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Turbulence Models for Practical Applications
——Survey of Models Part I (Mixing-length Models and Energy-equation Models)——
乱流モデルの工学への適用
——乱流モデル概観　その1（混合距離モデルと1方程式モデル）——

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A survey is given of relatively simple turbulence models which are in use for solving practical flow problems. The models all employ the eddy viscosity/diffusivity concept and include algebraic mixing-length models, energy equation models employing a differential transport equation for the turbulent kinetic energy k. The merits and shortcomings of these models are discussed. More elaborate models such as the k-ε two-equation model and the algebraic second moment closure model are presented in part II of the paper.

1. INTRODUCTION

In spite of considerable advances in the direct solution of the time-dependent Navier-Stokes equations and in large-eddy simulation techniques, the only economically feasible way to solve practical turbulent flow problems is still the use of statistically averaged equations governing mean-flow equations. In these equations, the transport of momentum, heat and mass by the turbulent motion is represented by correlations between fluctuating quantities. Because of the appearance of these terms, the mean-flow equations are not closed and a turbulence model is necessary to determine these turbulent momentum, heat and mass fluxes before the equations can be solved.

A wide variety of turbulence models has been suggested over the years, ranging from simple algebraic expressions relating the turbulent fluxes to the mean-flow field to models employing differential transport equations for the individual turbulence correlations or even for their spectral distribution over the various element sizes contributing to the turbulent motion. However, the more complex models have so far been tested mainly for rather simple, idealised flow situations and have hardly been used in calculations of practical flow problems. Such calculations were carried out mostly with the aid of simpler models employing either algebraic expressions or differential transport equations only for the velocity scale of the turbulent motion or at most also for the length scale. The present paper reviews the available models of this type; for more complex models based on the transport equations for the individual turbulent fluxes the reader is referred to the literature (e.g. 2), 12), 13)). The present paper is further restricted to models for incompressible flows and concentrates on so-called high-Reynolds-number versions of the models which are not applicable in viscosity-affected regions very near walls. The treatment of these regions is discussed briefly in Part II. Examples of applications of the models reviewed are given in the third part of the paper.

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2. BASIC CONCEPTS

The reader is first reminded of the task of turbulence models. This is to determine the turbulent or Reynolds stresses $-\rho\overline{u_iu_j}$ and the turbulent heat or mass fluxes $-\rho\overline{u_i\varphi}$ appearing in the statistically averaged mean-flow equations. In tensor notation these read as follows:

continuity equation:

$$\frac{\partial U_i}{\partial x_i}=0 \tag{1}$$

momentum equation:

$$\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial U_i}{\partial x_j}-\overline{u_iu_j}\right)+g_i \tag{2}$$

temperature/concentration equation:

$$\frac{\partial \phi}{\partial t}+U_i\frac{\partial \phi}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\lambda\frac{\partial \phi}{\partial x_i}-\overline{u_i\varphi}\right)+S_\phi \tag{3}$$

For incompressible flows, these are the equations governing the mean-flow velocity components U_i, the mean pressure P and the scalar quantity ϕ which stands either for mean temperature or mean concentration. In general, an equation of state has to be added relating the density ρ to the quantity ϕ on which the density may depend. The lower case quantities u_i and φ are the fluctuating velocity components and scalar fluctuations respectively. The correlations between these fluctuating quantities (indicated by an overbar) represent the turbulent momentum and heat or mass fluxes.

