研　究　解　説

UDC 532.517.4

A Simple Second-Moment Closure for the Prediction of Turbulent Flows under the Action of Force Fields

——Part 1　Closure of the Second-Moment Equations——

外力が働く乱流場予測に対する簡便な乱れの2次モーメント完結モデルの応用

——その1　乱れの2次モーメント方程式の完結モデル——

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The paper considers the effects of force fields on turbulent shear flows and the extent to which observed phenomena can be accounted for by turbulence models developed by reference to flows unaffected by body forces. Especial emphasis is placed on second-moment closure as at this level the direct effects of force fields appear in the second-moment equations in a form that can be treated exactly. Applications of the model will be presented in a future issue of this journal.

1. Introduction

External force fields produce a bewildering variety of effects on turbulent shear flows reflecting both the intrinsic non-linearity endemic in turbulent flows and the subtle intercouplings that the force field may impart.

While no-one seriously supposes that any computational approach short of full simulation will completely and reliably capture the outcome of applying an arbitrary force field to turbulence, there is, nevertheless, considerable interest from both industrial and environmental standpoints in seeing whether relatively simple models can be devised that broadly account for the action of body forces over at least a limited range of turbulent flows. The present contribution attempts to throw light on this issue. The modelling level considered is that of second-mement closure. At present this is the highest order approximation that can be contemplated for the multi-dimensional, inhomogeneous flows of practical interest. The main attraction of modelling at this level is that the direct effect of the force field in question on the second-moment generation rates appears in the model in a from requiring no further approximation.

The present contribution limits attention to the simplest and most widely used form of second-moment closure. The model itself is developed in the following section while applications covering free flows and near-wall layers and involving several types of force field will be presented in the Part II, to appear in the following issue.

At the outset it will be helpful to distinguish two different types of effect of a body force on a turbulent flow. *Time-averaged* body forces applied to a shear flow may modify the velocity field in accordance with the *mean* momentum and this, indirectly, modifies the generation rates of the Reynolds stresses and other second moments. *Fluctuations* in body force contribute source or sink terms to the equation governing the turbulent velocity field and thus directly contribute to the rate of creation of the velocity-containing second moments. The reason for drawing this distinction is that, in the former case, eddy-viscosity models often lead to satisfactory flow-field predictions, e.g. Cotton and Jackson (1987), McGuirk and Spalding (1976). In the latter, however, the assumption of isotropic turbulent transport coefficients in the momentum and enthalpy equations will hardly ever capture the effects of a significant force field with the desired level of accuracy. Accordingly, the present contribution is concerned predominantly with cases where the main effects of the force field arise from the direct modification of the generation rate of the second moments.

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2. The Exact Second-Moment Equations

An exact describing the transport of the kinematic Reynolds stress, $\overline{u_iu_j}$, may be obtained by taking a fluctuating-velocity-weighted moment of the Navier-Stokes equation and averaging. The result may be expressed:

$$\frac{D\overline{u_iu_j}}{Dt}=-\underbrace{\left\{\overline{u_iu_k}\frac{\partial U_j}{\partial x_k}+\overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right\}}_{P_{ij}}+\underbrace{\left[\overline{f_iu_j}+\overline{f_ju_i}\right]}_{F_{ij}}$$
$$+\underbrace{\overline{\frac{p}{\rho}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right]}}_{\varphi_{ij}}\quad\underbrace{-2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}_{\epsilon_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{u_iu_ju_k}+\frac{\overline{pu_j}}{\rho}\delta_{ik}+\frac{\overline{pu_i}}{\rho}\delta_{jk}-\nu\frac{\partial\overline{u_iu_j}}{\partial x_k}\right]}_{d_{ij}}\qquad(1)$$

The symbols take their standard meaning but, in any event, are defined under Nomenclature. The first four groups of terms on the right of (1) express the action of source or sink terms on the generation rate of $\overline{u_iu_j}$ due respectively to mean strain (P_{ij}), the action of the body force (F_{ij}), the interaction between the fluctuating pressure and strain fields (φ_{ij}) and viscous dissipation (ϵ_{ij}). The final term expresses the diffusional transport rate of $\overline{u_iu_j}$ through velocity and pressure fluctuations and by molecular diffusion (d_{ij}).

