (or not linked) to the Young's modulus obtained from the seismic wave velocity. The strain-level dependent stiffness of soils and rocks, covering broad ranges of both E_{max} and the peak strength, $q_{max}=(\sigma_1-\sigma_3)_{max}$, is discussed in this paper, based on the results of laboratory tests in which the stiffness was measured in succession for a range of strain from 10^{-6} to that at q_{max}.

MONOTONIC LOADING TESTS PERFORMED

The details of the tests are summarized in Table 1. The data on hard rocks were reproduced from Noma et al. (1987). The specimens of the sands, having the gradings shown in Fig. 1, were prepared by pluviating dry grains through air, while the Hime gravel specimen was made by manually stacking the particles. The kaolin, reconstituted in slurry and pre-

Table 1 Summary of monotonic loading tests.

	Hard rocks 12		Soft rocks				Gravel and Sands				Clay
Sample	Kimachi sand-stone	Oya tuff	Sagara mudstone	Sagami-hara mudstone	CSS		Hime gravel	Silver Leighton Buzzard	Toyoura sand		kaolin
					1B2D	1B3U			dense	loose	
Test 1	U		TC (CU)			TC (CD)	TC (CD)	PSC (CD)			TC (CU)
Specimen	5φx10cm	5φx10cm	5φx10cm	6.5φx15	30φx60	30φx60	30φx60	8x16x20	4x8x10.5	4x8x10.5	5φx10
σ_3' 2	zero	zero	3.2	3.0	1.5	2.5	0.5	0.8	0.5	0.5	3.0
Initial condi. 3	N.A.	N.A.	w=8.9% γ=2.405	w=25.5% γ=1.986	w=35.0% γ=1.839	w=34.15% γ=1.851	e_0=0.548 D_r=64%	e_0=0.555 D_r=78%	e_0=0.670 D_r=83%	e_0=0.833 D_r=39%	e_0=1.657 w=62.1%
q_{max} 4	276.3	129.6	90.13	58.40	21.17	18.43	1.96	3.60	2.90	1.68	1.10
E_{max} 5	65,800	38,230	38,400	34,500	30,460	31,100	3540	4660	1720	1480	1095
$(\varepsilon_1)_e$ 6	0.105%	0.085%	0.005%	0.008%	0.007%	0.007%	0.002%	0.0015%	0.0001%	0.0001%	0.0002%
$(\varepsilon_1)_r$ 7	0.420	0.339	0.235	0.169	0.0695	0.0593	0.0554	0.0773	0.169	0.114	0.10
$(\varepsilon_1)_f$ 8	0.80%	0.55%	0.672%	0.395%	1.175%	0.430%	3.00%	2.79%	2.12%	3.63%	9.70%
X_f 9	1.91	1.63	2.86	2.33	16.9	6.58	54.2	36.1	12.6	31.8	97.0
c_1 10	N.A.	N.A.	N.D	N.D.	1.26	1.36	0.38	0.17	0.22	0.10	0.13
c_2 11	N.A.	N.A.	N.D	N.D.	1.05	1.26	1.08	1.19	1.57	1.55	1.09

N.B.: 1) TC: triaxial compression, PSC: plane strain compression, CU: consolidated undrained, CD: consolidated drained, U: unconfined compression.
2) confining pressure (constant during shear) in kgf/cm^2.
3) w: water content, γ: total unit weight in gf/cm^3, D_r: relative density.
4),5) units in kgf/cm^2. 6) elastic-limit strain. 7) reference strain ($=q_{max}/E_{max}$).
8) ε_1 at q_{max}. 9) normalized strain at q_{max}, $(\varepsilon_1)_f/(\varepsilon_1)_r$.
10),11) $c_1=(E_{max})_{hf}/E_{max}$, $c_2=(q_{max})_{hf}/q_{max}$, N.D.; not determined.
12) Noma et al. (1987).