As two-dimensional thin shear layers play an important role in the discussion below and also in the applications paper (part III and continued papers), the mean-flow equations are also given in a simplified boundary-layer form applicable to these flows (definition of symbols see Fig. 1).

continuity equation:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{4}$$

momentum equation:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial U}{\partial y}-\overline{uv}\right) \tag{5}$$

temperature/concentration equation:

$$U\frac{\partial \phi}{\partial x}+V\frac{\partial \phi}{\partial y}=\frac{\partial}{\partial y}\left(\lambda\frac{\partial \phi}{\partial y}-\overline{v\varphi}\right) \tag{6}$$

It can be seen that in this shear layers only the

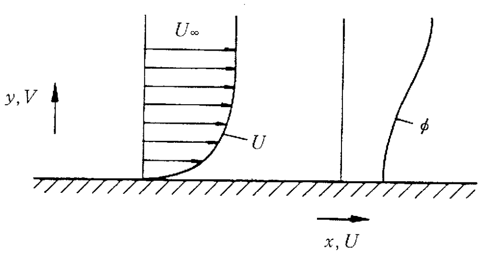

Fig. 1　2D boundary-layer flows

shear stress $-\rho\overline{uv}$ (representing the lateral transport of longitudinal momentum) and the lateral heat or mass flux $-\rho\overline{v\varphi}$ need to be determined by the turbulence model.

An important concept in all turbulence models, and in particular in the simpler ones considered here, is the characterisation of the local state of turbulence by only few parameters. For dimensional reasons, at least one velocity scale $\hat{V}$ and one length scale L (or alternatively a time scale $L/\hat{V}$) has to be used. These scales play a major role in the models described below. Once the parameters to characterise the state of turbulence have been chosen, the task of a turbulence model is i) to relate the turbulent stresses and heat or mass fluxes to the parameters chosen and ii) to determine the variation of the parameters over the flow field.

Another important concept, which is a significant part of nearly all the turbulence models discussed in this lecture, is the Boussinesq eddy viscosity concept. This assumes that, in analogy to the viscous stresses and to the heat or mass transfer by the molecular motion in laminar flows, the turbulent stresses and fluxes are proportional to the mean velocity and mean temperature/concentration gradients. For thin shear layers, there follows therefore:

$$-\overline{uv}=\nu_t\frac{\partial U}{\partial y}\qquad -\overline{v\varphi}=\Gamma_t\frac{\partial U}{\partial y} \tag{7}$$

where ν_t is the turbulent or eddy viscosity and Γ_t the turbulent or eddy diffusivity. In contrast to the molecular viscosity ν and the molecular heat conductivity or mass diffusivity λ, these quantities are not fluid properties but depend strongly on the state of the turbulence and may hence vary significantly over the flow field and also from one flow to another. The introduction of the eddy viscosity formula (7) is therefore by itself not a turbulence model; the main modelling problem is now shifted to the determination of the distribution of ν_t and Γ_t. Most models employ the Reynolds analogy between heat/mass transfer and momentum transfer and therefore assume the eddy diffusivity to be proportional to the eddy viscosity:

$$\Gamma_t=\frac{\nu_t}{\sigma_t} \tag{8}$$

where σ_t is the turbulent Prandtl or Schmidt number. For general flow situations, where six different components of the Reynolds stress $-\rho\overline{u_iu_j}$ and of the turbulent flux $-\rho\overline{u_i\varphi}$ prevail, the eddy viscosity/diffusivity concept may be expressed as

$$-\overline{u_iu_j}=\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij}$$

$$-\overline{u_i\varphi}=\Gamma_t\frac{\partial\phi}{\partial x_i} \quad (9)$$

The last term in the eddy-viscosity relation involving the Kronecker delta δ_{ij} is necessary to ensure that the sum of the three normal stresses (when $i=j=1, 2, 3$) is equal to twice the kinetic energy of the turbulence k (defined as $1/2\overline{u_iu_i}$).

Because the eddy viscosity concept assumes an analogy between the momentum transport by the turbulent motion and the transport by the molecular motion, it has frequently been criticised as physically unsound. However, in spite of the conceptual objections, the eddy-viscosity concept has often been found to work quite well in practice, simply because ν_t as defined by equation (7) can be determined to good approximation in many flow situations. For dimensional reasons, the eddy viscosity is proportional to a velocity scale $\hat{V}$ and a length scale L characterising the turbulent motion

$$\nu_t\propto\hat{V}L \quad (10)$$

and it is the distribution of these scales that can be approximated reasonably well in many flows.