The quantity f_i is the fluctuating body force per unit mass in direction x_i. Its precise form depends on the force field in question: the gravitational force is $\rho' g_i/\rho$ (ρ' being the fluctuation in density about its mean value), while the effective x_i-directed force arising from observing the motion in a coordinate frame rotating at angular velocity Ω_k is $2u_m\Omega_k\epsilon_{ikm}$, where ϵ_{ikm} is the third rank alternating tensor.

The resultant generation terms in (1) are thus:

Buoyancy:

$$F_{ij}=\frac{1}{\rho}\{g_i\overline{\rho' u_j}+g_j\overline{\rho' u_i}\}\qquad(2a)$$

Coordinate rotation (Coriolis) force:

$$F_{ij}=-2\Omega_k\{\overline{u_ju_m}\epsilon_{ikm}+\overline{u_iu_m}\epsilon_{jkm}\}\qquad(2b)$$

As asserted in the Introduction, these terms contain only second-moment products as unknowns, so if closure is at second-moment level the terms can be included without further approximation.

The corresponding transport equation for the scalar fluxes, obtained by multiplying the Navier-Stokes equations by the fluctuating scalar θ and adding it to the instantaneous energy equation multiplied by u_i, may be expressed:

$$\frac{D\overline{u_i\theta}}{Dt}=-\underbrace{\overline{u_iu_k}\frac{\partial\Theta}{\partial x_k}}_{P_{i\theta_1}}-\underbrace{\overline{\theta u_k}\frac{\partial U_i}{\partial x_k}}_{P_{i\theta_2}}+\underbrace{\overline{f_i\theta}}_{F_{i\theta}}$$
$$+\overline{\frac{p}{\rho}\frac{\partial\theta}{\partial x_i}}+(\lambda+\nu)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial\theta}{\partial x_k}}$$
$$-\frac{\partial}{\partial x_k}\left[\overline{u_i\theta u_k}+\frac{\overline{p\theta}}{\rho}\delta_{ik}\right]\qquad(3)$$

In the above, molecular contributions to diffusive transport have been dropped. In the case of a rotating reference frame, the body force term becomes

$$F_{i\theta}=-2\Omega_k\overline{u_m\theta}\epsilon_{ikm}\qquad(4)$$

while for buoyancy

$$F_{i\theta}=\frac{g_i\overline{\rho'\theta}}{\rho}\qquad(5)$$

Density fluctuations are here supposed to arise purely from the fluctuations in the scalar quantity Θ (whether it denote temperature or mass fraction of a particular species) so it is convenient to link the two more explicitly. To fix ideas, we suppose θ denotes temperature fluctuations and introduce a dimensionless volumetric expansion coefficient α:

$$\alpha\equiv-\frac{\Theta}{\rho}\frac{\partial\rho}{\partial\Theta}\Big|_P\qquad(6)$$

Then $F_{i\theta}=\alpha g_i\overline{\theta^2}/\Theta$ where for an ideal gas α is unity.

Thus, for buoyancy-modified flows, one must necessarily consider the approximation of the mean square temperature variance; in a second-moment closure that, too, would be found via its own transport equation. The exact transport equation for $\overline{\theta^2}$, obtained by multiplying the energy equation by θ and averaging, was first presented by Corrsin (1952) and runs:

$$\frac{D\frac{1}{2}\overline{\theta^2}}{Dt}=-\overline{\theta u_k}\frac{\partial\Theta}{\partial x_k}-\lambda\overline{\left[\frac{\partial\theta}{\partial x_k}\right]^2}-\frac{\partial}{\partial x_k}(\overline{\theta^2u_k}/2)\qquad(7)$$