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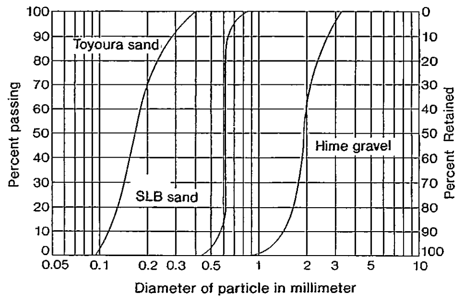

Fig. 1 Grain size distribution of granular materials.

consolidated one-dimensionally at a vertical stress of $1.5 kgf/cm^2$, had the plastic limit and plasticity index of 43.6% and 38.8, respectively. The trimmed specimen was reconsolidated in a triaxial cell to the confining pressure, σ_3', twice greater than the preconsolidation stress. All the specimens were isotropically consolidated. The natural sedimentary Sagara and Sagamihara mudstones were obtained by means of core sampling, and each was reconsolidated to σ_3' equal to the in-situ overburden pressure. The pre-mixed slurry of a cement-treated sandy soil (CSS) consisted of 1,177 kgf of Sengenyama sand with D_{50}=0.3mm and the maximum grain size of 1mm, 80 kgf of a cement, 110 kgf of mudstone powder and 520 kgf of sea water per volume of cubic meters. In the field experiment, it was casted through a tremie pipe in a huge ship-building dock filled with sea water. The triaxial specimens of CSS tested had a curing period of about three months. A great care was taken of the small strain measurement in which the axial strain was measured locally using a pair of local deformation transducers (LDT, Goto et al., 1991). The instrumentation employed for the plane strain compression (PSC) tests has been described by Shibuya et al. (1990). The strain measurements were utterly free from the effects of bedding error.

MAXIMUM YOUNG'S MODULUS AND PEAK STRENGTH

The deviatoric stress, $q=(\sigma_1-\sigma_3)$, versus the axial strain, ε_1, is shown in Fig. 2 for the two triaxial compression tests on CSS and kaolin. Note that E_{max}-values were taken as the initial linear portion defined within the ranges of ε_1 less than about 0.007% for CSS and 0.002% for kaolin. These materials exhibited virtually a linear elastic response in the event of the first unloading-reloading cycle that applied within the small strain range (Fig. 2). Thus, the limiting strain could mean the elastic-limit strain, $(\varepsilon_1)_e$, within which the response of soils is virtually linear elastic (i.e., $E_{max}=E_{sec}=E_{tan}$) (see Table 1). Note that in the PSC tests performed on isotropically consolidated specimens, the value of $(E_{max})_{PSC}=q/\varepsilon_1$ shown in Table 1 seems to be close to $(E_{max})_{TC}$, which is Young's modulus in triaxial compresion, since $(E)_{PSC}=(E)_{TC}/(1-\nu^2)$ for an isotropic material, and Poisson's ratio ν is at around 0.2 at small strains.

NORMALIZED STRESS-STRAIN RELATIONSHIP

The hyperbolic stress-strain relation using the true values of E_{max} and q_{max} is expressed as;

$$q=\frac{\varepsilon_1}{\{1/(c_1E_{max})\}+\{\varepsilon_1/(c_2q_{max})\}} \quad (1)$$

or

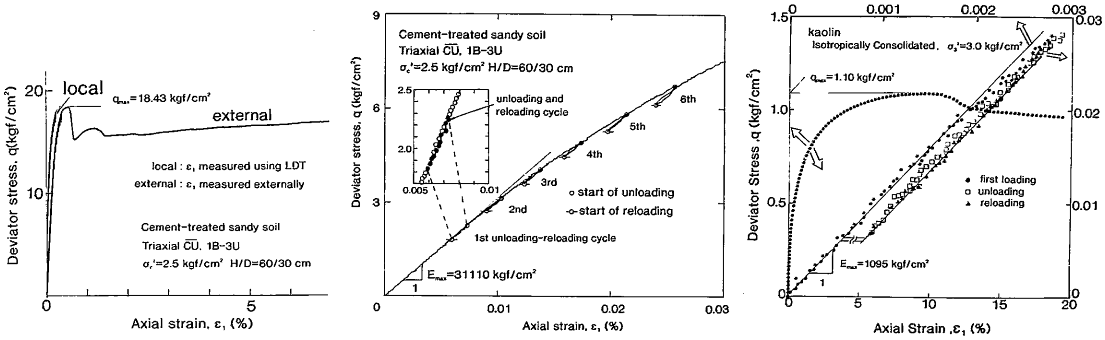

Fig. 2 Relationship between q and ε_1;
(a), (b) Cement-treated sandy soil (1B3U) and (c) kaolin.