It should be mentioned that the eddy viscosity/diffusivity concept breaks down in certain flow regions, for example where the shear stress $(-\rho\overline{uv})$ and the velocity gradient $\partial U/\partial y$ have opposite sign. In such regions, the eddy viscosity would have to be negative which is only mathematically possible but not physically meaningful. However, regions with negative eddy viscosity or diffusivity are, in most flows of engineering interest, rather small so that for practical purposes the local failure of the eddy viscosity/diffusivity concept is not of great consequence. The situation is different for geophysical flows, where large regions may be present in which the transport by the turbulent motion is against the gradient of the transported quantity. In equation (9), the eddy viscosity ν_t and the eddy diffusivity Γ_t have been introduced as scalar quantities so that they are the same for all stress components $\overline{u_iu_j}$ and flux components $\overline{u_i\varphi}$. This will in general not be the case; especially in complex situations with directional influences on the turbulence (e.g. gravitational or centrifugal forces), the assumption of an isotropic eddy viscosity may be too cude. Below, algebraic stress/flux models are introduced which remove this limiting assumption.

In most turbulent flow problems of engineering interest, the assumption of a constant eddy viscosity or diffusivity is not sufficient so that the turbulence model must provide the means of calculating the distribution of these quantities over the flow field. In the following and part II of the paper, three types of models permitting this are reviewed, namely mixing-length models not using any additional differential equations for turbulence quantities, one-equation models introducing a differential transport equation for the velocity scale of the turbulent motion, and two-equation models introducing an additional transport equation also for the turbulent length scale.

3. MIXING-LENGTH MODELS

The first model for describing the distribution of ν_t was proposed in 1925 by Prandtl[1]. This model is known as mixing-length hypothesis and relates the eddy viscosity directly to the mean-velocity field. It was designed for thin shear layer flows where the only significant velocity gradient is $\partial U/\partial y$. Prandtl postulated that the velocity scale $\hat{V}$ of the turbulent motion is equal to this velocity gradient times a length scale ℓ_m which is called mixing length. According to (10) there results for the eddy viscosity:

$$\nu_t=\ell_m^2\left|\frac{\partial U}{\partial y}\right| \quad (11)$$

The mixing length ℓ_m has to be prescribed empirically, and the success of the model for calculating simple shear layers lies in the fact that this can be done through relatively simple formulae for these flows. In free shear layers, for example, ℓ_m can be assumed constant across the layer and proportional to the layer width. The proportionality factor, i.e. the empirical constant in the turbulence model, depends however on the type of free flow considered (for actual values see 2)). In wall boundary layers, a ramp distribution is usually employed, with a linear distribution of $\ell_m=\kappa y$ near the wall and ℓ_m proportional to the boundary layer thickness δ in the outer region, i.e. $\ell_m=\lambda\delta$. The linear law is valid up to a point $y=\delta/\kappa$ in which ℓ_m reaches the value $\ell_m=\lambda\delta$ prescribed by the outer law. The proportionality constant κ is the von Kármán constant appearing in the logarithmic velocity distribution given below, and the linear variation of ℓ_m near the wall is consistent with this distribution. Patankar and Spalding[3] suggest $\varkappa=0.435$ and $\lambda=0.09$ for the empirical constants while Crawford and Kays[4] suggest $\varkappa=0.41$ and $\lambda=0.085$. Very close to the wall, where viscous effects play a role, the linear mixing-length relation has to be modified, and usually the following relation proposed by van Driest[5] is employed:

$$\ell_m=\varkappa y\left[1-\exp\left(-\frac{y(\tau_w/\rho)^{1/2}}{A\nu}\right)\right],\ A=26 \tag{12}$$

With this mixing-length distribution inserted into the eddy viscosity expression (11), the boundary layer equation (5) can be integrated right to the wall.

