研 究 解 説 UDC 532.517.4

SECOND-MOMENT CLOSURES FOR RECIRCULATING AND STRONGLY-SWIRLING FLOWS
——Part 1. Turbulence Models——
再循環流および強い旋回流に対する二次モーメント完結問題
——第1部：乱流モデル——

Michael A. LESCHZINER* and Toshio KOBAYASHI**
マイケル。レシュチナー・小 林 敏 雄

Second-moment closures are turbulence models which consist of differential or algebraic equations, each governing the balance between generation, redistribution, dissipation and transport of a related Reynolds stress or a scalar flux. The paper argues the potential advantages of using such closures in preference to eddy-viscosity models in the presence of body forces brought about by fluid recirculation, swirl and buoyancy. Some basic forms of second-moment closure are introduced, and their numerical implementation is discussed.

1. INTRODUCTION

Impressive progress has been made over the past decade in the area of direct and large-eddy simulation of turbulent flow, with the latter begining to yield results not only valuable from a fundamental viewpoint but also helpful in the engineering context[1,2]. Yet, it is the conventional approach, based on time- or ensemble-averaged formulations, which has maintained a strongly dominant position - and seems set to continue doing so - in the field of computing complex industrially-relevant flows in which information is sought on mean transport of heat, mass and momentum, and on the distribution of design-related global parameters such as wall pressure, skin friction and heat-transfer coefficients.

A statistically-averaged mathematical framework is based on the notion that any instantaneous flow quantity - say, velocity, pressure, density and enthalpy - may be represented as the sum of a time- (or ensemble-) mean value and a related turbulent fluctuation. Insertion of this decomposition into the instantaneous flow equations, followed by an averaging process - involving time-integration or event summation, results in a set of equations governing the distribution of averaged flow quantities. In the case of momentum components, this process 'transfers' the Navier-Stokes to the Reynolds equations.

The principal merit of the above approach is *economy*, particularly if attention is focused on statistically steady and two-dimensional flow, for in such circumstances, the computational task reduces from a fully three-dimensional time-dependent solution to a two-dimensional iterative one. The penalty of averaging the parent equations is, however, an important loss of information, reflected by the appearance of *unknown* double correlations - termed 'second moments'. In the case of momentum, these correlations are the 'Reynolds stresses', while inclusion of scalar transport gives rise to further unknown correlations of scalar and velocity fluctuations representing 'turbulent fluxes'. A pre-requisite for the solution of the averaged equations is thus a mechanism yielding the unknown stresses and fluxes in terms of known or determinable quantities. This is what is known as the 'Turbulence Model'.

The key concept underlying most traditional and well-established approaches to modelling turbulence effects is the quasi-linear relationship between stress and strain, analogous to that used for laminar flow, but in which a flow-dependent 'eddy viscosity' takes the place of the fluid viscosity. Cartesian-tensor

*Visiting Scientist (University of Manchester Institute of Science and Technology)
**Dept. of Mechnical Engineering and Navel Architecture, Institute of Industrial science, University of Tokyo

notation permits the relationship between all six independent Reynolds stresses in a 3D incompressible flow, $\rho \overline{u_i u_j}$, and their associated strains to be collectively written as:

$$-\rho\overline{u_iu_j}=\mu_t\left[\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right]-\frac{2}{3}\rho k\delta_{ij} \quad (1)$$

where the last term containing the turbulence energy, $k=0.5\ (u_1^2+u_2^2+u_3^2)$, has been included to account for the fact that the sum of the normal stresses must amount to $2k$, even if all strains are zero. In analogy to (1), any scalar flux (that of enthalpy, say) is related linearly to the associated gradient of the mean scalar value, with the diffusivity being the proportionality factor and formed by the ratio of the above eddy viscosity and an assumed Prandtl/Schmidt number.