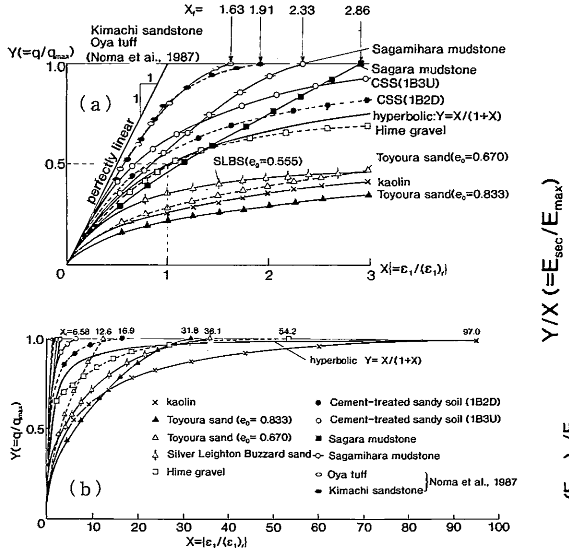

Fig. 3　Normalized stress-strain relationship; (a) small strain range and (b) full range.

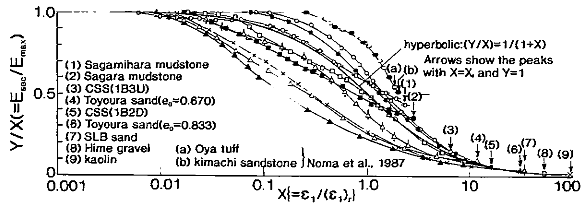

Fig. 4　Secant stiffness (Y/X) versus strain.

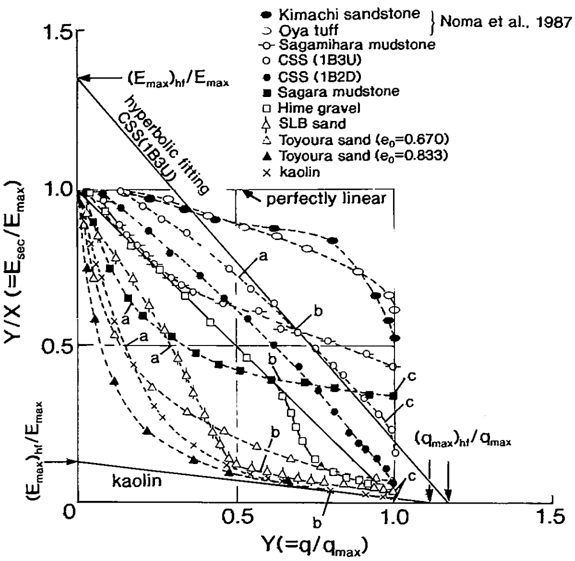

Fig. 5　Secant stiffness versus stress level.

$$Y=\frac{X}{(1/c_1)+(1/c_2)X} \qquad (1')$$

$(Y=q/q_{max},\ X=\varepsilon_1/(\varepsilon_1)_r$, and $(\varepsilon_1)_r=q_{max}/E_{max})$ where c_1 and c_2 are the coefficients of correction for E_{max} and q_{max}, respectively, and $(\varepsilon_1)_r$ stands for reference strain. In Eqs. (1) and (1′), the relation when $c_1=c_2=1.0$ will be called the *original hyperbolic equation* (i. e., $Y=X/(1+X)$).

The stress-strain relations were examined using Y and X (Figs. 3～6). The secant modulus, Y/X, is equivalent to E_{sec}/E_{max} (c.f., $E_{sec}=q/\varepsilon_1$). The symbol of X_f represents the peak strain when q_{max} was mobilized (i.e., at Y=1, see Table 1), and the value of (Y/X) at failure (Y=1.0) is $1/X_f$ (Figs. 4 and 5). The (X/Y) versus Y relations (Fig. 5) are a projection of 3D plots shown in Fig. 6. Note that the perfectly-linear material will have a constant ratio of (X/Y)=1.0 throughout shearing (Figs. 3～5). Note that (i) none of the stress-strain relations was properly fitted by using the original hyperbolic function, (ii) the original hyperbolic function underestimated the stiffness of rocks and overestimated that of clay and sands, and (iii) the materials having larger q_{max} or E_{max} exhibited a less degree of non-linearity.

The results of hyperbolic fitting in a plot of ε_1 versus ε_1/q are shown in Fig. 7, in which the fitted $(E_{max})_{hf}$ and $(q_{max})_{hf}$ correspond to the inverse of the intersect at the axis of $\varepsilon_1=0$, and of the inclinaiton of the fitted straight line, respectively. Note that (i) for the natural sedimentary mudstones, the relationship was non-linear, hence both of $(E_{max})_{hf}$ and $(q_{max})_{hf}$ were not determined, and (ii) for the other materials, the relationship was apparently linear, and the values of $(E_{max})_{hf}/E_{max}{}'(=c_1)$ were 1.2～1.4 for CSS, about 0.4 for gravel and 0.1～0.2 for sands and clay (see Table 1). A couple of examples to explain the meaning of the obtained values of c_1 and c_2 (= $(q_{max})_{hf}/q_{max}$) are shown for Fig. 5, in which the fitted straight lines are shown for the extreme cases of CSS 1B3U) and kaolin. When the stress-strain relations between two points of 'b' and 'c' are considered (Figs. 5 and 6), the fitted ranges were relatively close to the peak (i.e., the value of 'Y' larger than 0.7 for CSS and 0.6 for kaolin). Note also that irrespective of the kind of the materials, the hyperbolic fitting overestimated the q_{max} value (see Table 1).