Another popular model is that proposed by Cebeci and Smith[6], who use van Driest's mixing length relation (12) near the wall, but do not strictly employ the mixing length hypothesis in the outer region where they assume the eddy viscosity itself to be constant and related to the velocity distribution according to the following expression:

$$\nu_t=\alpha\left|\int_0^\infty(U_\infty-U)\,dy\right| \tag{13}$$

where U_∞ is the free stream velocity. This formula was developed for general shear layers and can be expressed as

$$\nu_t=\alpha U_\infty\delta^*,\qquad \delta^*=\int_0^\infty\left(1-\frac{U}{U_\infty}\right)dy \tag{14}$$

where δ^* is the displacement thickness. Cebeci and Smith suggest a constant value of $\alpha=0.0168$ for boundary layers with high Reynolds numbers, but introduce an empirical Reynolds-number function for boundary layers at low Reynolds numbers and also additional empirical formulae to allow the calculation of laminar-turbulent transition. In flows with local separation zones, the use of (13) leads to difficulties as the free-stream velocity U_∞ is not known a priori. For such situations, Baldwin and Lomax[7] proposed a different model for the outer region which has recently become popular in computational aerodynamics. This model relates the eddy viscosity to the maximum of a function involving the mean vorticity and to the distance from the wall of the point at which this maximum occurs. In wake situations, the eddy viscosity is related to the maximum velocity difference across the wake and to the vorticity at the point where the function mentioned above has its maximum. Visbal and Knight[8] have carried out a critical examination of the Baldwin Lomax model by performing test calculations for shock-boundary-layer interaction flow on a ramp. They concluded that, even for this case with a relatively small separation region, the model did not perform too well. They found that the function involving the mean vorticity does not always have an unambiguous maximum, that the model does not account for any history effects and that altogether the concept of an algebraic eddy viscosity is not very adequate for complex separated flow fields. In flows over bluff bodies with larger separation zones is not clear whether the distance to the base wall is in fact a relevant length scale, and along which sections a velocity difference should be formed. Hence, the Baldwin Lomax model is likely not to be very suitable for flows with larger separation zones.

Effects of buoyancy and rotation

Body forces like those due to buoyancy or streamline curvature and rotation have been found to influence the mixing-length distribution significantly. Empirical relations were developed to account for the influence of buoyancy on the mixing length. The buoyancy effect is thereby characterised by the gradient Richardson number

$$R_i=-\frac{g}{\rho}\frac{\partial\rho/\partial y}{(\partial U/\partial y)^2} \tag{15}$$

where y is the vertical direction. For stable stratification ($R_i>0$) the so-called Monin-Oboukhov relation

$$\frac{\ell_m}{\ell_{m_0}}=1-\beta_1 R_i \tag{16}$$

is often employed, where ℓ_{m_0} is the mixing length distribution in the corresponding non-buoyant flow. The empirical constant β_1 is of the order of 7 (value of β_1 ranging from 5 to 10 have been reported in the literature). For unstable conditions ($R_i<0$) the KEYPS formula

$$\frac{\ell_m}{\ell_{m_0}}=(1-\beta_2 R_i)^{-1/4} \tag{17}$$

is commonly used, with $\beta_2\cong 14$. Bradshaw[9] demonstrated the close analogy between bouyancy and curvature effects on the turbulence and found that relations (16) and (17) also allow to describe the influence of streamline curvature on the mixing-length distribution. In this case, the Richardson number is defined as the ratio of centrifugal to inertial forces:

$$R_i = \frac{U_s/R_c}{\partial U_s/\partial n} \tag{18}$$

where U_s is the velocity component along the streamlines, n is the direction normal to them and R_c is the local radius of curvature of the streamlines. Bradshaw[10] reports that the constant β_1 in relation (16) lies in the range 6 to 14 and depends relatively strongly on the flow situation considered.

