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Small Strain Behaviour of Sands in Plane Strain Compression
——Part II Stress dilatancy relations——
平面ひずみ圧縮状態での砂の微小ひずみでの挙動
——その２ ストレス・ダイレイタンシー関係——

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1. Introduction

It is widely acknowledged that the stress-dilatancy relation of sands at shear strains larger than, say, 0.1% (domain II) obeys the rule proposed by Rowe (1962), which is R=KD (Fig. 1). However the response at small strains (domain I) is still ambiguous; i.e., three types of responses represented by paths A, B and C have recently been observed by Hettler and Vardoulakis (1984), Pradhan et al. (1989) and Negussey and vaid (1990), respectively. This paper describes the results of plane strain compression tests equipped with the small strain measurements (Shibuya et al., 1990), by using which the stiffness and stress-dilatancy relation of three kinds of sands were examined for a wide range of shear strain covering domains both I and II.

2. Plane strain compression tests performed

The grain size distributions of Onahama, Toyoura and silver Leighton Buzzard (SLB) sands are shown in Fig. 1 of Park et al. (1990). Note that the grain shape is angular to sub-angular for Onahama and Toyoura sands and round for SLB sand, respectively. Rectangular specimens of Onahama sand had the same dimensions with Toyoura sand (4 cm in width, 8 cm in length and 10.5 cm in height.). These were prepared by means of wet tamping method with the water content, w, of 2~4 %. The specimens of Toyoura and SLB sands were prepared by raining the air-dried grains through air. In all the tests, the specimens were sheared under drained conditions with σ_3' kept constant (see Shibuya et al., 1990 for the details).

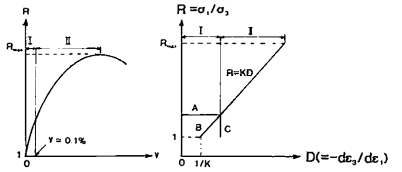

Fig. 1 Stress dilatancy relations of sands in general.

3. Discussions

(a) Stiffness

The stress-strain relationships of SLB and Toyoura sand specimens have respectively been shown in Figs. 7 and 8 of Shibuya et al. (1990). The effect of overconsolidation on the stress-strain relation is shown for dense SLB sand specimens (Fig. 2). In these tests, the overconsolidation with the overconsolidation ratio (OCR) equal to 5.33 was applied along the isotropic stress path to reach a common stress state with $\sigma_3 = 0.15$ kgf/cm^2 prior to the subsequent drained shear. The post-peak stress-strain relations were derived based on an assumption with zero volumetric

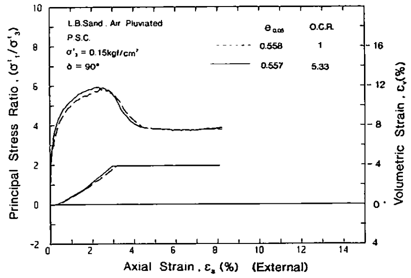

Fig. 2 Overall stress-strain relationship of dense silver Leighton Buzzard sand specimens.

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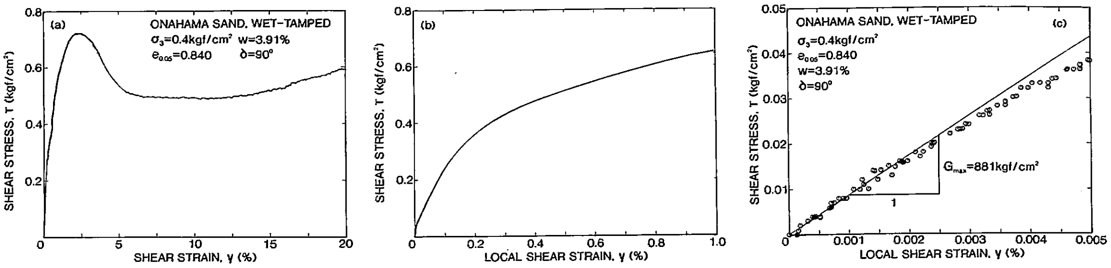


(a) maximum scale for γ equal to 20%, (c) maximum scale for γ equal to 0.005%.

(b) maximum scale for γ equal to 1%,

Fig. 3 Stress-strain relationship of a loose Onahama sand specimen.

strain increment. The interpretation of the pre-peak behaviour, which are described in the following figures, was made using the results of direct local strain measurements.

Figure 3 shows the stress-strain relationship of a loose specimen of Onahama sand ($e_{0.05}=0.840$) when sheared using a constant cell pressure of 0.4 kgf/cm^2. As has been observed in the test performed on SLB sand (Fig. 7d of Shibuya et al., 1990), the maximum shear modulus, G_{max}, can well be defined for the extremely small trains with γ less than about 2×10^{-5} (Fig. 3c).