Dimensional reasoning dictates that the eddy viscosity be proportional to a velocity scale and a length scale of the turbulent motion, and the most obvious parameters to adopt is the *rms* value of the turbulence energy, k, and a macro-length scale, L, representing the large energy-containing eddies which interact most intimately with the mean strains. It is readily appreciated that both quantities cannot simply depend on local flow properties alone but must be influenced by processes in neighbouring locations, with convective and diffusive transport providing the 'bridges'. It is this realization which has led to the formulation of a number of "two-equation models of turbulence"[3], all containing a modelled transport equation for turbulence energy and a second equation for L or some combination of k and L (e.g. vorticity $\omega=k^{1/2}/L$ or dissipation $\varepsilon=k^{3/2}/L$). The k-ε model of Launder and Spalding[4] is the version which has established itself as the most popular model for computing a wide range of flows - from simple 2D free shear flows[5] to very complicated 3D recirculating flow in jet engine combustors[6] and rotating impeller passages[7].

While the above models have been found to return satisfactory numerical solutions in many situations, they do not perform well in flows in which body forces - arising from strong curvature, recirculation, swirl and buoyancy - play an important role. Such body forces interact differently with different normal and shear stresses, and this selective, or rather discriminatory, influence connot be captured by use of a model which relates all stresses to the mean field via a single isotropic parameter.

It is helpful to illustrate the above interaction by focusing on the specific example of streamline curvature in the plane 2D shear flow shown in Fig. 1. Exact transport equations for the Reynolds stresses can be derived, and these will be introduced later. Here, it is merely necessary to accept that exact stress-generation terms contribute to these equations, and that, for the present thin shear flow, the rates of generation of the stresses $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ can be expressed, in terms of stream-line adapted co-ordinates, as follows:

$$P_{\overline{uu}}=-2\,\overline{uv}\left[\frac{\partial U}{\partial r}+\frac{U}{R}\right] \quad (2)$$

$$P_{\overline{vv}}=4\,\overline{uv}\,\frac{U}{R} \quad (3)$$

$$P_{\overline{uv}}=-\overline{v^2}\,\frac{\partial U}{\partial r}+\left\{\left[2\overline{u^2}-\overline{v^2}\right]\frac{U}{R}\right\} \quad (4)$$

A number of useful qualitative observations may be made by reference to the above equations; We note first that the secondary strain U/R tends to increase $P_{\overline{uv}}$ and hence $\overline{uv}$, since $2\overline{u^2}$ typically exceeds $\overline{v^2}$ by a factor 4. This stress is negative, however, and curvature will consequently reduce the *magnitude* of uv. Second, the influence of U/R is much stronger than one might expect, for its multiplier in $P_{\overline{uv}}$ is $(2\overline{u^2}-\overline{v^2})$ which is much larger than $\overline{v^2}$, the multiplier of the primary strain. That $\overline{v^2}$ is relatively small follows from the observation that $P_{\overline{vv}}$ is not only low but, in fact, negative. Of course, despite the negative production, $\overline{v^2}$ cannot itself be negative, and this is ensured by so-called "pressure-stain-interaction" processes which continuously 'feed' energy from $\overline{u^2}$ to $\overline{v^2}$. The stress $\overline{u^2}$ can well afford this loss, for it is generated at a high rate due to the interaction between uv and the primary stain $\partial U/\partial r$. The final

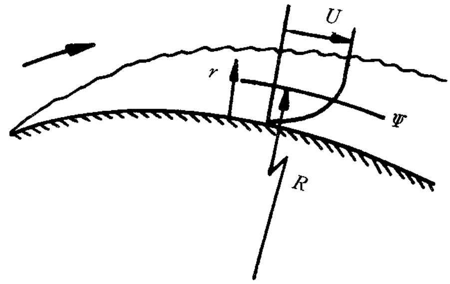

Fig. 1　Curved 2D thin shear flow

point to note is that the curvature-induced reduction in $\overline{v^2}$ leads to a further reduction in the magnitude of $\overline{uv}$ through the product of $\overline{v^2}$ and the primary strain in $P_{\overline{uv}}$. In summary then, it may be concluded that curvature selectively attenuates $\overline{v^2}$ (but slightly increases $\overline{u^2}$) leading to a substantial reduction in $\overline{uv}$. In addition, $\overline{uv}$ is reduced directly by U/R. Similar arguments apply to other curved or swirling flows (where thrbulence may either be attenuate or amplified, depending on the sense of curvature), as well as to situations in which buoyancy gives rise to terms analogous to those associated with curvature.