Discussion

Mixing-length models have been used successfully to calculate many thin shear layer flows. The extensive testing has, however, also brought to light the limitations of these models, in particular the lack of universality of the empirical input. One shortcoming of the model is that it is based on the implied assumption that turbulence is in a local state of equilibrium, which means that, at each point in the flow, turbulence energy is dissipated at the same rate as it is produced, so that there can be no influence of turbulence production at other points in the flow or at earlier times. Hence, mixing-length models cannot account for transport and history effects of turbulence. In grid turbulence, for example, the turbulence is generated by the wakes directly behind the grid and is then convected downstream by the mean motion. The mixing-length model cannot account for this transport and yields zero turbulence ($\nu_t = \Gamma_t = 0$) because the mean velocity is uniform in the downstream region. In channel flow, the turbulence is produced mainly near the walls and is transported to the central region by diffusion due to the turbulent fluctuations; the mixing-length model neglects this transport and therefore predicts zero turbulence at the centerline.

In conclusion, mixing-length models are not very suitable when convective and diffusive transport of turbulence or history effects are important. Further, although the basic relation for thin shear layers can be extended for general flows (see e.g. 2)), the model is of little use in complex flows because of the great difficulties in specifying the distribution of the mixing length ℓ_m. However, for many simple shear layers where ℓ_m can be specified empirically, the mixing-length model is a practically useful and therefore popular tool.

4. ENERGY-EQUATION MODELS

In order to account for the transport and history effects of turbulence, models were developed which solve differential transport equations for turbulence parameters. The simplest models of this kind use a transport equation for a suitable velocity scale of the turbulent motion. The physically most meaningful scale is $\sqrt{k}$, where $k = 1/2\overline{u_i u_i}$ is the kinetic energy of the turbulent motion per unit mass, which is a measure of the intensity of the fluctuations in the three directions. An exact equation for k can be derived from the Navier Stokes equations and reads for high Reynolds numbers:

$$\underbrace{\frac{\partial k}{\partial t}}_{\substack{\text{rate of}\\ \text{change}}} + \underbrace{U_i \frac{\partial k}{\partial x_i}}_{\substack{\text{convective}\\ \text{transport}}} = \underbrace{-\frac{\partial}{\partial x_i}\left[\overline{u_i\left(\frac{\partial u_j \partial u_j}{2} + \frac{p}{\rho}\right)}\right]}_{\text{diffusive transport}} \underbrace{-\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{\substack{P=\text{production}\\ \text{by shear}}} \underbrace{-\beta g_i \overline{u_i \varphi}}_{\substack{G=\text{buoyant production/}\\ \text{destruction}}} \underbrace{-\nu \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}}}_{\substack{\varepsilon=\text{viscous}\\ \text{dissipation}}} \tag{19}$$

This equation is the trace of the exact equation for $\overline{u_i u_j}$ given, for example, in 2). The rate of change of k is balanced by the convective transport due to the mean motion, the diffusive transport due to velocity and pressure fluctuations, the production of k by interaction of Reynolds stresses and mean-velocity gradients, and the dissipation of k by viscous action into heat. In buoyant flows there is also production or destruction of k due to buoyancy forces; the term G represents an exchange between the turbulent kinetic energy k and the potential energy (β is the volumetric expansion coefficient). In viscosity-affected regions, an additional molecular diffusion term is present.

The exact k-equation (19) is of no use in a turbulence model because new unknown correlations appear in the diffusion and dissipation terms. In order to obtain a closed set of equations, the following model assumptions are usually introduced for these terms:

$$-\overline{u_i\left(\frac{u_j u_j}{2} + \frac{p}{\rho}\right)} = \frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i} \tag{20}$$

$$\varepsilon = c_D \frac{k^{3/2}}{L} \tag{21}$$

where σ_k and c_D are empirical constants. Relation (20) involves a gradient-diffusion assumption and the dissipation relation (21) follows from dimensional analysis if one assumes that the amount of dissipated turbulent energy is determined by the energy-containing large-scale motion at high Reynolds numbers. With (20) and (21) and the eddy viscosity and diffusivity expressions (9) for $\overline{u_i u_j}$ and $\overline{u_i \varphi}$ introduced, the k-equation reads:

$$\frac{\partial k}{\partial t}+U_i\frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \underbrace{\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{P} \underbrace{+\beta g_i\frac{\nu_t}{\sigma_t}\frac{\partial \phi}{\partial x_i}}_{G} \underbrace{- c_D\frac{k^{3/2}}{L}}_{\varepsilon} \qquad (22)$$