It is instructive to compare the intercouplings among the second-moment equations arising (with one exception noted below) from their generative agencies and in particular how, for a buoyancy-affected flow, these couplings differ according to whether the flow is vertical or horizontal. Figure 1 relates to two-dimensional shear flows in which x_1 is the direction of the mean flow and x_2 is the direction in which gradients of velocity and temperature occur. We note that the solid lines interconnecting various second-moment components are the same for both the horizontal and the vertical shear flow: these represent the couplings arising from mean velocity and temper-

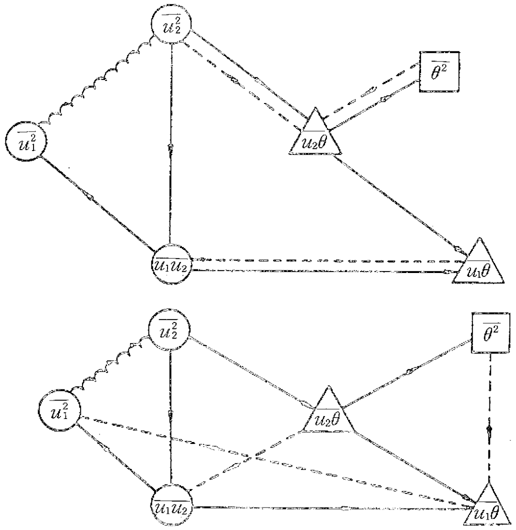

Fig. 1　Buoyant couplings through generation terms in vertical and horizontal shear flows
--- buoyant coupling; —— coupling through mean/velocity or temperature qradients
a) horizontal flow　b) vertical flow

ature gradients. Thus, velocity fluctuations down the velocity gradient $(\overline{u_2^2})$ create shear stress $(\overline{u_1u_2})$ which in turn creates streamwise fluctuations $(\overline{u_1^2})$. Here we note the exceptional connection: there is no direct link between $\overline{u_2^2}$ and the other stress equations. Turbulent fluctuations in the direction of the mean velocity gradient are sustained only by the action of the pressure-strain process in deflecting fluctuating energy from the streamwise direction; this pressure-strain transfer is shown by the wavy line. Considering the scalar fluxes, a flux down the temperature gradient is produced by $\overline{u_2^2}$ while a streamwise flux, $\overline{u_1\theta}$, is generated both by the shear stress (interacting with the temperature gradient) and $\overline{u_2\theta}$ (interacting with $\partial U_1/\partial x_2$). Likewise $\overline{\theta^2}$ is created by the product of the down-gradient heat flux and the temperature gradient itself. Notice that in the above system the velocity field is uncoupled from the scalar field.

The situation is quite different when buoyant contributions, shown by broken lines in Figure 1, are included. The nature of the intercoupling, however, is very different for vertical than horizontal flows. For a horizontal flow the buoyant feedback is directly into the component already acting as the generative agency, while in a vertical flow the interconnections are more scattered. These two very different coupling patterns help one to see why any model based upon the notion of isotropic diffusion coefficients is unlikely to handle both vertical and horizontal flows even if, by suitable empirical correlations, one of the cases could be represented adequately.

3. Closure of the Second-Moment Equations

3.1　Preliminary Remarks

While the generation and convective transport rates of the second moments require no approximations, the remaining processes in the transport equations cannot be handled exactly at second-moment level and must be "modelled." Surrogate forms devised to imitate the real process will incorporate at least some of the formal characteristics of the tensors they replace: dimensional homogeneity; rank, symmetry and contraction properties of the original form; invariance to the coordinate frame adopted for monitoring the flow development. One may also wish to insist that the approximation should give exactly the correct result in certain limiting cases where the magnitude of the original correlation is known (e.g. in isotropic turbulence). These formal constraints, however, need to be weighed against the benefits of simplicity (from both conceptual and computational standpoints) and what might be regarded as the inherent limitations of a second-moment closure that disregards such features as intermittency. Particularly within the context of industrial flows the interwoven principles of diminishing returns and receding influence should always be borne in mind. As it turns out, a very simple formulation, used increasingly over the past decade, leads to a decisively greater width of predictive accuracy than any eddy-viscosity-based scheme and, indeed, that the only other set of proposals to have been extensively used. It is this simple formulation that is described in the succeeding paragraphs and for which results will be presented in Part II.