The fact that some essential elements of the interaction between curvature and turbulence can only be explained by reference to the individual stress-generation terms lends strong support to the assertion that a turbulence model expected to yield a high degree of generality must be based on equations describing the processes affecting the balance of each Reynolds stress (and, if appropriate, flux) separately. Exact forms of these equations can be derived by somewhat lengthy manipulations and combination of the Navier-Stokes, the Reynolds and the analogous scalar-property equations[8,9]. Adopting, at this stage, a simple descriptive representation, one may write the stress and flux equations in the following form:

$$\begin{aligned} \text{Convection } (\overline{u_iu_j}) &= \text{Diffusion } (\overline{u_iu_j}) \\ &+ \text{Production } (\overline{u_iu_j}) \\ &+ \text{Pressure-strain } (\overline{u_iu_j}) \\ &- \text{Dissipation } (\overline{u_iu_j}) \qquad (5) \end{aligned}$$

$$\begin{aligned} \text{Convection } (\overline{u_ic}) &= \text{Diffusion } (\overline{u_ic}) \\ &+ \text{Production } (\overline{u_ic}) \\ &+ \text{Pressure-scrambling } (\overline{u_ic}) \\ &- \text{Dissipation } (\overline{u_ic}) \qquad (6) \end{aligned}$$

that is, each equation represents a balance between physical processes which, apart from pressure/strain and pressure/scalar-gradient interaction, are familiar from mean-flow considerations. In the above equations, when written in their full, mathematically correct form, convection and (most importantly) production need not be modelled, for both only contain mean-flow quantities and the stresses (or fluxes) themselves. The remaining terms, however, contain higher-order moments (for example, triple correlations of the form $\overline{u^2v}$ and $\overline{p\partial u/\partial x}$) or indeterminable correlations such as the product of strain fluctuations. It is this which necessitates approximations to be postulated if the stress and flux equations are to be closed at second-moment level. Of course, these approximations are certain to introduce errors into the equations, therby eroding their capabilities. Yet, the expectation is that the retention of the exact production terms for each stress, coupled with reasonably good modelling proposals for diffusion, pressure-strain and dissipation, would ensure a high level of generality.

Previous applications of stress/flux models to curved and buoyant boundary-layer type flows[10,11] have, indeed, shown that the models return the correct response to the anisotropy-promoting agents. Few studies have focused on more complex recirculating flow, however, and the little evidence which has emerged from these studies is inconclusive. In a number of cases[12,13], the response of the turbulence closures has been completely masked by numerical errors provoked by the use of the first-order upwind approximation within a hybrid central/upwind-differencing scheme for convection. These errors are particularly damaging in the context of stress closures which do not naturally yield diffusivity-containing second-order terms enabling the central-differencing part of the hybrid scheme to operate without loss of iterative stability. The absence of such numerically stabilizing terms appears also to have seriously hindered the use of stress closures in combination with accurate, numerically non-diffusive discretization schemes. In some recent instances[14-18], stability problems have been overcome either due to some artificial diffusion introduced through the use of implicit Euler-type time-marching or other stability-promoting measures.

The present paper reports the efforts towards computing complex recirculating and strongly swirling flows with stress closures. The review starts with a statement of the turbulence models under consideration. There follows an outline of the numerical framework, with particular emphasis placed on a brief description of some generally applicable numerical measures, specifically designed to promote numerical stability when stress closures are used in conjunction with non-diffusive discretization within the finite-volume method.

2. THE TURBULENCE MODELS

Four types of turbulence models have been used in computations to be presented in Part 2, and all are time-averaged formulations. One closure, an unweighted (as opposed to density-weighted) stress-transport model (RSTM), may be considered the 'master' or 'parent' version from which two other unweighted forms may be derived by introducing additional modelling assumptions or simplifications. The fourth model is a density-weighted version of the stress-transport model.

The most general co-ordinate framework used in the computations is curved-orthogonal, and the models may, in principle, be written in terms of these co-ordinates. However, such a form is unnecessarily complicated for the present purpose of outlining the principles adopted; indeed, it would impair transparency. Instead, the models are introduced in terms of Cartesian-tensor notation. Also, while the complete models contain swirl-related contributions and consist of coupled equations for the stresses, fluxes and, most generally, the variance of the turbulent scalar fluctuations, the forms reported here exclude swirl and scalar contributions. Flux and variance equations only come into play in the case of two swirling-flow calculations to be presented, and appropriate reference will be made later to papers containing complete model descriptions. Finally, in one swirling-flow case, strong density variations occur due to the mixing between jets of very different densities. This has necessitated the use of density-weighted (Favre-averaged) form of the stress/flux-transport closure with inclusion of additional density-related terms. Here again, this special form is not given, but reference is made to a related publication.