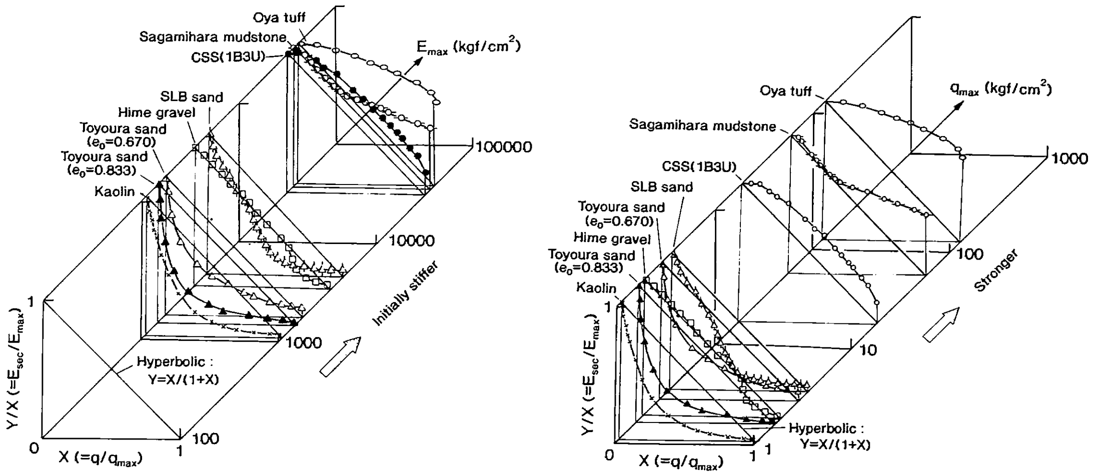

Fig. 6 Three dimensional plots of (Y/X), (Y) and (q_{max} and E_{max}).

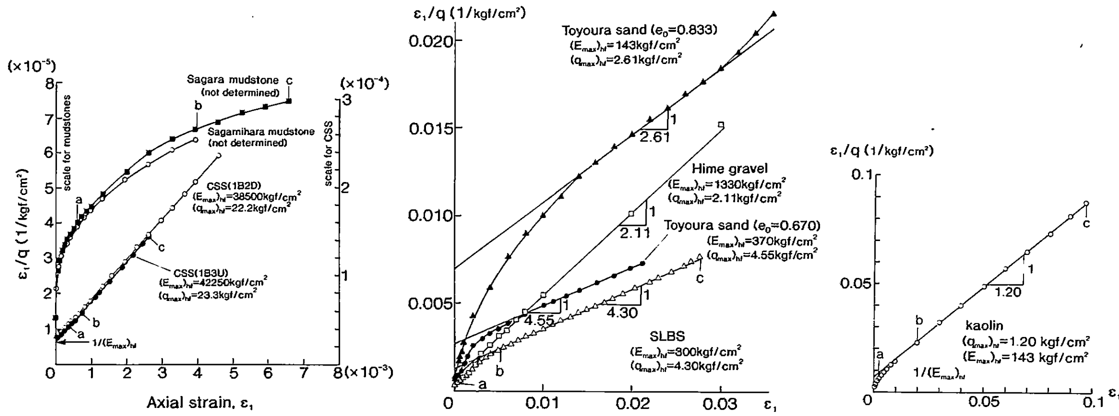

Fig. 7 Results of hyperbolic fitting; (a) Soft rocks, (b) gravel and sands and (c) kaolin.

CONCLUSIONS

1) The original hyperbolic stress-strain relation was found inappropriate to model the non-linear stress-strain relation of any of the geotechnical engineering materials examined.

2) The ratio of E_{max} estimated from the conventional hyperbolic fitting to the measured E_{max} was 1.2~1.3 for an artificial soft rock, about 0.4 for gravel and 0.1~0.2 for sands and clay, and not determined in case of natural soft rocks.

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