3.2　Non-Dispersive Pressure Interactions

The pressure-strain correlation contains within it three types of process. This may be seen by forming a Poisson equation for the pressure fluctuation p (by taking the divergence of the Navier-Stokes equation and subtracting the mean part):

$$\frac{\partial^2 p}{\partial x_\ell^2} = -\frac{\partial^2 (u_\ell u_m - \overline{u_\ell u_m})}{\partial x_\ell \partial x_m} - 2\frac{\partial U_\ell}{\partial x_m}\frac{\partial u_m}{\partial x_\ell} + \frac{\partial f_\ell}{\partial x_\ell} \quad (8)$$

On integrating (8) and multiplying each side by the instantaneous strain and averaging, one obtains an expression for the pressure-strain correlation φ_{ij} which, away from the vicinity of rigid boundaries, may be written:

$$\varphi_{ij}=\frac{1}{4\pi}\int\Big\{\overline{\left(\frac{\partial^2 u_\ell u_m}{\partial x_\ell \partial x_m}\right)'\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}$$
$$+2\left(\frac{\partial U_\ell}{\partial x_\ell}\right)'\overline{\left(\frac{\partial u_m}{\partial x_m}\right)'\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}$$
$$-\left(\frac{\partial f_\ell}{\partial x_\ell}\right)'\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\Big\}\frac{dVol}{\boldsymbol{r}} \tag{9}$$

where the primed quantities are evaluated at distance $\boldsymbol{r}$ from the point in question and the integration extends over all space (though in practice the contribution is limited to distances from the point comparable with the turbulent macroscale). The corresponding equation for $\varphi_{i\theta}$ can obviously be formed in an analogous way. While in Part II we shall consider a little more closely the details of the integral in (9), we note here simply that it comprises a contribution involving only fluctuating velocities (φ_{ij1}), one containing linear mean-strain elements multiplying double-velocity products (φ_{ij2}) and one involving the fluctuating body force (φ_{ij3}). Since these three contributors to the integral clearly arise from quite distinct processes, they will require separate approximation. Considerable efforts are now being made to devise widely valid forms that broadly follow or extend the general direction advocated by Lumley (1978). In this section, however, simple intuitive forms are presented that are only loosely connected with the three integrals in (9). One of the most enduring proposals in second-moment closure is Rotta's (1951) linear return-to-isotropy proposal for φ_{ij1}:

$$\varphi_{ij1}=-c_1\epsilon a_{ij} \tag{10}$$

where a_{ij} is the dimensionless stress anisotropy tensor, $a_{ij}\equiv(\overline{u_iu_j}-\frac{1}{3}\delta_{ij}\overline{u_ku_k})/k$ and $\epsilon\equiv\overline{\nu(\partial u_i/\partial x_k)^2}$ is essentially the kinematic rate of dissipation of turbulence energy. For decaying anisotropic turbulence, with dissipation processes assumed isotropic (viz § 3.3), equation (10) produces a return to isotropy for $c_1>1$. If a constant value is to be chosen for this coefficient, the optimum choice seems to be close to 2.0.