The basic form of the parent closure is the version of Gibson & Launder[8] (based on that of Launder et al[19]), and this consists of modelled transport equations for the stresses u_iu_j, which may be written

$$\frac{\partial U_k\overline{u_iu_j}}{\partial x_k}=\frac{\partial}{\partial x_k}\left[C_s\,\overline{u_ku_l}\,\frac{k}{\varepsilon}\,\frac{\partial\overline{u_iu_j}}{\partial x_l}\right]+P_{ij}+\phi_{ij}-\frac{2}{3}\delta_{ij}\varepsilon \qquad (7)$$

where U_k are mean-velocity components in the directions x_k;

$$P_{ij}\equiv-\overline{u_iu_k}\,\frac{\partial U_j}{\partial x_k}-\overline{u_ju_k}\,\frac{\partial U_i}{\partial x_k} \qquad (8)$$

is the generation of the stress $\overline{u_iu_j}$, ϕ_{ij} controls the re-distribution of turbulence energy, $k=0.5\overline{u_iu_i}$, among the normal stresses, and $2/3\ \delta_{ij}\varepsilon$ stands for the rate of dissipation of the normal stresses by the action of viscosity. In the above, convection and generation are exact, while the remaining terms are models of exact expressions which cannot be used in their original form as they contain third-order correlations. Thus, diffusion is modelled by a generalized gradient approximation, while dissipation is assumed to be isotropic, with each normal stress dissipated at the same rate, $2/3\varepsilon$, where ε is determined from its own transport equation,

$$\frac{\partial U_k\varepsilon}{\partial x_k}=\frac{\partial}{\partial x_k}\left[C_t\,\overline{u_ku_l}\,\frac{k}{\varepsilon}\,\frac{\partial\varepsilon}{\partial x_l}\right]+C_{\varepsilon,1}\,\frac{\varepsilon}{2k}\,P_{kk}-C_{\varepsilon,2}\,\frac{\varepsilon^2}{k} \qquad (9)$$

Finally, the redistribution term, ϕ_{ij}, modelling the interaction between turbulent fluctuations of pressure and stains, consists of three contributions, namely Rotta's linear 'return to isotropy' term,

$$\phi_{ij,1}=-\frac{C_1\varepsilon}{k}\left[\overline{u_iu_j}-\frac{1}{3}\,\delta_{ij}\,\overline{u_ku_k}\right] \qquad (10)$$

the 'isotropization of production' term,

$$\phi_{ij,2}=-C_2\left[P_{ij}-\frac{1}{3}\,\delta_{ij}\,P_{kk}\right] \qquad (11)$$

and the 'wall-reflection' terms,

$$\phi_{ij,w}=C_{1,w}\,\frac{\varepsilon}{k}\left(f_l\,\overline{u_ku_m}n_kn_m\delta_{ij}-\frac{3}{2}f_l\,\overline{u_ku_i}n_kn_j-\frac{3}{2}f_l\,\overline{u_ku_j}n_kn_i\right)+C_{2,w}\left(f_l\phi_{km,2}n_kn_m\delta_{ij}-\frac{3}{2}f_l\phi_{ik,2}n_kn_j-\frac{3}{2}f_l\phi_{jk,2}n_kn_i\right) \qquad (12)$$

where the suffix 'l' takes the same numerical value as 'k' with *no summation* implied in this instance only, and n_i is the unit vector normal to the i-direction. The above is, essentially, a vectorial interpretation of a model combining suggestions by Shir[20] and Gibson and Launder[8], and is meant to account for the simultaneous influence of 'x'-and 'y'-directed walls on normal-stress anisotropy. This influence is ex-

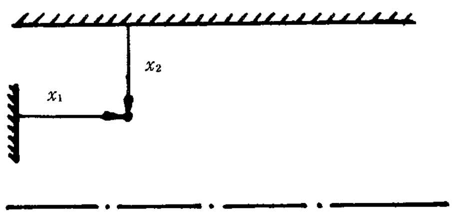

Fig. 2 Wall-related distances in functions f_1 and f_2 (equation 13)

pressed via the functions (applicable to plane cases only),

$$f_1=\frac{k^{1.5}}{2.5\varepsilon}\left(\frac{1}{x_L}+\frac{1}{x_R}\right) \qquad f_2=\frac{k^{1.5}}{2.5\varepsilon}\left(\frac{1}{y_B}+\frac{1}{y_T}\right) \tag{13}$$

which represent ratios of a turbulence length scale to equivalent wall distances, where x_L, x_R, y_T and y_B are actual distances to surrounding walls, as illustrated by the example shown in Fig. 2.