For the specimens of Onahama and SLB sands, the relationships between secant shear modulus, G_{sec} or G_{sec},/G_{max}, and γ are shown in Figs. 4 and 5, respectively. It is interesting to note that, for both the sands, the initial linear portion was observed for γ up to about 2×10^{-5}, then the stiffness decreased as γ increased. Note also that the effect of OCR has little effect on the value of G_{max}, however the rate of decrease in stiffness was larger for the normally consolidated (NC) specimen (Fig. 5).

(b) Stress-dilatancy relations

The relationships between the principal stress ratio, $R=\sigma_1'/\sigma_3'$, and the principal strains are shown for the tests on Onahama and SLB sands in Figs. 6 and 7, respectively. Figures 8 and 9 show the plot of R versus D for the Onahama and SLB sand tests, respectively. For each data point shown in these figures, the strain increment ratio was obtained by means of linear approximation that applied for the relation of ε_1 versus ε_3 (see Figs. 6b and 7b). The range of strains to give each value of D was varied with the shear strain level (see the insets in Figs. 8 and 9). Each value of D was then plotted against the mean value of R for the range of data analysed. For the specimen of Onahama sand, the values of $D=-(\varepsilon_3/\varepsilon_1)$ increased continuously from 0.23 at the beginning of shear with γ less than 0.005 % to 1.72 at near the peak (Fig. 6). Note that the initial value of D for γ from zero to 0.0025% was 0.3 for both NC and OC specimens of SLB sand (see Fig. 7b and point '1' in Fig. 9). However, as shear progressed, the effect of OCR was noticealbe in that the OC specimen exhibited a tendency of being more dilatant than the NC specimen. At large strains near failure, the values of D were

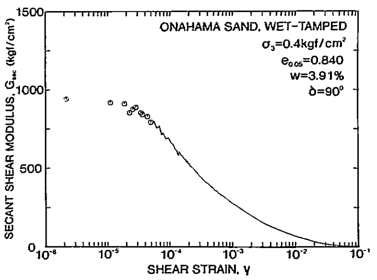


Fig. 4 Variation of secant shear modulus for the Onahama sand specimen.

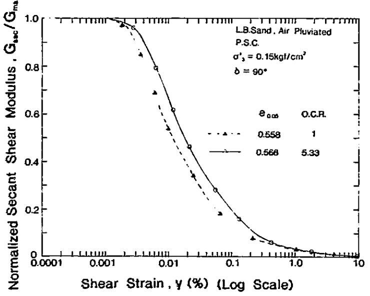
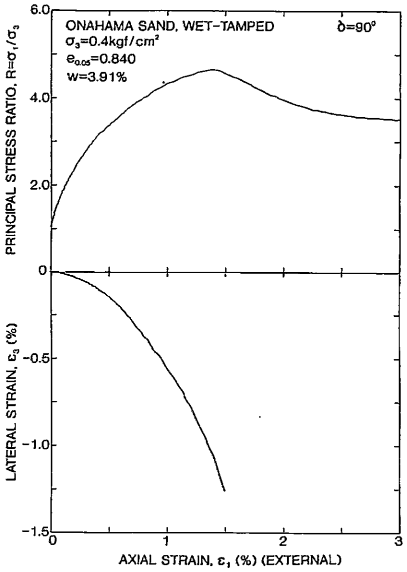


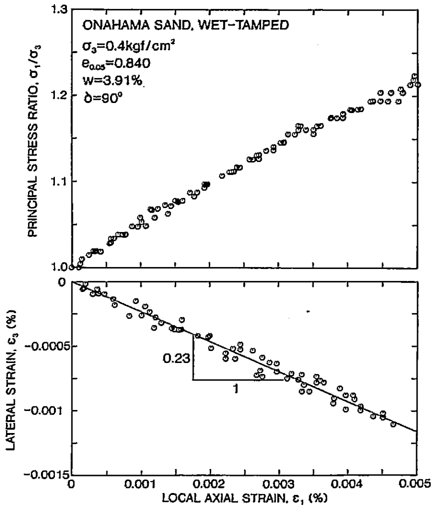
Fig. 5 Variation of secant shear modulus for the silver Leighton Buzzard sand specimens.

2.36 and 2.5 for the NC and OC specimens, which in turn give the angle of dilation, $\nu_{13}=-\arcsin\{(1-D)/(1+D)\}$ of 23.9 and 25.4 degrees, repectively. It is interesting to note that the angle of ν_{13} for the corresponding directional shear test was about 21 degrees (Wong and Arthur, 1985).

Fig. 10 shows a similar relationship of the test performed on dense Toyoura sand (refer to Fig. 8 of Shibuya et al., 1990).