The best simple model for φ_{ij2} has come to be known as the Isotropization of Production (IP) model. First proposed by Naot et al (1970)—though as a *replacement* for φ_{ij1} rather than as an addition to it—its form is directly parallel to (10):

$$\varphi_{ij2}=-c_2(P_{ij}-\frac{1}{3}\delta_{ij}P_{kk}) \tag{11}$$

If c_2 is chosen as 0.6, equation (11) satisfies Crow's (1968) exact result for isotropic turbulence:

$$\varphi_{ij2_{iso}}=0.4k\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$$

A wide range of pairings of c_1 and c_2 have in fact been adopted in the literature, though nearly all closely satisfy the interrelation $(1-c_2)/c_1=0.23$ (see Launder, 1985, 1988 for a discussion). The writer's group normally adopts the value 0.6 indicated by isotropic turbulence.

An obvious extension of the IP idea is to suppose

$$\varphi_{ij3}=-c_3(F_{ij}-\frac{1}{3}\delta_{ij}F_{kk}) \tag{12}$$

which was first adopted by Launder (1975a) in considering the behaviour of a horizontal, stably stratified shear flow. This form too is exact for isotropic turbulence, though in this case the coefficient is smaller, 0.3 (Lumley, 1975; Launder, 1975b). The usual *practice* in adopting (12) is to choose the same value for c_2 and c_3 and, as noted later, there is some experimental support for this idea, at least in shear flows.

Strictly, the above models for φ_{ij2} and φ_{ij3} have a fundamental weakness that, in relation to (11) was first noticed by Mjolsness (1979). In a rotating reference frame the tensor P_{ij} depends on the rate of rotaion; moreover (and this is the crucial point) so does $(P_{ij}+F_{ij})$. Thus, even with the same values adopted for c_2 and c_3, the predicted behaviour of, say, an axisymmetric swirling jet will depend upon the rotation rate of the frame of reference used to examine the flow. This is plainly nonsense! In order to avoid such anomalies, the model of φ_{ij} needs to be expressed in terms of *objective tensors*. The obvious (though not the only) way of modifying P_{ij} and F_{ij} so that they do form a frame-indifferent tensor is by introducing the convective transport tensor $C_{ij}\equiv U_k\partial\overline{u_iu_j}/\partial x_k$. Another line of thought that would also lead to the inclusion of C_{ij} is that, in formulating the Poisson equation for p, the "mean-strain" contributor arises from two elements in the fluctuating velocity equation, one of which is associated with "production" and the other "convection". Thus, a rational and simple form of the IP model reinterpreted to provide a form independent of the observer's notion is:

$$(\varphi_{ij2}+\varphi_{ij3})=-c_2(P_{ij}+F_{ij}-c_{ij}-\frac{1}{3}\delta_{ij}[P_{kk}+F_{kk}-c_{kk}] \qquad (13)$$

This modification, as will be seen in Part II, of the paper has brought great improvement to the prediction of swirling flows while producing little change from the behaviour provided by equations (11) and (12) in simple shear flows.

Precisely parallel approximations have been adopted in the corresponding terms in the scalar flux equations. Thus

$$\varphi_{i\theta}=-c_{1\theta}\frac{\epsilon}{k}\overline{u_i\theta}-c_{2\theta}P_{i\theta 2}-c_{3\theta}F_{i\theta} \qquad (14)$$

The first term on the right of (14) was contributed by Monin (1965). Regarding the second term, only the part of the generation of heat flux associated with mean velocity gradients, $P_{i\theta 2}$, is included—not mean temperature gradients—in accord with the indication of the Poisson equation for p. The usual practice has been to adopt a value of approximately 0.5 for both $c_{2\theta}$ and $c_{3\theta}$ (Owen, 1973; Launder, 1975a), though in the isotropic limit it may be shown that $c_{3\theta}$ takes the value $\frac{1}{3}$ (Lumley, 1975; Launder, 1975b).