The second closure is a so-called algebraic stress model (ASM), and the particular version presented here arises upon the replacement of the differential stress-convection and diffusion terms by Rodi's proposal[21],

$$(C_{ij}-D_{ij}) \rightarrow \frac{\overline{u_iu_j}}{k}(C-D)=\frac{\overline{u_iu_j}}{k}(P-\varepsilon) \tag{14}$$

where C, D, P and ε represent convection, diffusion, production and dissipation of turbulence energy, respectively. There terms arise in the transport equation for the turbulence energy,

$$\frac{\partial U_k k}{\partial x_k}=\frac{\partial}{\partial x_k}\left(C_s\,\overline{u_ku_l}\,\frac{k}{\varepsilon}\frac{\partial k}{\partial x_l}\right)+P-\varepsilon \tag{15}$$

which is simply half the sum of the transport equation of the normal stresses $\overline{u_i^2}$. The essential feature to note here is that, once k and ε (and hence P) have been determined from the differential equations (15) and (9), equations (7), incorporating the model (14), couple the stresses algebraically.

The final and simplest unweighted closure, used here merely to generate 'reference predictions' against which stress-closure calculations can be compared, is the Boussinesq-viscosity k-ε model (EVM)[4]. This assumes stress-strain relationships involving an isotropic eddy viscosity,

$$-\overline{u_iu_j}=\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right) \tag{16}$$

The viscosity is related, by dimensional arguments, to the turbulence energy and its dissipation rate via

$$\nu_t=C_\mu\frac{k^2}{\varepsilon} \tag{17}$$

where k and ε are determined from equations (15) and (9) with $C_s\,\overline{u_ku_l}\,\frac{\partial k}{\partial x_l}$ and $C_\varepsilon\,\overline{u_ku_l}\,\frac{\partial \varepsilon}{\partial x_l}$ replaced by $C_\mu\;k\,\frac{\partial k}{\partial x_k}$ and $C_\mu/\sigma_\varepsilon\;k\,\frac{\partial \varepsilon}{\partial x_k}$, respectively, where σ_ε is a Prandtl number.

Extended forms of the Reynolds-stress-transport equations for uniform-density swirling flow may be found in refs[22,23], while mass-averaged forms for swirling flow, including scalar-flux and variance equations, are documented in ref[24]. Finally, a version of the algebraic stress closure applicable to general-orthogonal co-ordinates is given in ref[25].

Table 1　Coefficients appearing in three closures

C_1	C_2	C_s	C_t	C_μ	σ_t	$C_{\varepsilon,1}$	$C_{\varepsilon,2}$	$C_{1,w}$	$C_{2,w}$
1.8	0.6	0.22	0.16	0.09	1.3	1.45	1.92	0.5	0.3

3. NUMERICAL IMPLEMENTATION

Discretization of the transport equations governing mean and turbulence properties is based on the staggered finite-volume approach. The principles of this approach, as well as the underlying rationale for staggering the volumes pertaining to mass conservation and momentum components U and V, are well known and will not be pursued here. A logical extension of the above rationale, previously used by Pope[13] and adopted here, is to stagger the locations of the stresses (and fluxes, if applicable) and their associated volumes, as shown in Fig. 3.

The main advantage of this practice is increased numerical stability - a result of the strong coupling established between the stresses (or fluxes) and the associated 'primary' strains (or scalar gradients). Focusing on the shear stress $\overline{u_1u_2}$ as an example, it will be observed that stress is located such that the discrete velocities used to approximate the strain $(\partial U/\partial r+\partial V/\partial x)$ straddle this stress centrally. A useful consequence of this practice is that the stresses are located such that no interpolation is involved in evaluating stress differences required for the finite-volume equations.