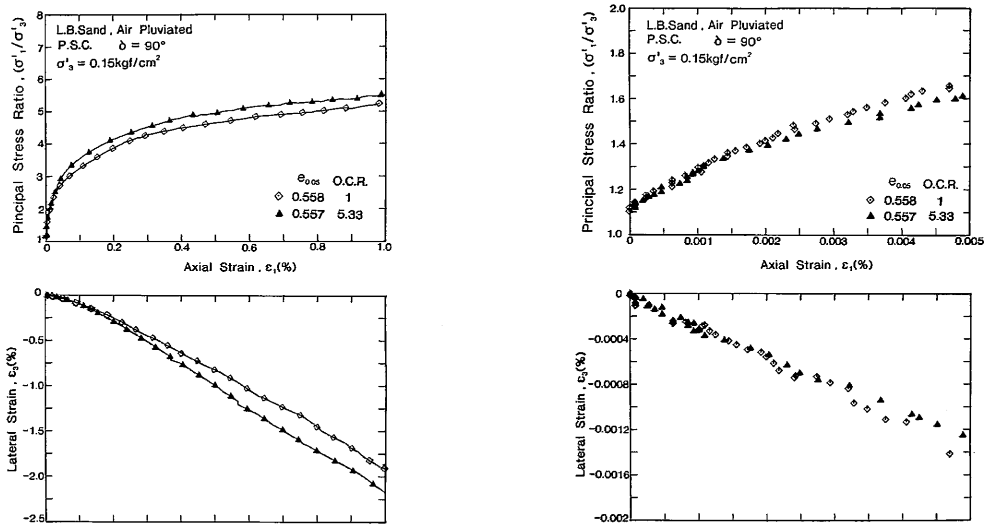
From Figs. 8~10, it may be seen that the stress-dilatancy relations obeyed the Rowe's equation, R=KD, in a broad sense, irrespective of the sample prepraration method, density, and the size and shape of grains. It may be noticed, however, that the relations at the very beginning of shearing deviate somehow to the upperside of the overall linear relations of R=KD. This may be due to the elastic strain increments, $d\varepsilon^e$, which were rather constant at around





(a) maximum scale for axial strain equal to 3%, (b) maximum scale for axial strain equal to 0.005%.

Fig. 6 Relationship between principal stress ratio and principal strains for the Onahama sand specimen.



(a) maximum scale for axial strain equal to 1%, (b) maximum scale for axial strain equal to 0.005%.

Fig. 7 Relationship between principal stress ratio and principal strains for the silver Leighton Buzzard sand specimens.

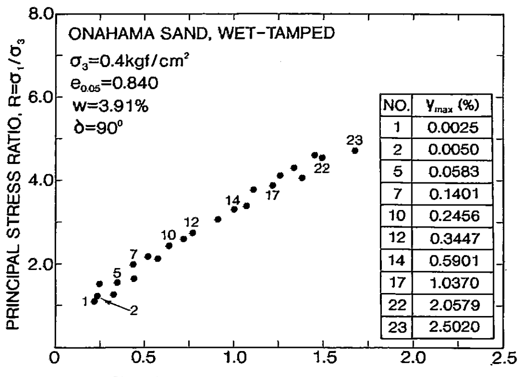


Fig. 8 Stress-dilatancy relationship of the Onahama sand specimen.

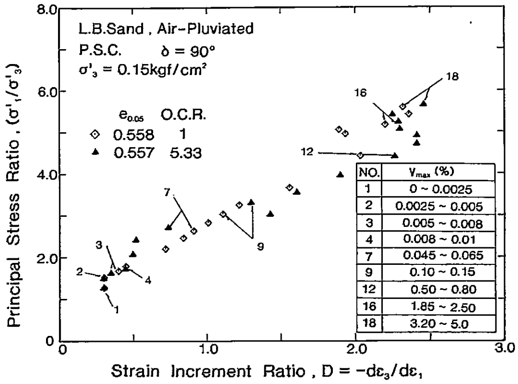


Fig. 9 Stress-dilatancy relationship of the silver Leighton Buzzard sand specimens.

0.3. It is expected that the measured relations using plastic strain increments obey the relation of R=KD in a better way.

4. Conclusions

(1) The linearity in the stress-strain relationship appeared for a range of shear strain less than 2×10^{-5}.

(2) In the case of SLB sand specimens, the maximum shear modulus, G_{max}, was scarcely affected by the isotropic over consolidation, but the effect of OCR resulted in slightly stiffer and more dilative behaviour for the shear strain from 10^{-4} onwards.

(3) For the range of γ from 10^{-5} to those at peak, the Rowe's stress-dilatancy relation seems to be a good approximation for air-dried Toyoura and SLB sands, and wet-tamped Onahama sand.

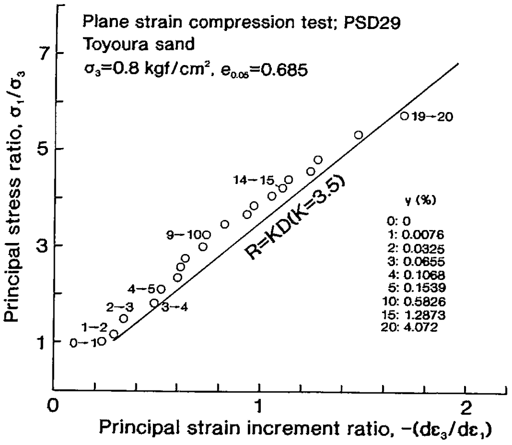


Fig. 10 Stress-dilatancy relationship of the Toyoura sand specimen.

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