In the neighbourhood of a wall, an additional term must be added to φ_{ij} and $\varphi_{i\theta}$ to account for the way pressure reflections from the boundary "interfere" with the energy-transfer processes. Formally we can attribute this "wall-echo" process to the contribution of a surface integral that is missing from the exact representation of φ_{ij} (which is, of course, absent in free turbulence). This wall contribution seems to be responsible for the very different effects of a stable stratification on free and near-wall turbulence. No theories as such are available, but wall effects are usually accounted for (Shir, 1973; Gibson and Launder, 1978) by using the unit vector normal to the wall, n_k, as a device for preferentially damping velocity fluctuations in that direction. Thus, in terms of the pressure-strain correlation, a wall correction φ^w_{ij} is added of the form

$$\begin{aligned}\varphi^w_{ij}=\{&c'_1(\overline{u_ku_m}n_kn_m\delta_{ij}-\frac{3}{2}\overline{u_ku_i}n_kn_j\\&-\frac{3}{2}\overline{u_ku_j}n_kn_i)\epsilon/k+c'_2[\varphi_{km2}\\&+\varphi_{km3})n_kn_m\delta_{ij}-\frac{3}{2}(\varphi_{ik2}+\varphi_{ik3})n_kn_j\\&-\frac{3}{2}(\varphi_{jk2}+\varphi_{jk3})n_kn_i]\}\left[\frac{k^{3/2}}{\epsilon x_n}\right]^a\end{aligned} \qquad (15)$$

The quantity $(k^{3/2}/\epsilon x_n)$ multiplying the whole term is the ratio of the turbulent length scale at a point to the distance from the wall; as this ratio becomes smaller, wall influences diminish. The exponent "a" has usually been taken as unity, though Naot and Rodi (1982) report improved agreement in predicting turbulence driven flows in rectangular ducts from taking $a=2$.

3.3 Second-Moment Dissipation Rates

The usual (albeit not unchallenged) view is that the very fine scale eddies, which are essential to account for the destruction of turbulence energy by viscous action, are formed by a large number of interactions in which large eddies are successively broken down into finer scale motions. As this breakdown proceeds, the strong directional orientation imprinted on the larger eddies by the mean strain field gradually gets lost. Thus, by the time the scales are small enough for significant kinetic energy to be dissipated (implying an eddy Reynolds number of order unity) the motions are *isotropic*. In this event, the stress and scalar flux dissipation rate are given by:

$$\epsilon_{ij}=\frac{2}{3}\delta_{ij}\epsilon$$
$$\epsilon_{i\theta}=0 \qquad (16)$$

The determination of the energy dissipation rate itself is one of the weakest points in second-moment closure. While an exact transport equation for ϵ can be readily obtained from the Navier-Stokes equation, the resultant equation does not in practice form a useful starting point. The reason is that the important quantities appearing in it all relate to interactions among the finest scales of motion present. Yet, only in a legalistic sense is the rate of energy dissipation controlled by these processes. The *real* controlling factor is the rate that energy "cascades" from large- to small-scale eddies. The interactions producing that transfer are larger-scale, essentially inviscid motions. Accordingly, in formulating a surrogate transport equation for ϵ, one relies heavily on analogy, intuition and experiment. The form usually adopted may be written:

$$\frac{D\epsilon}{Dt}=c_\epsilon\frac{\partial}{\partial x_k}\left[\frac{\overline{u_ku_\ell}}{\epsilon}k\frac{\partial\epsilon}{\partial x_\ell}\right]+\frac{0.5\epsilon}{k}[c_{\epsilon 1}P_{kk}+c_{\epsilon 3}F_{kk}]-c_{\epsilon 2}\frac{\epsilon^2}{k} \qquad (17)$$

The three terms on the right of (17) are respectively diffusive, generative and dissipative in character. The diffusion model shown is the generalized gradient-diffusion hypothesis (GGDH) of Daly and Harlow (1970). The value usually adopted for c_ϵ—about 0.18—is typical of those chosen when applying this submodel to represent the turbulent diffusive transfer of other quantities.