The convective fluxes appearing in any finite

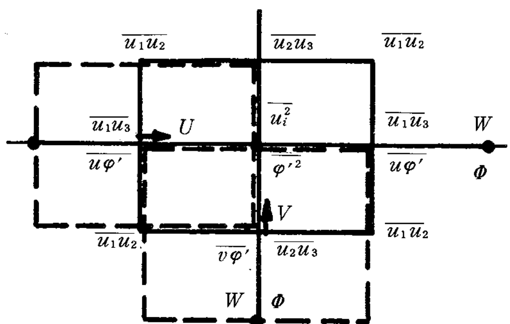

Fig. 3　Staggered stress and flux arrangement

-volume equation are approximated by the power -law differencing scheme (PLDS) of Patanker[26] or, alternatively, by the quadratic upstream-weighted scheme (QUICK) of Leonard[27]. The latter has been used as a means of indicating the degree of grid -independence, for this scheme is, in contrast to PLDS, unaffected by artificial diffusion. The numerically non-diffusive character of the quadratic scheme, combined with the absence of naturally -arising second-order (physical) diffusion terms, and the intense coupling between the equations, necessitated the introduction of a series of special algorithmic measures designed to enhance numerical stability and hence convergence. One important measure, documented in detail in ref[28], involves the representation of a particular portion of each Reynolds stress by means of an associated apparent viscosity. As shown in ref.[28], such a representation is offered by the discretized version of the related stress-transport equation which equates the stress to a collection of additive contributions, one of which consists of a group of unconditionally positive quantities multiplying the primary strain which is associated with the stress in question. A similar practice is adopted in the case of the algebraic model, only that in this case attention is focused directly on the algebraic equation governing the stress. An analogous strategy is also applied to flux equations.

A second measure which has been found to be beneficial to stability in the case of stress-transport computations entails a discrimination between positive and negative contributions to the source of any one discretized equation at all grid points, and allocating contributions according to

$$S_{ij}=S_{U,ij}+S_{P,ij}\,\overline{u_iu_j} \quad \text{(no summation on } i,\ j) \tag{18}$$

where S_U combines all positive and S_P all negative contributions.

Finally, an important stability-promoting practice adopted in conjunction with the algebraic model involved a coupled solution of the normal-stress equations, with the coefficients of the equations re -arranged in such a way that the system always returns positive values for the stresses. Following the solution of this system, the remaining shear-stress equations are solved explicitely. Details of this practice may again be found in ref[28].

4. CONCLUDING COMMENTS

An outline has been provided of recent progress made in computing complex recirculating and strongly swirling flows with Reynolds-stress closures and non-diffusive discretization within the finite-volume framework. Many results reported suffice to illustrate that stress closures provide a superior representation of turbulent transport processes to that returned by the Boussinesq-viscosity model. This applies both to mean-flow and turbulence features, the latter being of particularly importance in the context of heat transfer and scalar transport.

(Manuscript received, March 2, 1988)

REFERENCES

1) Moin P and Kim J, -Numerical investigation of turbulent channel flow, J Fluid Mech, Vol 118, p 341, 1982.
2) Kobayashi T, Kano M, Ishihara T and Saga T, - Large eddy simulation of turbulent flow in two -dimensional channel with rectangular turbulence promoters, IIS Report 213, Vol 33, No 3 University of Tokyo, 1987.
3) Launder B E and Spalding D B, - *Lectures in Mathematical Models of Turbulence*, Academic Press, 1972.
4) Launder B E and Spalding D B, - The numerical computation of turbulent-flow, Comp Meths Appl Mech Eng, Vol 3, p 269, 1974.
5) Launder B E, Morse A P, Rodi W and Spalding D B, - The prediction of free shear flows - A comparison of the performance of six turbulence models, Proc. NASA Langley Free Turbulent Shear Flow Conf., Vol 1, NASA SP 320, 1973.
6) Coupland J and Priddin C H, - Modelling the flow and combustion in a production gas turbine combustor, Proc. 5th Symp. on Turbulent Shear Flow, p 10.1, 1985.
7) Bosman C and Ramatalla M A F, - The calculation of turbulent flow effects in a centrifugal impeller, Proc. I Mech E Conf. on Computational Methods in Turbomachinery, 1984.
8) Gibson M M and Launder B E, - Ground effects on pressure fluctuations in the atmospheric boundary layer, J Fluid Mech, Vol 86, p 491, 1978.
9) Launder B E, Reynolds W C, Rodi W, Mathieu J and Jeandel D, - Turbulence models and their applications, CEA-EDF INRIA Ecole d'ete d'analyse numerique, Editions Eyrolles, 1984.
10) Scheuere G and Rodi W, - Calculation of curved shear layers with two-equation turbulence models, Phys Fluids, Vol 26, p 1422, 1983.
11) Hossain M S and Rodi W, - A turbulence model for