This is the high-Reynolds number form of the transport equation for k used in most energy-equation models. The majority of these models employs the eddy viscosity concept, which with $\sqrt{k}$ introduced as velocity scale yields according to (10):

$$\nu_t = c'_\mu \sqrt{k}\, L \qquad (23)$$

where c'_μ is an empirical constant. This formula is known as the Kolmogorov-Prandtl expression. In the energy-equation model introduced so far, $c'_\mu c_D \cong 0.08$ and $\sigma_k=1$ appear to be reasonable values for the empirical constants (the individual values of the constants c'_μ and c_D are not important but only their product). It should be noted that the above model is not applicable to the viscous sublayer near walls. A low-Reynolds-number version applicable also in this region is introduced below in Part II of the paper as part of a two-layer model.

Bradshaw et al.[11] converted the boundary-layer form of the k-equation (19) into an equation for the shear stress $\overline{uv}$ by assuming that in these flows $\overline{uv}$ is proportional to k ($\overline{uv}=0.3k$ is used). Hence, in their model the eddy viscosity concept is not used. Bradshaw et al's modelled k-equation is some what different from (22); in particular they did not make use of the gradient diffusion model (20) but assumed that the diffusion flux of k is proportional to a bulk velocity. Bradshaw et al's model was applied successfully in many wall-boundary-layer calculations but cannot be used in a straightforward manner in shear layers where the shear changes sign (e.g. in duct flows, jets, wakes).

Length-scale determination

The Kolmogorov-Prandtl expression (23) and also the dissipation relation in the k-equation (22) contain the length scale L which needs to be specified to complete the turbulence model. In energy-equation models, the distribution of this scale is described empirically. Usually simple empirical relations are adopted similar to those used for the mixing length ℓ_m. In buoyant situations, the relations (16) and (17) introduced for the mixing length can be used also to account for the effect of buoyancy on the length scale L. The empirical relations work quite well for simple shear layers, while in more complex flows L is no easier to prescribe than the mixing length ℓ_m. For this reason, the application of energy-equation models was so far limited mainly to shear-layer flows. Various authors have tried to develop formulae for calculating L in general flow situations, and some test calculations have been carried out for separated flows. A discussion on this can be found in Rodi[2], who concluded that these formulae were insufficiently tested and also rather complex and expensive of computing time. The trend has therefore been to use two-equation models which determine the length scale from a second transport equation.

5. CONCLUDING REMARKS

In the past, mixing-length models have been used widely and with considerable success for calculations of simple shear layers, and a great amount of experience has been collected on the empirical specification of the mixing-length distribution in such flows. The mixing-length hypothesis and related algebraic eddy viscosity models are, however, not very suitable whenever turbulence transport and history effects are important, and they are of little use for flows more complex than shear layers because of the great difficulties in specifying the mixing-length distribution in such flows. Further, extra effects on turbulence, such as those due to body forces, can be accounted for in an entirely empirical way only. One-equation models employing a transport equation for the kinetic energy of turbulence account for transport and history effects and are therefore superior to mixing-length models for such non-equilibrium shear layers where the length-scale distribution can be prescribed realistically; they are, however, not very suitable for complex flows where an empirical length-scale determination is difficult. Two-equation models which determine the length-scale from a second transport equation are required.

Two-equation models and Algebraic stress/flux models will be given in Part II of this paper.

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