The source and sink terms in this equation are the critical terms in the equation, a change in one of the coefficients by only a few percent altering the rate of growth of a free shear flow by typically four times as much. In flows unaffected by force fields the normal approach is to take $c_{\epsilon1}$ and $c_{\epsilon2}$ as constants (the usual values are 1.44 and 1.92 respectively). This practice brings difficulties when buoyant flows are considered, however, because then the coefficient $c_{\epsilon3}$ is assigned different values depending upon whether the shear flow is directed horizontally or vertically (compare Hossain and Rodi, 1982, and McGuirk and Papadimitriou, 1988). This is clearly an unsatisfactory state of affairs—especially if one is concerned with recirculating flows, which are sometimes horizontal and sometimes vertical. An alternative approach is to make one or both of the coefficients depend upon the dimensionless anisotropy of the stress field. The second invariant $A_2 \equiv a_{ij}a_{ij}$ is the commonly used measure of anisotropy and some use is also beginning to be made of the third invariant $A_3 \equiv a_{ij}a_{jk}a_{ki}$. In fact, in the first such proposal, Lumley and Khajeh-Nouri (1974) suggested that if $c_{\epsilon2}$ were made a function of $A_2$the coefficient $c_{\epsilon1}$ could be set to zero, a choice they preferred on physical grounds. Subsequent studies by Lumley's group reinstated the turbulence energy generation rate (e.g. Zeman and Lumley, 1979) but with a coefficient $c_{\epsilon1}$ of approximately 0.5, i.e. about one third of that adopted when $c_{\epsilon1}$ and $c_{\epsilon2}$ are constant. Current work at UMIST takes $c_{\epsilon1}$ and $c_{\epsilon3}$ equal to unity and takes $c_{\epsilon2}=1.92/(1+0.6A^{\frac{1}{2}}A_2)$ where $A=1-\frac{9}{8}(A_2-A_3)$. Ince (personal communication) has found this adaptation greatly inproves the prediction of buoyant plumes.

Usually where temperature fluctuations are to be computed ϵ_θ, the dissipation rate of $\frac{1}{2}\overline{\theta^2}$, is obtained by relating it to the kinetic energy dissipation rate via:

$$\epsilon_\theta = R\frac{\frac{1}{2}\overline{\theta^2}}{k} \tag{18}$$

where R is just the ratio of dynamic to scalar time scales. In simple shear flows, where temperature and velocity gradients occur in the same regions in space, a constant value for R seems to work well—a value of approximately 0.5 being commonly chosen. To handle other circumstances, transport equations for ϵ_θ have been proposed, for example by Lumley (1978), Zeman and Lumley (1979), Newman et al (1981), Elghobashi and Launder (1983), Jones and Musonge (1985). None of the proposed versions has yet been subject to a sufficiently wide ranging set of test flows to form a clear view of the satisfactoriness or otherwise of the recommended forms of transport equation. An alternative strategy, intermediate in complexity between the two approaches noted above, is to correlate R as a function of the invariants of the stress or heat flux fields. For example, Haroutunian and Launder (1988) have shown that the choice $R^{-1}=1.2+2.3A_{2\theta}$ leads to the correct level of temperature fluctuations in jets and plumes (where $A_{2\theta}$ is the heat-flux invariant $\overline{u_i\theta}\ \overline{u_i\theta}/k\overline{\theta^2}$).

3.4 Diffusion

While rather elaborate models of the second-moment diffusion processes have been put forward (Ettestad and Lumley, 1984; Dekeyser and Launder, 1984), in practice most workers adopt the generalized gradient-diffusion hypothesis

$$d_\varphi = c_\varphi \frac{\partial}{\partial x_k}\left[k\frac{\overline{u_k u_\ell}}{\epsilon}\frac{\partial\varphi}{\partial x_\ell}\right] \tag{19}$$

where d_φ denotes "net turbulent transport rate of φ", the quantity φ standing for the second-moment product in question. The idea underlying the use of such simple models is that often the predicted mean flow is insensitive to the transport model adopted for the second moments. It is thus seen as an ineffective use of computer resource to adopt a comprehensive model for the third-order moments responsible for diffusive transport.