buoyant flow and its application to vertical buoyant jets, *Turbulent Buoyant Jets and Plumes*, Ed W Rodi, HMT Series, Pergamon Press, 1982.

12) Naot D, - Iterative procedure for simultaneous calculation of stress transport turbulence model in two-dimensional elliptic fields, *Computational Techniques in Transient and turbulent flow*, Eds C Taylor and K Morgan, p 192, 1982.

13) Pope S B and Whitelaw J H, - The calculation of near-wake flows, J Fluid Mech., Vol 73, p 9, 1976.

14) El Tahry S H, - Application of a Reynolds stress model to engine flow calculations, Flow in Internal Combustion Engines, II, ASME, Ed T Uzkan, p 39, 1984.

15) Prud'homme M and Elghobashy S, - Prediction of wall-bounded turbulent flows with an improved version of the Reynolds-stress model, Proc. 4th Symp on Turbulent Shear Flows, Karlsruhe, p 17, 1983.

16) Jones W P and Marquis A J, - Calculation of axisymmetric recirculating flows with second-order turbulence model, Proc. 5th Symp. on Turbulent Shear Flows, Cornell University, p 20.1, 1985.

17) McGuirk J J, Papadimitriou C and Taylor A M K P, - Reynolds-stress model calculations of two -dimensional plane and axisymmetric recirculation flows, paper presented at 5th Symp. on Turbulent Shear Flows, Cornell University, 1985.

18) Boysan F, Weber R and Swithenbank J, - Modelling coal-fired cyclone combustors, Combustion and Flame, Vol 63, p 73, 1986.

19) Launder B E, Reece G J and Rodi W, - Progress in the development of a Reynolds-stress turbulence closure, J Fluid Mech, Vol 68, p 537, 1975.

20) Shir C C, - A preliminary numerical study of atmospheric turbulent flows in the idealised planetary boundary layer, J Atmos Sci, Vol 30, p 1327, 1973.

21) Rodi W, The prediction of free boundary layers by use of a two-equation model of turbulence, PhD thesis, University of London, 1972.

22) Leschziner M A, - Numerical implementation and performance of Reynolds-stress closures in finite -volume computation of recirculating and strongly swirling flows, Report TFD/87/8, UMIST, Dept Mech Eng, Thermofluids Div, 1987. Also, lecture notes in Introduction to the Modelling of Turbulence, von Karman Institute, Brussels, May 1987.

23) Hogg S and Leschziner M A, - Computation of highly swirling confined flow with a Reynolds -stress turbulence model, Report TFD/86/12, UMIST, Dept Mech Eng, Thermofluids Div, 1986. (Accepted for publication in J AIAA, 1987).

24) Hogg S and Leschziner M A, - Second-moment -closure calculation of strongly swirling confined flow with large density gradients, Report TFD/87/10, UMIST, Dept Mech Eng, Thermofluids Div, 1987. (Submitted to Int J Heat Fluid Flow, 1987).

25) Kadja M, - Computation of recirculating flow in complex domains with algebraic Reynolds-stress closure and body-fitted meshes, Ph.D. Thesis, University of Manchester, 1987.

26) Patankar S V, - *Numerical Heat Transfer and Fluid Flow*, McGraw Hill, 1980.

27) Leonard B P, - A stable and accurate convective modelling procedure based on quadratic upstream interpolation, Comp Meths Appl Mech Eng, Vol 19, p 59, 1979.

28) Huang P G and Leschziner M A, - Stabilization of recirculating flow computations performed with second-moment closure and third-order discretization, Proc 5th Symp on Turbulent Shear Flows, Cornell University, p 207, 1985.