Sometimes even simpler models are adopted for handling the second-moment transport equations. The ultimate step in simplifying the transport model is the so-called *algebraic* second-moment transport hypothesis which expresses the transport of a second moment in terms of the transport of kinetic energy. If this approximation is applied to both convection and diffusion processes, the closure is reduced to an ASM (algebraic second-moment) model in which only k and ϵ among turbulence quantities need to be found from transport equations. This approach has been extensively used in computing buoyancy modified shear flows, especially by Professor Rodi's group in Karlsruhe (e.g. Hossain and Rodi, 1982). A philosophical reason for *not* approximating stress transport in this way is that by so doing one is representing a process described by a non-objective tensor in terms of an objective tensor. It says, at the very least, that this approach should be avoided in swirling free flows where transport effects are large (Fu et al, 1988).

4. Concluding Remark

The present paper has presented the model of turbulence that has been widely applied to predict the behaviour of turbulent flows modified by buoyancy. In Part II of the paper, to appear in a forthcoming issue of SEISAN-KENKYU numerous applications of the model will be presented.

The paper is based on lectures presented at the Institute of Industrial Science, Tokyo in August 1987 and at the 2nd European Turbulence Conference, Berlin in September 1988. My appreciation goes to all my colleagues in the CFD group at UMIST whose discoveries have been included in this paper. The camera-ready manuscript has been prepared with great care and skill by Mrs. L.J. Ball.

(Manuscript received, November 25, 1988)

5. Nomenclature

a_{ij}	dimensionless anisotropic Reynolds stress $(\overline{u_iu_j}-\frac{1}{3}\overline{u_ku_k})/k$
A, A_2, A_3	invariants of Reynolds stress field; defined following eq. (20)
$A_{2\theta}$	hear flux invariant $\overline{u_i\theta}\,\overline{u_i\theta}/\overline{\theta^2}k$
$c's$	empirical coefficients
D	space between plates or diamenter of pipe
f_i	fluctuating body force per unit mass
F_{ij}	source due to body force in $\overline{u_iu_j}$ transport equation
$F_{i\theta}$	source due to body force in $\overline{u_i\theta}$ transport equation
g_i	gravitational acceleration vector
k	turbulent kinetic energy
L	Monin Obukhov length scale
n_i	unit vector normal to wall
P, p	mean, fluctuating parts of pressure
P_{ij}	generation rate of $\overline{u_iu_j}$ by mean shear
$P_{i\theta 1}, P_{i\theta 2}$	generation rates of $\overline{u_i\theta}$ by mean temperature and velocity gradients respectively
R	time scale ratio, see eq. (18)
R_f	flux Richardson number
R_i	gradient Richardson number
R_o	rotation number $\Omega D/\nu$
Re	pipe Reynolds number
U	streamwise component of mean velocity
U_i, u_i	mean, fluctuating components of velocity in direction x_i
$\overline{u_iu_j}$	Reynolds stress tensor
v'	rms velocity fluctuation normal to wall
w	tangential component of mean velocity
x	streamwise coordinate
x_i	Cartesian space coordinate (x_1 denotes stream direction; x_2, x_3 have variable meanings explained in text)
α	dimensionless volumetric expansion coefficient
$\Delta\Theta$	excess of temperature above free-stream value
ϵ	dissipation rate of turbulence energy
ϵ_{ij}	dissipation rate of $\overline{u_iu_j}$
ϵ_θ	dissipation rate of $\frac{1}{2}\overline{\theta^2}$
Θ, θ	mean, fluctuating parts of temperature
ρ, ρ'	mean, fluctuating density
σ_t	turbulent Prandtl number
ν	kinematic viscosity
φ	generalized dependent variable
φ_{ij}	pressure-strain correlation
$\varphi_{i\theta}$	pressure/temperature gradient correlation
Ω	angular velocity
Ω_k	coordinate rotation vector

Subscripts

o	value at origin or under neutral or non-rotating conditions

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